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# COMPUTATIONAL ANALYSIS OF ACOUSTIC HORNS USING AN MFS MODEL

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**Abstract**

In this paper, a computational model based in the Method of Fundamental Solutions (MFS) is presented for the study of acoustic horns with axisymmetric configuration. The model is based in the definition of two subdomains, connected by an interface in which continuity boundary conditions are imposed. It allows the calculation of the acoustic response (in terms of acoustic pressure) of a horn attached to an infinite, plane rigid screen. These conditions are similar to the laboratorial conditions used for the experimental characterization of these devices in terms, for instance, of their directivity. Solutions calculated by the proposed method are compared with those computed using a finite-element approach; additionally, the computed directivity is compared with experimental results, obtained in the laboratory.

**Keywords:** acoustic horns, Method of Fundamental Solutions, axisymmetric, experimental validation.

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## 1 Introduction

The fundamentals of acoustic horns were widely developed by Euler, Lord Rayleigh and Webster. Webster was the first to introduce the concepts of specific acoustic impedance and the analogous acoustic impedance, both widely used concepts in acoustic analysis [1]. This author established what is known as Webster's horn equation which, in the strictest sense, is only applicable to the three following waveguide structures: the plane wave tube, and the conical and cylindrical horns. Based on the works carried out by Webster (1919) and Salmon (1946), the audio engineering community has designed different types of horns analyzing their cut-off frequency and proper size. As an example, D. Keele [2] has developed a frequency dependent constant directivity device, achieved by joining an exponential or hyperbolic throat segment, for driver loading, with two conical mouth segments, for directivity control.

Due to the need to design horn systems capable of improved efficiency and directional characteristics, different numerical methods have been used to simulate and optimize different types of configurations. Among those methodologies, the three most widely employed numerical techniques are the Boundary Element Method (BEM), the Finite Difference Method (FDM) and the Finite Element Method (FEM).

Morita et al. [3] and Beltran [4] used the Finite Element Analysis (FEA) to model axisymmetric horns. Within the analysis, Morita et al. used a radiation boundary condition, corresponding to an analytical integral equation defined over the mouth of the horn, that assumes that the horn is mounted in an infinite baffle. This approach reduced the numerical complexity of the system, taking into account the effect of the variation of the acoustic particle velocity across the mouth of the horn being modelled. Beltran used a non-reflective boundary condition on a spherical surface around the horn's mouth, only requiring meshing the outside region in front of the horn. He also modeled the mechano-acoustic interaction of the compression driver diaphragm coupled to the horn throat, for different material properties, showing the usefulness and advantages of a FEA approach to the design of the complete radiating system.

Hodgson and Underwood [5] used a BEM scheme to compute the throat impedance and the far field pressure, demonstrating good agreement with experimental data; they also analyzed the correlation between the impedance peaks and the far field pressure response peaks. Noreland et al. [6] and Udawalpola et al. [7] applied a gradient-based optimization algorithm to improve the transmission properties of a variable mouth acoustic horn, by computing the reflection spectra using FEM and BEM methodologies. These authors remarked the advantages of using a BEM model, since it is not necessary to remesh or modify the mesh volumes within the optimization processes. On the other hand, Makarski [8] proposed a computational tool based on the BEM and the Fundamental Mode decomposition, with its focus on the professional development of horn loudspeakers, in which the electroacoustic transducer and the horn are treated separately.

Morgans [9] describes an optimization method, based on BEM and FEM, for the design of constant beamwidth horn loaded loudspeakers. Using the Transmission Matrix Method, Lampton [10] established a numerical approach, which consists of discretizing complex geometry of electroacoustic systems using small cascade-elements network. Each acoustic component can be described by a pair of pressure and volume velocity ports at the input and output of the element, being these related by a 2x2 matrix. Adopting the Transmission Matrix approach, McLean et al. [11] studied the throat impedance characteristics of constant directivity horns, showing a good agreement between experimental data and computed results.

In this work, the authors aim to present a very efficient and accurate numerical model for the solution of problems related with acoustic horn analysis. The proposed approach is based on the use of the Method of Fundamental Solutions (MFS), adopting a sub-region technique to simulate the behavior of a horn mounted on a rigid screen. The method has some advantages when compared with other approaches, specially concerning its computational efficiency and accuracy.

This paper is organized as follows: first, the mathematical formulation of the governing equations in sound propagation is briefly outlined, indicating the fundamental solutions for pressure and particle velocity (both for the 2D, 3D and axisymmetric cases), and formulating the Method of Fundamental Solutions (MFS) in the frequency domain; then, a MFS model is described for the analysis of a radiating acoustic horn embedded in a rigid infinite screen; finally, a comparison of the sound directivity computed using the MFS and the FEM for an axisymmetric horn configuration is presented, together with an experimental validation of the computed results.

# 2 Mathematical formulation

## 2.1 Governing equations

The propagation of sound within a homogeneous acoustic space can be mathematically represented in the frequency domain by the Helmholtz differential equation,

$$\nabla^2 p + k^2 p = 0 \tag{1}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ in the case of a 3D problem, and $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ for the 2D case; $p$ is the acoustic pressure, $k = \omega / c$ the wave number, $\omega = 2\pi f$ the angular frequency, $f$ the frequency and $c$ the sound propagation velocity within the acoustic medium.

For the 3D case, assuming a point source placed within the propagation domain, at point $\boldsymbol{x_0}$ with coordinates $(x_0, y_0, z_0)$, it is possible to establish fundamental solutions $G$ for the sound pressure and $H$ for the particle velocity at a point $\boldsymbol{x}$ with coordinates $(x, y, z)$, which can be written respectively as

$$G^{3D}(\boldsymbol{x}, \boldsymbol{x}_0, k) = \frac{\mathrm{e}^{-\mathrm{i}kr}}{r} \tag{2}$$

$$H^{3D}(\boldsymbol{x}, \boldsymbol{x}_0, k, \vec{n}) = \frac{1}{-\mathrm{i}\rho\omega} \frac{(-\mathrm{i}kr - 1)\mathrm{e}^{-\mathrm{i}kr}}{r^2} \frac{\partial r}{\partial \vec{n}} \tag{3}$$

A special case can be considered when an axisymmetric problem is analyzed, for which it is also possible to establish a fundamental solution. For that case, assuming that the source is also axisymmetric (e.g. a circular ring) and aligned with the $x$ axis, the required fundamental solutions become

$$G^{Axi}(\boldsymbol{x}, \boldsymbol{x}_0, k) = \int_0^{2\pi} \frac{\mathrm{e}^{-\mathrm{i}kr(\theta)}}{r(\theta)} \mathrm{d}\theta \tag{4}$$

$$H^{Axi}(\boldsymbol{x}, \boldsymbol{x}_0, k, \vec{n}) = \frac{1}{-\mathrm{i}\rho\omega} \int_0^{2\pi} \frac{(-\mathrm{i}kr(\theta) - 1)\mathrm{e}^{-\mathrm{i}kr(\theta)}}{r^2(\theta)} \frac{\partial r(\theta)}{\partial \vec{n}} \mathrm{d}\theta \tag{5}$$

Equations (4) and (5) correspond, in practice, to the integration of a 3D source, given by equations (2) and (3), along the circle centred on the $x$ axis and passing through $(x_0, y_0, 0)$, being $r(\theta) = \sqrt{y^2 + {y_0}^2 - 2y\, y_0 \cos(\theta) + (x - x_0)^2}$ .

Therefore, it becomes obvious that if $(x_0, y_0, 0)$ and $(x, y, 0)$ coincide, then $r(0) = 0$, and hence a singularity occurs in the integrands of equations (4) and (5). However, if the source and receiver points are apart, then those integrals become non-singular, and can be easily approximated using standard numerical integration techniques.

Similarly, the concept of image-sources can be applied to 3D and axisymmetric problems. Since the present paper deals mostly with the latter, it is important to define the corresponding solution. Considering, for that case, a rigid vertical plane located at $x$=0, the fundamental solution can be written as

$$G^{Axi}_{sym}(\boldsymbol{x}, \boldsymbol{x}_0, k) = \int_0^{2\pi} \left( \frac{\mathrm{e}^{-\mathrm{i}kr(\theta)}}{r(\theta)} + \frac{\mathrm{e}^{-\mathrm{i}kr_1(\theta)}}{r_1(\theta)} \right) \mathrm{d}\theta \tag{6}$$

where $r_1(\theta) = \sqrt{y^2 + {y_0}^2 - 2y\, y_0 \cos(\theta) + (x + x_0)^2}$ .

### 2.2 The MFS for the analysis of an emitting horn embedded in a rigid infinite screen

To apply the MFS to the analysis of an emitting horn, embedded within a rigid screen of infinite extent, consider the configuration schematically represented in Figure 1a. In that figure, at $x=L$ a rigid surface is considered, simulating an infinite rigid screen. To analyze this system using the MFS, a model with two sub-domains is considered, as described in Figure 1b, in which sub-domain $\Omega_1$ incorporates the geometry of the horn, while sub-domain $\Omega_2$ simulates the semi-infinite space and the rigid screen. The common interface, along which continuity of pressure and particle velocity must be imposed, is designated as $\Gamma_c$, while $\Gamma_1$ and $\Gamma_v$ correspond to surfaces along which the normal velocity is prescribed. To simulate the horn problem, null velocities are prescribed in $\Gamma_1$, while a unit velocity is prescribed in the vibrating surface $\Gamma_v$.

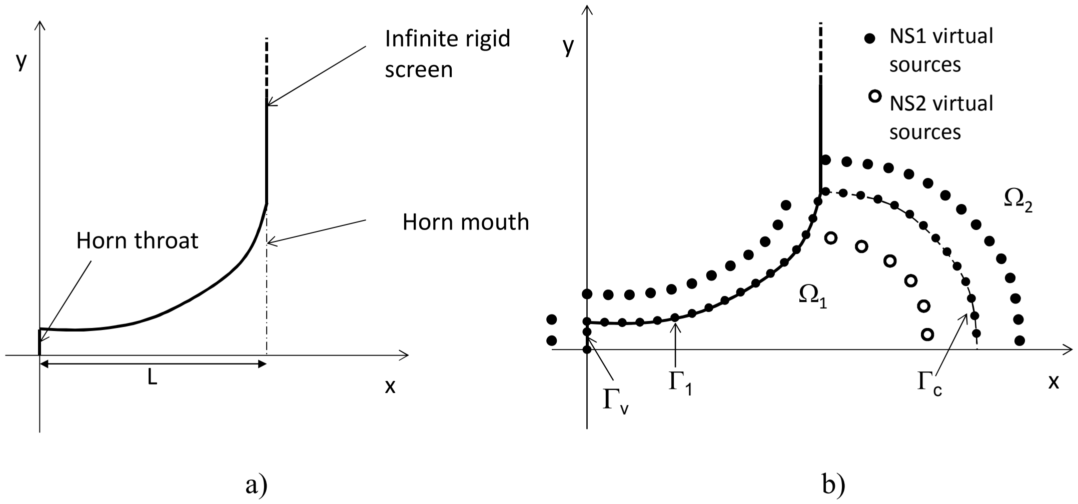



Figure 1 – a) Schematic representation of the problem geometry; b) Distribution of virtual sources and collocation points in the MFS model with two sub-domains.

If an axisymmetric configuration is considered, the corresponding fundamental solutions to be used are those of equations (4) and (6), and the pressure field is then defined by

$$p(\boldsymbol{x},k)_{\Omega_1} = \sum_{j=1}^{NS1} Q_j G^{Axi}(\boldsymbol{x},\boldsymbol{x}_{1,j},k) \text{ for } \boldsymbol{x} \text{ in } \Omega_1 \tag{7}$$

$$p(\boldsymbol{x},k)_{\Omega_2} = \sum_{j=1}^{NS2} P_j G^{Axi}_{sym}(\boldsymbol{x},\boldsymbol{x}_{2,j},k) \text{ for } \boldsymbol{x} \text{ in } \Omega_2 . \tag{8}$$

# 3 Application example

As an application example, the optimized axisymmetric acoustic horn design proposed in the works of Noreland et al. [6] and Udawalpola et al. [7] was considered. The proposed shape of the horn was obtained numerically by these authors with the aim of having optimal radiation efficiency at frequencies in the range 1.6–9.05 kHz, while satisfying a convexity constraint on the flare. The geometry of this horn is schematically displayed in Figure 2, exhibiting a throat radius of 0.0193 m, a mouth radius of 0.1500 m, and a length of 0.1615 m (note the curvature at the edges of the mouth of the horn).

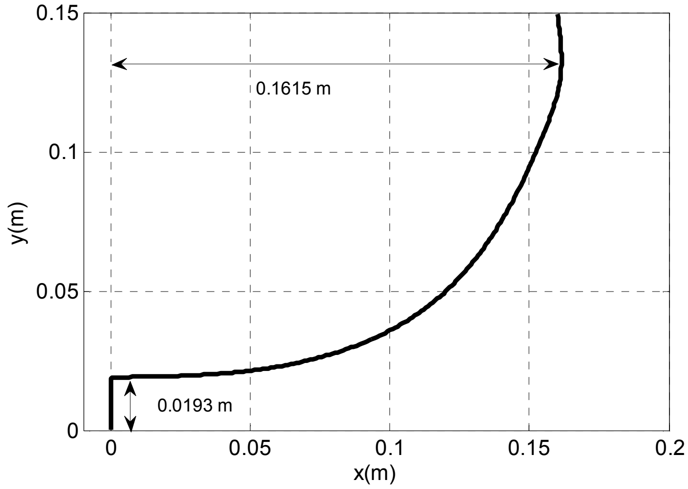



Figure 2 – Geometry of the optimized axisymmetric horn proposed by Noreland et al. [6].

The horn was modelled using the proposed axisymmetric MFS model, making use of a total of 113 boundary points to discretize its rigid surface. It is worth remarking that the optimized horn exhibits a very subtle geometry, thus requiring a larger number of points to correctly define its shape; indeed, an incorrect definition of this shape results in relevant changes in the computed results. Beyond the mouth of the horn, an infinite rigid screen is considered and incorporated in the model by means of the virtual sources technique. Figure 3 depicts the directivity results computed for frequencies of 500 Hz, 2000 Hz and 8000 Hz, using both the MFS model and a finite element commercial software. For the latter, the field was discretized making use of a fine mesh, with at least 12 elements per wavelength for a frequency of 8 kHz. As shown in the directivity plots, an excellent agreement exists between both solutions, although with a lower computational effort in the case of the MFS. It can be noted, in that plot, that the optimized horn exhibits regular curves, without any observable oscillations along the angular range.

To validate the numerical results shown in Figure 3, the directivity of the horn was measured in laboratory, mounting the horn in a large rigid screen, simulating the theoretical infinite rigid surface of the model. The directivity was measured in anechoic conditions, at a distance of 1m from the mouth of the horn, as described in [12]. In order to obtain the directivity pattern, sound pressure level was registered using a measurement system, and performing sound pressure level evaluations at angular intervals of 5º. In Figure 4a, a photography of the horn tested in the laboratory is depicted. Figure 4b exhibits a comparison of results for frequencies of 1 kHz, 4 kHz, 8 kHz and 10 kHz. The results illustrated in that figure clearly allow concluding that the agreement between numerical results and experimental data is good, although existing some differences between both sets. For the lower frequencies, of 1 kHz and 4 kHz, the trend of the numerical and experimental results is similar, but differences of around 1 dB can be seen even at receivers placed at low angles. For higher frequencies, a good agreement can be observed for receivers placed at angles up to 60º, without significant deviations between the experimental and numerical result. It is only beyond this angle, for which the decay is over 20 dB, that significant differences (above 1 dB) are registered; these larger differences can be attributed to limitations of the experimental setup used for the directivity measurements.

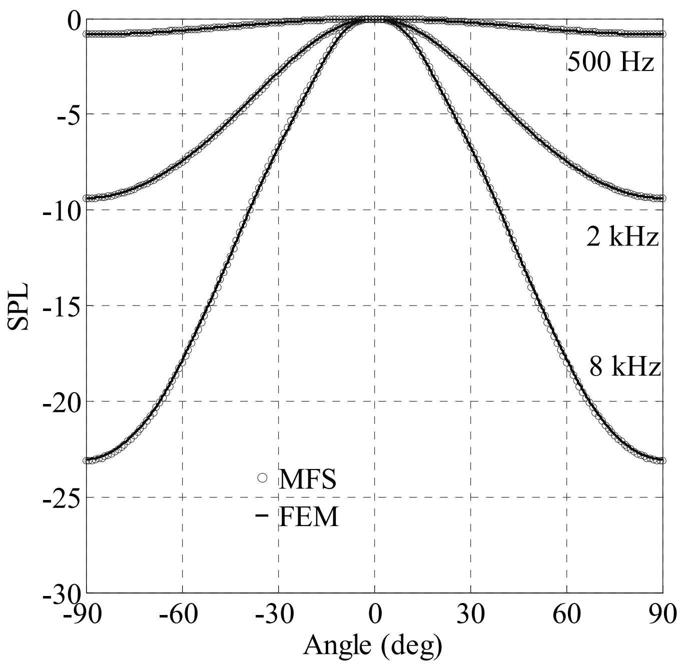
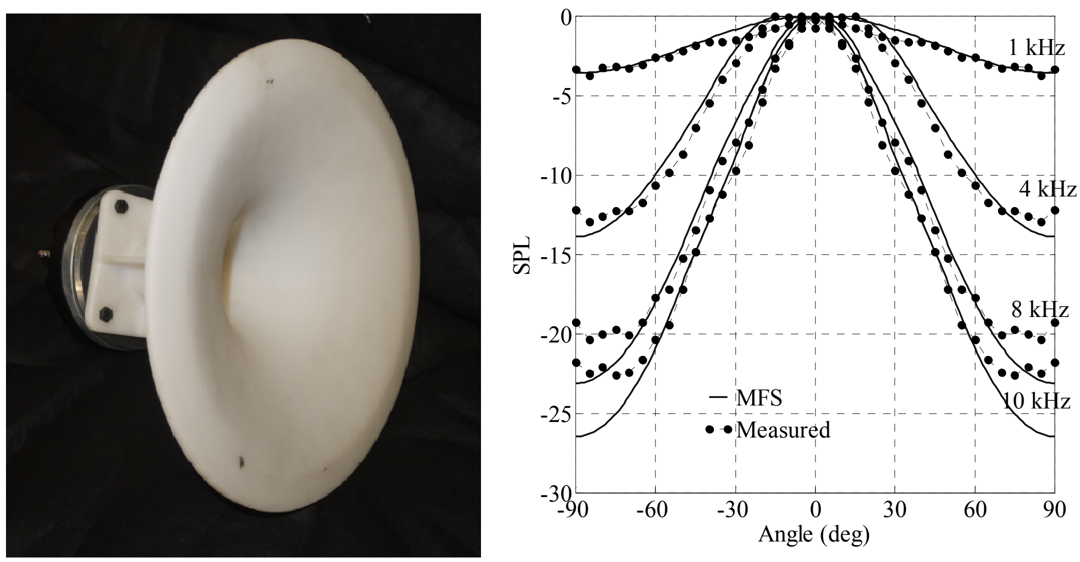



Figure 3 – Comparison of directivity results calculated using FEM and using the proposed MFS model.




a)

b)

Figure 4 – Validation of the numerical model using laboratory measurements: a) horn tested in the laboratory; b) measured and computed directivity results.

# 4 Conclusions

In the present paper the Method of Fundamental Solutions was used to model the behavior of acoustic horns with axisymmetric geometry. The developed model was based on the use of two sub-regions, one of them containing the rigid horn, and the other corresponding to a semi-infinite space. The joint use of these two sub-regions, connected by imposing the necessary continuity of acoustic pressure and normal particle velocities, allows simulating the case of an acoustic horn embedded within a rigid screen, a scenario that is frequent in laboratory analysis of such devices.
The model was here tested to analyze the behaviour of an optimized non-trivial horn geometry in terms of directivity at different frequencies. For the tested case, an excellent agreement with a commercial finite element software was observed. Comparison with experimental results obtained for the same geometry also revealed a very good agreement, indicating that the method performs well and can be a valuable analysis tool.

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