acústica 2012
VIII Congresso Ibero-americano de Acústica
1-3 outubro Évora - Portugal

MAXIMIZING THE AREA OF THE ‘QUIET ZONE’ PRODUCED BY A SECONDARY SOURCE IN FREE FIELD

Yves J.R. Gounot

Depto. Matematica Aplicada, Universidade Federal Fluminense, Rio de Janeiro, Brasil
ygounot@im.uff.br

Resumo
O dimensionamento da ‘zona de silêncio-10 dB’ gerada em campo livre pelo mais simples sistema de controle ativo de ruído — composto de uma fonte primária, uma secundária e um microfone de erro — foi investigado em um estudo prévio. Mostra-se, aqui, como e em que medida esta zona de silêncio pode ser alargada pela simples introdução de microfones de erro adicionais. Mostra-se que a largura desta zona medida na posição dos microfones, X_{mic}, pode ser aproximada por uma função linear da largura do arranjo, envolvendo ainda o número de microfones e a posição relativa da fonte secundária e dos microfones. Foi também encontrado que a *elasticidade* da zona (sua máxima variação em X_{mic}) não depende significativamente do número de microfones utilizados no arranjo.

Palavras-chave: controle ativo de ruído, zona de silêncio, campo livre.

Abstract
The dimensioning of the ‘10 dB-quiet zone’ obtained with the simplest free-field active noise control system — which consists of one primary source, one secondary source and one microphone, has been discussed quantitatively in a previous study. It is shown here how and to what extent this quiet zone can be enlarged by simply using additional error microphone(s). It has been found that the width of the quiet zone taken at the microphone position, X_{mic}, can be approximated by a linear function of the array size, of the number of microphones and the relative position of the secondary source and microphones. It is also shown that the *elasticity* of the quiet zone (its relative maximum variation in X_{mic}) does not significantly depend on the number of microphones used in the array.

Keywords: active noise control, quiet zone, free field.

PACS no 43.50.Ki, 43.50.Rq

1 Introduction

The initial purpose of active noise control (ANC) was to reduce an unwanted sound pressure field at low frequency, for which conventional passive techniques do not satisfactory perform. While currently the one-dimensional ANC problem can be considered resolved and technically overcome (see for example [1]), this is not the case of the three-dimensional free-field problem. One of the main challenges of ANC, which principle is the cancellation/attenuation of sound pressure at some places by using secondary sources emitting out of phase sound with same frequency as the one emitted by the primary source, is to optimize the positioning of the secondary sources in order to produce the best sound attenuation possible. Ideally, the best ANC system, which should perform a significant attenuation in all directions (and referred to as 'total control'), is generally obtained when the secondary sources are placed very close to the primary one, namely at a distance up to half wavelength at the frequency of interest, what allows absorption of sound power [2]. However, often practical situations don't enable this condition and, accordingly, only a 'local control' can be achieved. In order to evaluate the efficiency ANC system, one usually consider the size (and shape) of the 'quiet zone', which can be defined as the spatial region in which significant (habitually greater than 10 dB) pressure attenuation is obtained [3-5]. While various studies have already investigated the quiet zone obtained with some specific secondary source arrangements and described the impact of the relative positioning of these sources and microphones [4, 5], others have studied the best way to solve the minimization problem [6]. In a previous study [7], the dimensioning of the quiet zone that can be actually expected with the simplest ANC systems, ANC_{111}, — which consists of one primary source, one secondary source and one error microphone – has been numerically provided. While the main results are briefly reported here, the focal aim of the present paper is to investigate and answer the following question: how and to what extent this quiet zone, still produced by a single secondary source, can be enlarged?

2 Theoretical background and performance of the ANC_{111}

Quiet zone (or acoustic shadow as denominated in Ref. [5]) can be defined as the spatial region in which significant pressure attenuation is obtained by introducing an ANC system. The ANC theoretical background can be briefly described as follows. Let's consider a pressure field, p_p, emitted by a primary source modeled by n_p point sources with source strengths $\mathbf{q}_P = (q_{P1}, q_{P2}, ..., q_{Pn_p})$. At some places where an attenuation of the sound pressure is desired, n_M microphones are positioned. The attenuation is achieved by superposing to $\mathbf{p}_p$ a 'destructive' secondary pressure field, $\mathbf{p}_S$, produced by n_S secondary sources which strengths are denoted by $\mathbf{q}_S = (q_{S1}, q_{S2}, ..., q_{Sn_p})$. The total pressure field, p, to be minimized at the n_M microphone positions is given by

$$\mathbf{p} = \mathbf{p}_P + \mathbf{p}_S = \mathbf{Z}_{PM}\mathbf{q}_P + \mathbf{Z}_{SM}\mathbf{q}_S, \tag{1}$$

where $\mathbf{Z}_{PM}$ (respectively, $\mathbf{Z}_{SM}$) denotes the transfer impedance matrix between the primary sources (respectively, secondary sources) and the microphone positions. Since all the sources are here modeled as monopole point sources, the transfer matrix elements are given by [2, 4]

$$Z_{sm} = \mathrm{j}\omega\rho_0 \frac{\mathrm{e}^{-jkr_{sm}}}{4\pi r_{sm}}, \tag{2}$$

where ω is the angular frequency, ρ_0 is the uniform mean density of the propagation domain and r_{sm} stands for the distance between source s and microphone m. In this study, the cost function J to be minimized is the sum of the squared complex pressures at the microphone positions

$$J = \mathbf{p}^H\mathbf{p} \tag{3}$$

where the superscript ^{H} stands for the Hermitian transposed. As shown in Ref. 4, the source strengths of the secondary sources which minimize J are given by

$$\mathbf{q}_S^* = -\left(\mathbf{Z}_{SM}^H\mathbf{Z}_{SM}\right)^{-1}\mathbf{Z}_{SM}^H\mathbf{Z}_{PM}\mathbf{q}_P \tag{4}$$

By substituting $\mathbf{q}_S^*$ in Eq. 1, the total sound pressure can be computed at any point $\mathbf{x}$ in the domain, as well as the attenuation (or gain) provided by this ANC system, which is expressed in dB by

$$Attenuation(\mathbf{x}) = 20\log\left(\left|p(\mathbf{x})\right|/\left|p_P(\mathbf{x})\right|\right) \qquad (5)$$

As it is evidenced in Eq.2, the impedance matrixes and, therefore, the resulting computations (the optimized source strengths and the pressure attenuation fields) depend on the term kr_{sm}, i.e., on the relative distances between the sources and the microphones and also on the wavelength of interest.

Let's now consider the 'simplest' ANC system, denoted by ANC_{111}, consisting of one primary source, one secondary source and one microphone, and in which the secondary source and the microphone are aligned with the primary source positioned at the origin, at some distance from the primary source, respectively r_{ss} and r_{mic}. Both primary and secondary sources are modeled as point sources (monopoles) and emit a pure-tone sound, which wavelength is denoted by λ. The performance of a given ANC system is evaluated through the size of the quiet zone that the system will produce. The quiet zone considered in this study is, as commonly used [3-5], the region in which the pressure attenuation is higher than 10 dB and referred to as QZ_{10}. The dimensioning of this spatial region (in the *xy* plane containing the sources and microphone) is given by two selected indicators expressed in terms of λ:

- X_{mic} : the zone width, taken at the microphone position
- Y: the zone axial dimension, i.e., its depth.

Figure 1a shows the pressure attenuation field obtained with ANC_{111} for $r_{SS} = 5\lambda$ and $r_{mic} = 20\lambda$ (the primary source, the secondary source and the microphone positions are represented by a red star, a circle and a cross, respectively) and Figure 1b illustrates the corresponding QZ_{10} and its two size indicators, X_{mic} and Y.

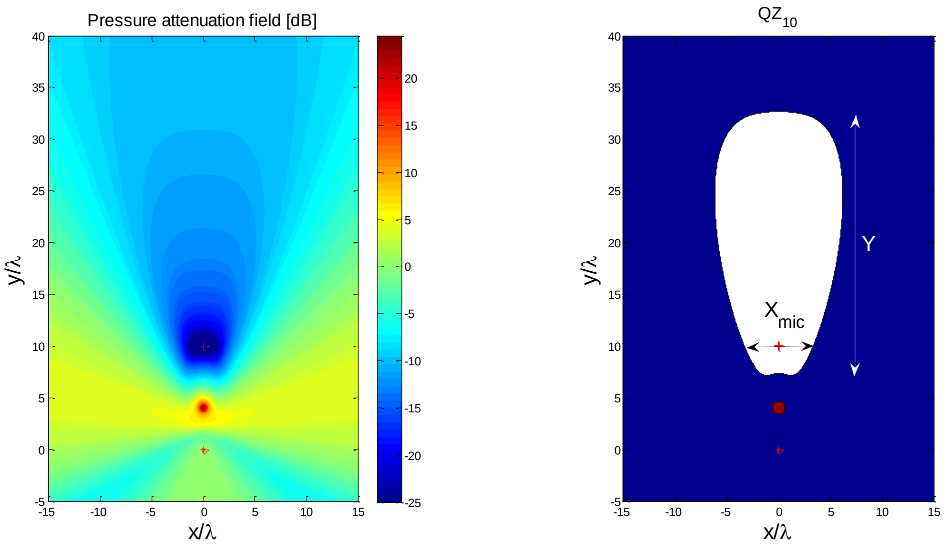

Figure 1 – Field of sound pressure attenuation (a) obtained with ANC_{111} with $r_{SS} = 4\lambda$ and $r_{mic} = 10\lambda$ and illustration of corresponding QZ_{10} (b) primary source (*),secondary source (o) and microphone (+)

It has been investigated how (quantitatively) the quiet zone evolves as the position of the secondary source varies between the primary source and the microphone, i.e., when r_{SS} passes from 0 to r_{mic}. Figure 2 shows four pressure attenuation fields (and their associated QZ_{10}) obtained with ANC_{111} for $r_{mic} = 20\lambda$ corresponding to four different positions of the secondary source. When the secondary source is relatively close to the primary one ($r_{SS} \sim \lambda$), the control can be described as 'semi-total', since

QZ_{10} covers a vast zone corresponding to roughly half the propagation domain. As the secondary source moves away from the primary one, the 'semi-total' aspect vanishes: all the QZ_{10} dimensions lessen, until showing a limited and bounded aspect, a shell-shaped region (see Fig 2c), with an axial dimension Y always significantly larger than its width. As r_{SS} keeps on increasing, the quiet zone keeps on shrinking, up to show a circular (spherical in 3D-space) aspect roughly centered at the microphone position, and whose radius decreases pointing to the microphone position.

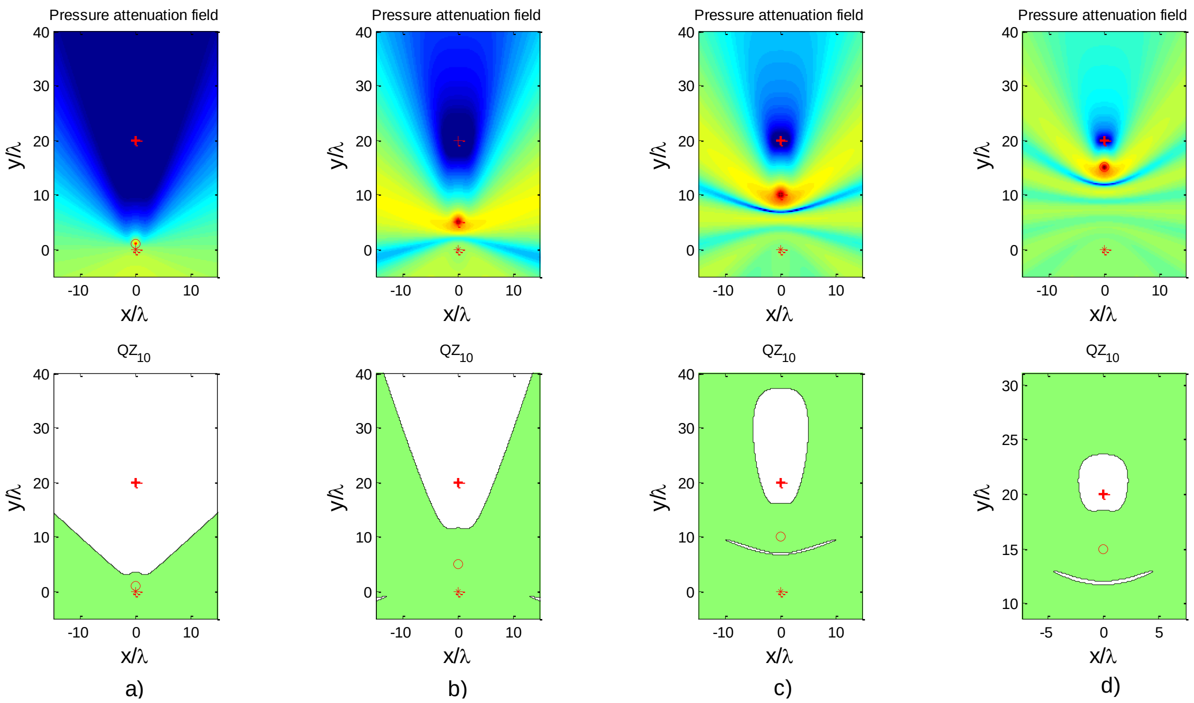

Figure 2 – Pressure attenuation fields (1st line) and QZ_{10} (2nd line) obtained for $r_{mic} = 20\lambda$ and with a) $r_{SS} = \lambda$, b) $r_{SS} = 5\lambda$, c) $r_{SS} = 10\lambda$, d) $r_{SS} = 15\lambda$.

Given that the QZ_{10} axial dimension Y is always larger than X_{mic}, and therefore not as much challenging as X_{mic}, we focused on the latter indicator, its evolution as a function of secondary source position being shown in Figure 3. All curves show a similar aspect: an initial strong decreasing phase (for $r_{SS} \leq 2\lambda$) is observed, in which, though, all X_{mic} values are quite significant. Note that this initial phase corresponds to the one in which the total output power passes from its minimal to its maximal values (shown in [7]). Then, for $r_{SS} \geq 2\lambda$, the curves show a phase in which the slope slows down as r_{SS} increases. Another general feature is that, for a given R_{SS} value, X_{mic} increases with R_{mic} and it has been found that a good approximation for X_{mic} is given by

$$X_{mic} \approx \frac{2R_{mic}}{(R_{SS})^{0.75}}, \tag{6}$$

This approximation — which curves are plotted with dashed lines in the Fig.3—, can be used to determine, for a required size of the quiet zone (i.e., a wanted minimum dimension for QZ_{10}), the highest acceptable value for R_{SS}, informing, thus, up to what distance from the primary source the secondary source should be positioned.

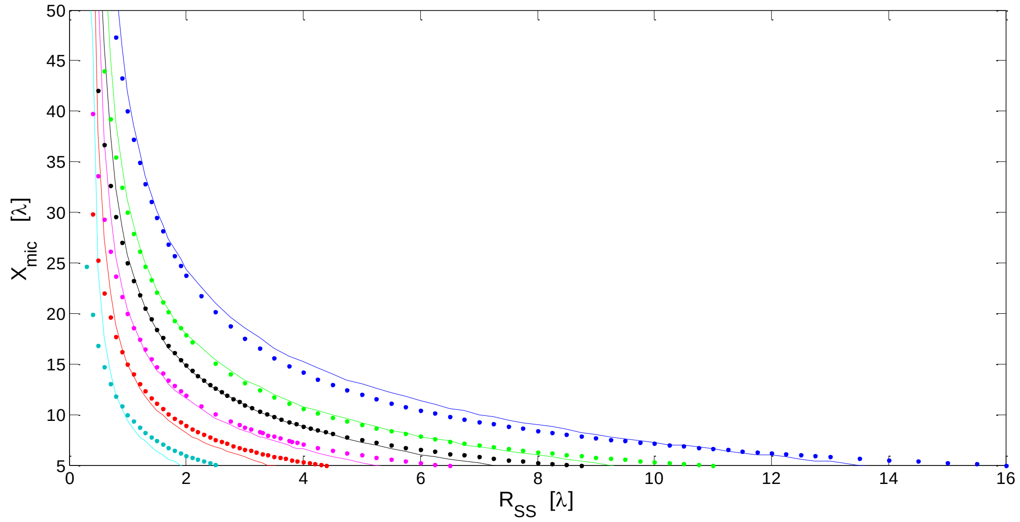

Figure 3 – Width of the QZ_{10} width, X_{mic}, against secondary source position for 6 microphone positions r_{mic} : 5λ (—), 7.5λ (—) 10λ (—),12.5λ (—) 15λ (—), 20λ (—), Dashed lines: approximation curves given by Eq. 6.

3 On the *Elasticity* of the Quiet Zone

After investigating the way QZ_{10} varies as the separating distance between first and secondary sources increases, the following question arises: can and how the quiet zone - still provided by a single secondary monopole - be enlarged? Since, as it has been recalled in the previous section, the zone axial dimension (Y) can easily reach a quite significant value (see also [7]), the present investigation is focused on the most constraining and challenging QZ_{10} dimension, which is its width at microphone position. The investigation is based on the following methodology. A second error microphone is placed at the same distance r_{mic} as the first one, the two microphones being symmetrically positioned relatively to the axis constituted by the primary and secondary sources. The new ANC system will be referred to as ANC_{112}. Similarly, will be also investigated the systems ANC_{113} and ANC_{114}, which are obtained by using a total of, respectively, 3 and 4 microphones, placed as shown in Fig. 4.

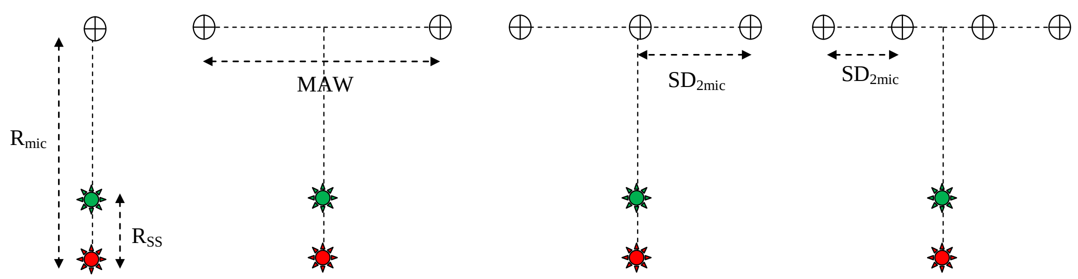

Figure 4 – Schematic representation of the ANC systems under study: ANC_{111}, ANC_{112}, ANC_{113} and ANC_{114}.

For each of these three new systems, one can look at two coupled parameters: the microphone array width (MAW) and the separating distance between 2 microphones (SD_{2mic}), both being expressed in term of λ, wavelength of the sound emitted by the primary (and secondary) source.
What will be investigated in this section is the influence of the size of the microphone array on the QZ_{10} expansion, for given distances r_{SS} and r_{mic}. A first general trend, which can be drawn from the ANC_{112} case, is illustrated in Figure 5 for $r_{SS} = 10λ$ and $r_{mic} = 20λ$.

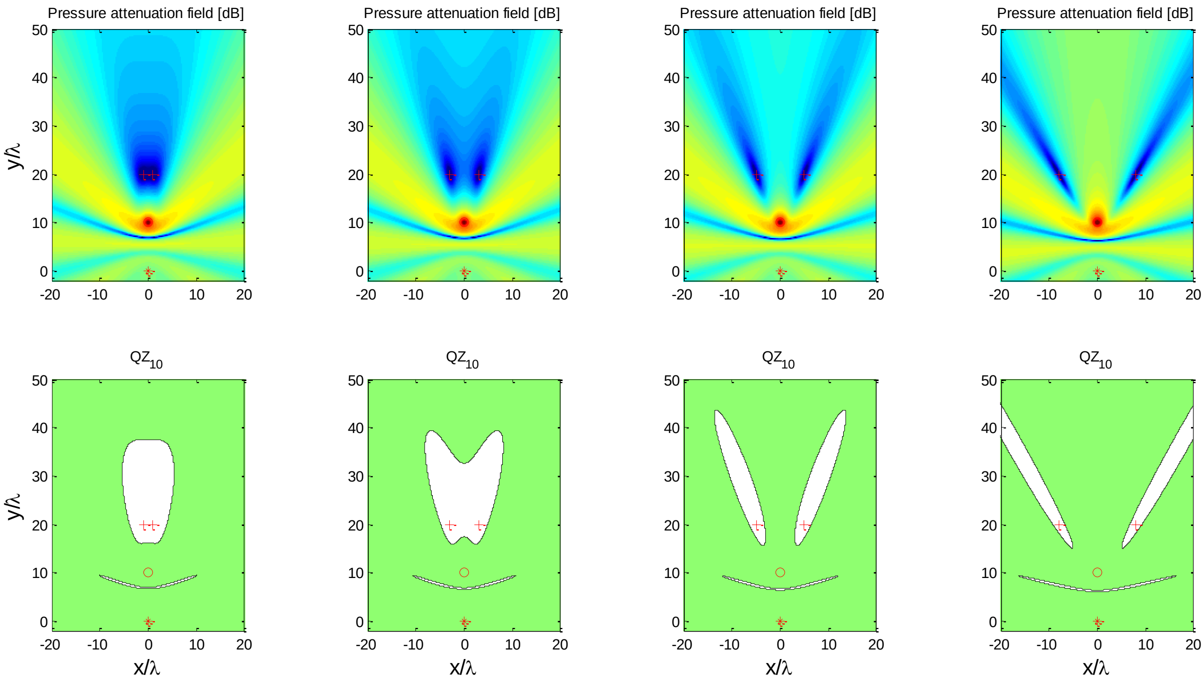

Figure 5 – Evolution of the pressure attenuation field (first line) and QZ_{10} (second line) provided by ANC_{112} (for $r_{SS} = 10λ$ and $r_{mic} = 20λ$) for
a) MAW = 2λ, b) MAW = 6λ, c) MAW = 10λ and d) MAW = 16λ.

When MAW is zero, which means that the microphones are superposed, one gets the same QZ_{10} which is provided by the ANC_{111} system described in section 2. As the MAW increases, the shape of QZ_{10} shows different stages. First, a heart-shaped (or V-shaped) region is observed, mainly due to the fact that, while its width goes increasing, its axial dimension Y goes reducing. This axial dimension Y keeps on decreasing until it engenders two separated QZ_{10} — as in a cell division process —, each one surrounding, neighboring around a microphone. Each one of these two new QZ_{10} has the shape of a beam which axes is given by the position of the secondary source and the error microphone. As MAW keeps on increasing, the two beams get more spaced, spanned, and each one of these beams gets more elongated (see Fig. 5c-5d). Also, it appears that the pressure attenuation observed in the in-between region is getting lower and lower (see the change in color intensity in pressure fields Fig. 5c-5d).

What we are interested in in this section corresponds to the first stage of the evolution, what we will call the *elasticity* of the quiet zone: in other words, we are interested in the following issue: to what extent MAW can be increased (up to a maximum value MAW*) so that QZ_{10} be enlarged in an unbroken, uninterrupted way along the microphone line/axes. The investigation has been systematically carried out for 24 cases: two r_{mic} values (10λ and 20λ), for each case, four r_{SS} values (1.25λ, 2.5λ, 5λ, 7.5λ and 2.5λ, 5λ, 10λ and 15λ, respectively) and for each one of these 8 cases, the QZ_{10} obtained with ANC_{112}, ANC_{113} and ANC_{114} has been investigated. Fig. 6 shows, for $r_{mic} = 20λ$, the

X_{mic} dimension as a function of the Microphone Array Width (a) and also as a function of the distance between two microphones (b).

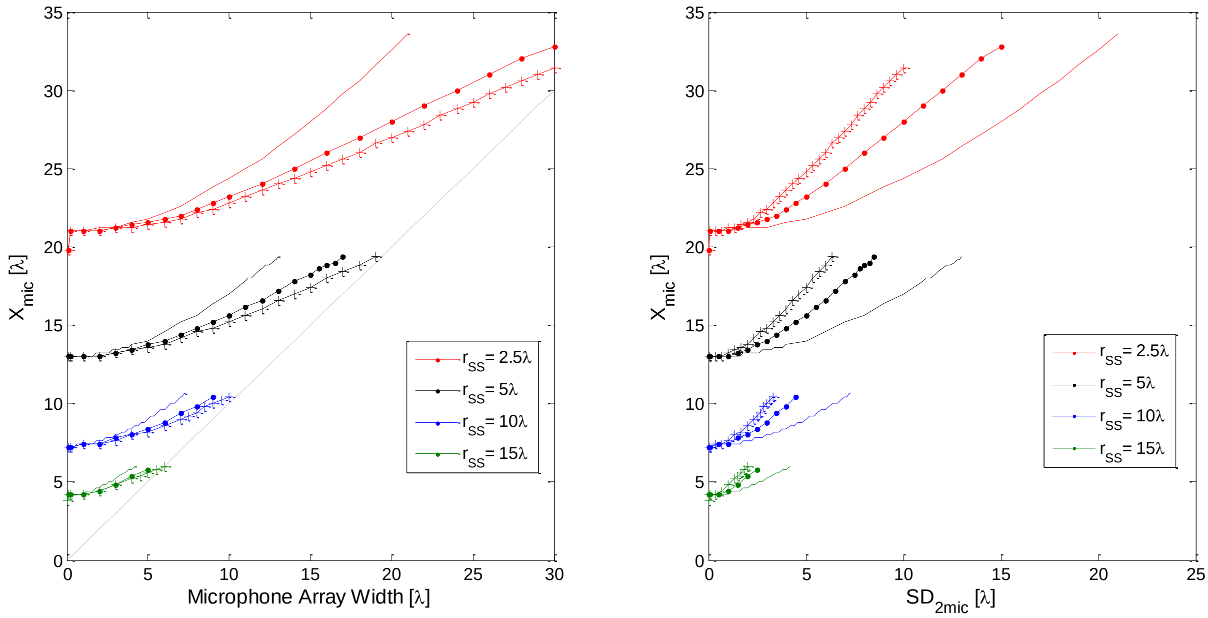

Figure 6 – Evolution of X_{mic} with a) the Microphone Array Width and b) the separating distance between two microphones for ANC_{112} (—), ANC_{113} (•) and ANC_{114} (+) for $r_{mic} = 20\lambda$.

General comments and main results can be summarized as follows:

a) First of all, the green line in Fig. 6a, which materializes ‘$X_{mic} = MAW$’, calls attention to the fact that the width of the quiet zone is always larger than the microphone array;
b) All the curves show roughly a similar shape, the quiet zone width increasing with MAW (of course, while $MAW \leq MAW^*$). The curves are shifted, the same shift than the one provided with ANC_{111} being observed: the nearer the secondary source (from the primary source) the larger X_{mic}, and therefore, the higher the *elasticity*;
c) As for this limiting upper value MAW*, data show that, while it is inversely proportional to r_{SS} and directly proportional to $2r_{mic}$, it increases with the number of microphones used in the array.
d) For a given r_{SS} value, the ANC_{112}, ANC_{113} and ANC_{114} curves are almost merged for MAW lower than 5λ, and only begin to differentiate after this MAW value. It appears then that, for a given MAW value, the width of the quiet zone obtained with ANC_{112} is larger than with ANC_{113}, which is larger than with ANC_{114}.

Quantitatively, it has been found that the value of X_{mic} provided by ANC_{11m} — where m is the number of microphones used in the array — can be suitably approximated by a linear function of MAW (Eq. 7a) in which the slope α_m is, in turn, roughly given by Eq. 7b,

$$X_{mic} \approx \alpha_m \, MAW + \beta \,, \qquad \alpha_m = -0.1m + 0.7 \qquad (7a\text{-}7b)$$

As for the y-intercept of the linear approximation of X_{mic}, β, since it corresponds to the theoretical value of X_{mic} provided by ANC_{111}, it logically doesn't depend on the number of microphones (m) used in the array, but only on the ratio of r_{mic} to r_{SS}, as given in Eq. 6.

Let's define the indicator *elasticity* of the quiet zone, *Elast* (expressed in %), as the relative band value of X_{mic} , i.e, as the ratio of $(X_{mic})_{max} - (X_{mic})_{min}$ to $(X_{mic})_{min}$, for MAW varying from 0 to its maximal acceptable value MAW*. Results show that *Elast* does not depend significantly on the number of microphones m used in the array, since the maxima (and minimum) values of X_{mic} are roughly the same for all m. The *elasticity* increases as r_{SS} decreases, i.e., as the secondary source gets closer to the primary source. This means that the closer the secondary source (from the primary source) the higher the potential increase of the quiet zone (engendered by enlarging the microphone array). Quantitatively, it has been found that a suitable approximation for the measure *Elast* is given by

$$Elast \approx 3.2 \frac{r_{mic}}{r_{SS}} + 40 \tag{8}$$

As for the main information that can be extracted from the curves "X_{mic} against the separating distance between 2 microphones" (shown in Fig.6b), it addresses the highest value that SD_{2mic} can take, SD_{2mic}^{*}, given that it informs on the maximal acceptable distance between two adjacent microphones in the array (beyond which the QZ_{10} ceases to be continuous). Note that, for ANC_{112}, the curves coincide with the ones in Fig.6a, since in this particular case, $MAW = SD_{2mic}$. The most important result is that SD_{2mic}* decreases as the distance primary/secondary source increases, and also as the number of microphones increases. In other words, the separating distance between 2 microphones must lessen as the array gets farther from the primary source and also as one adds microphones in the array.

4 Conclusões

The dimensioning of the '10 dB quiet zone' produced by the simplest active noise control system, ANC_{111} — which consists of one primary source, one secondary source and one microphone — has been investigated; a good approximation for X_{mic}, the zone width at microphone position, has been expressed as a function of the position of the microphone and the secondary source and the wavelength of interest.

It was shown here that the width of the quiet zone provided by ANC_{111} can be significantly enlarged by using one (or more) additional error microphone(s), forming thus an array of two (or more) microphones. It is shown that the zone width increases with the array size, MAW, as long as MAW does not exceed a certain value, MAW*, beyond which separated and smaller quiet zones are formed. This limiting upper value for the array size, MAW*, is inversely proportional to r_{SS} and proportional to $2r_{mic}$, and also increases with the number of microphones used in the array.

It was found that the value of X_{mic} provided by ANC_{11m} — where m is the number of microphones used in the array — can be suitably approximated by a linear function of MAW. In this linear approximation, the y-intercept corresponds to the value of X_{mic} provided by ANC_{111} (which logically doesn't depend on the number of microphones), which means that, for a given size of the array, X_{mic} will increase both as the secondary source gets closer to the primary source and as the microphone array moves away. As for the slope of the X_{mic} linear approximation, it only depends on m, the higher the number of microphones the lower the slope. This means that, for a given array size, the width of the quiet zone obtained with ANC_{112} is even larger than with ANC_{113}, which is larger than with ANC_{114}. While the 2 microphone-array is therefore recommended (if one wants to optimize merely the width of the quiet zone), on the other side, adding microphones in the array will contribute to maintain a fine pressure attenuation level in the axial, central region, guarantying thus significant values for the quiet zone depth.

As for the *elasticity* of the quiet zone — defined as the propensity for X_{mic} to increase with the array width —, the results show that it does not depend on the number of microphones used, and a simple function (of the positions of the secondary source and of the microphone array) is provided as a suitable approximation for this quantity.

Referências

[1] Canévet, G.; Mangiante, G. Absorption acoustique active et anti-bruit à une dimension, *Acustica*, 30, 1974, pp. 40-48.

[2] Elliott, S.J.; Joseph, P.; Nelson, P.A.; Johnson, M.E. Power output minimization and power absorption in the active control of sound, *Journal of the Acoustical Society of America*, 90, 1991, pp. 2501-2512.

[3] David, A.; Elliott, S. J. Numerical studies of actively generated quiet zones, *Applied Acoustics*, 41, 1994, pp. 63-79.

[4] Guo, J.; Pan, J.; Bao, C. actively created quiet zones by multiple control sources in free space, *Journal of the Acoustical Society of America*, 101, 1997, pp. 1492-1501.

[5] Wright, S.E; Vuksanovik, B. Active control of environmental noise, I., *Journal of Sound and Vibration*, 190, 1996, pp. 565-585.

[6] Tseng, W.K.; Rafaely, B.; Elliott, S.J. Local active sound control using 2-norm and ∞-norm pressure minimization, *Journal of Sound and Vibration*, 234, 2000, pp. 427-439.

[7] Gounot, Y.J.R. On the dimensioning of the quiet zone obtained with simple active noise control systems in free field. *18th International Congress on Sound and Vibration*, Rio de Janeiro, 10-14 of July 2011.