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DESIGN AND OPTIMIZATION OF SOUND DIFFUSERS USING RBF-BASED SHAPES AND GENETIC ALGORITHMS

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ABSTRACT

The optimization of diffusers' design has been a topic of intense research in the last years. In this paper, the authors propose an alternative technique to define new shapes of sound diffusion configurations, based on the use of radial basis functions (RBF). In addition, to allow the definition of optimal surface shapes for a given frequency band, a genetic algorithm is used. The diffusion coefficient is computed within the optimization procedure using the Kirchoff integral equation and the Boundary Element Method (BEM). A set of results is presented and discussed, namely the influence of the geometric constraints imposed due to the future production of panel samples.

Keywords: Acoustic diffusers; RBF; Genetic Algorithms; Kirchoff integral equation; BEM; FDTD.

RESUMO

A otimização de difusores tem sido um tema de pesquisa intensa nos últimos anos. Neste artigo, os autores propõem uma técnica alternativa para definir novas formas para dispersar eficientemente o som, com base no uso de funções de base radial (RBF). Além disso, para permitir a definição de superfícies otimizadas, para uma determinada banda de frequências, um algoritmo genético é usado. O coeficiente de difusão é calculado dentro do procedimento de otimização, usando a equação integral Kirchoff e o Método dos Elementos Fronteira (BEM). Um conjunto de resultados de aplicação são apresentados e discutidos, nomeadamente a influência das restrições geométricas impostas devido ao futuro fabrico de amostras.

Palavras-chave: Difusores acústicos; RBF; Algoritmos genéticos; Equação integral de Kirchoff; BEM; FDTD.

1. INTRODUCTION

In order to control reflections inside rooms with greater acoustic requirements, such as theatres, concert halls or auditoria, and to increase its diffuseness, it is usual to cover them with sound diffusers panels. A significant number of the acoustic diffusers commercially available are based on the phase grating diffusers or Schroeder-type diffusers. Their appearance is greatly underestimated by the architects and users of rooms for which they are projected. Variations of this type of acoustic diffusers are available on the market, often designed for aesthetic reasons rather than as a result of optimizing their performance. In recent years, significant efforts have been made to find other kind of surfaces with good properties in scattering the sound. The main objective of this work is to demonstrate the possibility of developing innovative acoustic diffuser solutions with optimized acoustic performance, whose shape is generated by the use of radial basis functions (RBF) and that are based on the most modern techniques of numerical modelling (BEM) and optimization (Genetic Algorithm), which can be more organic (*i.e.* curvilinear) and aesthetically more appreciated and better accepted.

This work follows a previous approach proposed by the authors [1]. The development of the shape of the surfaces, the evaluation of their efficiency and their optimization follow the same strategy: the shape is defined based on the use of RBF; for the performance optimization, the diffusion coefficient (as defined in [2]) is being considered as the "*figure of merit*", for a given band of frequencies, and a genetic algorithm is used; the diffusion coefficient is calculated within the optimization procedure, using the integral Kirchoff equation and the boundary element method (BEM); a comparison of results obtained with FDTD is also presented.

For this work, the number of height levels of the RBF control points was extended and some geometric constraints introduced to the solutions due to the future manufacture of samples and prototype solutions.

Next, the proposed mathematical formulation is briefly described, including the numerical strategy for analyzing the diffuser, the definition of its geometry and the shape optimization procedure. Then, a set of application results are presented and discussed.

2. IMPLEMENTED METHODOLOGY

2.1 DEFINITION OF THE GEOMETRY

Simple curvilinear shapes are defined, based on a number of control points, ***NC***, through which a smooth mathematical curve is defined, and optimized towards the diffusive behavior and aesthetic requirements. To define this organic shape, an interpolation scheme based on the use of Multi-Quadric Radial Basis Functions (***MQ RBF***) is adopted. This type of function depends only on the distance between a point of origin (center of RBF) and a destination point, ***r***, and on a free parameter, ***c***, and is defined as:

$$\phi(\underline{x}) = \sqrt{r^2 + c^2} \tag{1}$$

A possible interpolation scheme can be assembled using a set of ***NC*** RBFs, each one centered at one control point. More details about this definition can be seen in reference [1].

2.2 SOUND DIFFUSION COEFFICIENT

Sound diffuser performance is usually quantified by means of the Sound Diffusion Coefficient (as defined in [2]), which gives an idea of the capacity of a diffusing device to spread sound energy in space. This parameter is evaluated from the polar scattering diagram of a given diffuser configuration, by means of the equation:

$$d_\theta = \frac{\left(\sum_{i=1}^{n} 10^{L_i/10}\right)^2 - \sum_{i=1}^{n}\left(10^{L_i/10}\right)^2}{(n-1)\sum_{i=1}^{n}\left(10^{L_i/10}\right)^2} \tag{2}$$

To normalize this diffusion coefficient, it is compared with that of a flat plate with the same size. The purpose of this normalization is to remove edge diffraction scattering effects due to the limited size of the sample under analysis. The normalized diffusion coefficient, is thus defined as [2]:

$$d_{\theta,n} = \frac{d_\theta - d_{\theta,flat\_plate}}{1 - d_{\theta,flat\_plate}} \tag{3}$$

In this work, the analysis of the sound diffusers is performed numerically, and so the Sound Pressure Level (SPL) at different receiver positions (L_i) is evaluated using well established numerical tools.

2.3 NUMERICAL METHOD

The Boundary Element Method (BEM) is usually considered one of the best tools for computational diffuser analysis, since it allows describing the geometry of a given problem just by discretizing the boundary surfaces of any objects that exist in the propagation medium. The generated shape is discretized in a number of straight segments, always ensuring that a minimum of 10 segments per wavelength is used. Although this is a well-established method for the analysis of diffusers, it can still be quite time-consuming to use if an optimization process (where hundreds or thousands of configurations may be analyzed) is to be implemented, as it is the case in the present research work. In those cases, a simplified strategy, corresponding to Kirchoff's approximation, may also be adopted. In practice, this is only valid if the interaction between surfaces is neglected, and it is accepted if the analysis is not performed in the low-frequency range. The Kirchoff's approximation is quite simple to implement, and allows for a very fast analysis of different configurations. To know more about how these numerical methods were implemented see references [1, 3].

2.4 SHAPE OPTIMIZATION

This optimization is based on the use of a Genetic Algorithm, and it is described in a simplified manner in the flowchart of Figure 1. At the end of this process, a final organic (smooth) shape is obtained, defined in terms of RBF superposition, with optimal performance for the selected frequency band.

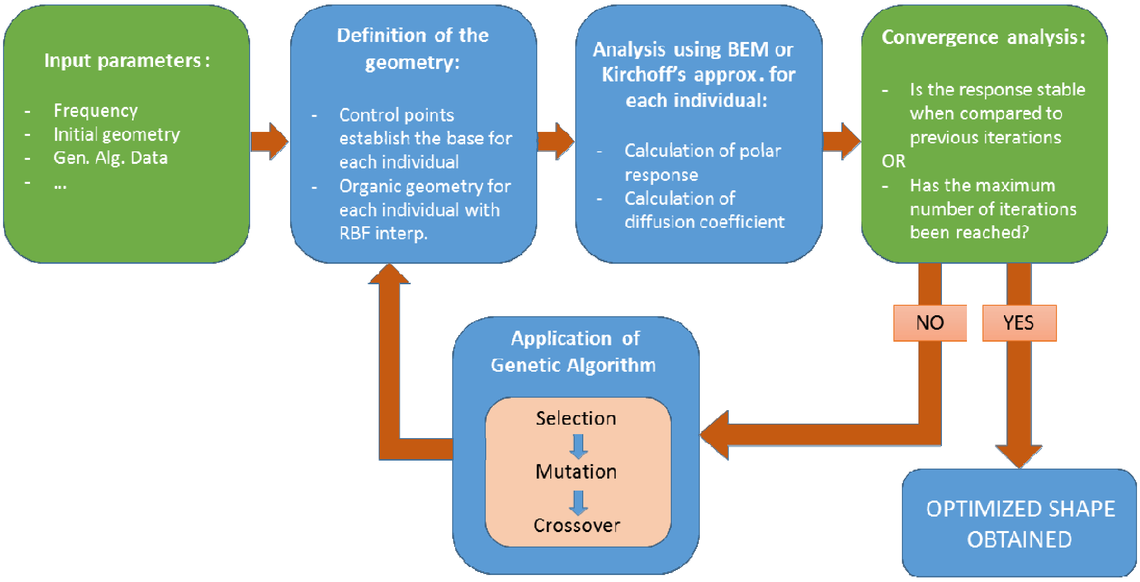


Figure 1 – Flowchart of the calculation/optimization process.

A population of individuals (diffusers) is randomly formed, and the characteristics of each individual are determined by their genes. When designing diffusers, the genes are simply a set of numbers that describe the surface (control points of the RBFs). In this work, these genes were coded in a 3 bit set that allows the definition of eight different height levels, *i.e.* 2^3. Each individual (or diffuser design) has a fitness value (or figure of merit) that indicates how good it behaves at diffusing sound: the numerically evaluated ***diffusion coefficient***. Over time, new populations are produced by combining (breeding) previous shapes, and the old population dies off. Descendants are produced by pairs of parents breeding, and they have genes that are a composite of their parents' genes (50% chance of the child's gene coming from each parent):

multi-point ***crossover***. If all that happens is a combination of the parents' genes, then the system never looks outside the parents' population for better solutions. To enable dramatic changes in the population of diffusers, ***mutation*** is also needed. This is a random procedure whereby there is a small probability of any gene in the child sequence being randomly changed, rather than directly coming from the parents. ***Selecting*** diffusers to die off can be done randomly, with the least fit (the poorest diffusers) being most likely to not be selected. By these principles, ***selection***, ***mutation*** and ***crossover***, the fitness of successive populations should improve in the optimization process. This is continued until the population becomes sufficiently fit so that the best shape produced can be classified as optimum.

3 RESULTS OBTAINED

3.1 INPUT PARAMETERS

The input data are: the frequency (octave) band to optimize (the number of frequencies within the octave band used to calculate the diffusion coefficient was 5); the dimension (width) of the sound diffuser was 0.60m; the number of control points for the RBF (also defines the horizontal spacing between them because they are equally spaced and the width of the diffuser is fixed) and the maximum height possible of each control point. For all the test cases, the initial configuration was a flat plate (null height, *i.e.*, y_i=0.0 m for all points).

At this stage, the aim wasn't to evaluate the influence of the genetic algorithm control parameters (selection, crossover and mutation), thus, for compactness it remained constant for all examples, and for each generation (iteration) a population (set of configurations) of 22 individuals (***npop***=22) was used.

For this work, 8 levels of height were used for the control points, coded in 3 bits. Thus, in the optimization process, genetic algorithms can "choose" a level between the minimum value (y=0) and the maximum value (y=***refv***).

On the other hand, in order to correspond to future constraints in the manufacture of prototypes, some geometric constraints were imposed on the control points at the extremities of the diffusers, *i.e.*, 1st (x_1=0) and last control point (x_{NC}=L=0.6m): to have the same height (regardless of the inclination of the tangent at those points) – referenced as "***h_equal***"; to have the same height and end slope (the tangents to the curve at these end points on both sides are equal, allowing the continuity of the diffuser shape when several unit modules are grouped together) – mentioned as "***compatible***". Additionally, a set of cases where there are no restrictions at all to the end control points, was also simulated – referred by "***free***".

For the calculation of the diffusion coefficient, $\boldsymbol{d_\theta}$, the sound source is positioned at 0° (normal incidence) facing the midpoint of the diffuser, at a distance of 500m. A set of 180 receivers are arranged in an arc of a circle with r=250 m, also centered with the diffuser. This calculation was performed for octave bands using 5 discrete frequencies per band (***nfreq_bands***=5).

3.2 ANALYSIS OF THE CONVERGENCE: BEM vs. KIRCHOFF

The first analysis was on the convergence of the optimization for a local maximum (maximum diffusion coefficient in the frequency band to optimize, $\boldsymbol{d_{max}}$). For this purpose, several consecutive tests were conducted without changing any input parameter, and then by checking the final geometric configuration (RBF) and the maximum diffusion coefficient obtained. The results presented in this section were obtained for the frequency band (octave) of 1000 Hz and with 15 cm of maximum height of the control points.

Several cases were tested considering for 5, 10 and 20 control points. For each one the results were analysed with the three types of geometric constraints of the extreme points: "***compatible***", "***h_equal***" and "***free***".

For illustrative purposes, Figure 2 shows the convergence evolution of the figure of merit for the two types of numerical analysis (BEM and Kirchoff) for 5 control points and without any geometric constraints ("***free***"). As can be seen, although it does not converge to a single solution, the method converges rapidly and the relative dispersion (understood as the difference between the maximum and minimum values relative to the mean value) is rather small.

As we can see, with only 40 iterations a value (on average) very close to the value obtained after 150 iterations has already been obtained. Several tests were performed, for different number of control points and geometric constraints, and the worst case was achieved in the BEM analysis with 20 control points (3.25%) - in fact, in general, it can be stated that the "speed" with which it converges increases with a decrease in the number of control points (as expected, since it reduces the universe of possible configurations). It was also found that the relative dispersion of values is small, increasing with the number of control points, reaching a maximum value of 5.6% in the case of 20 control points and with the "***compatible***" endpoints.

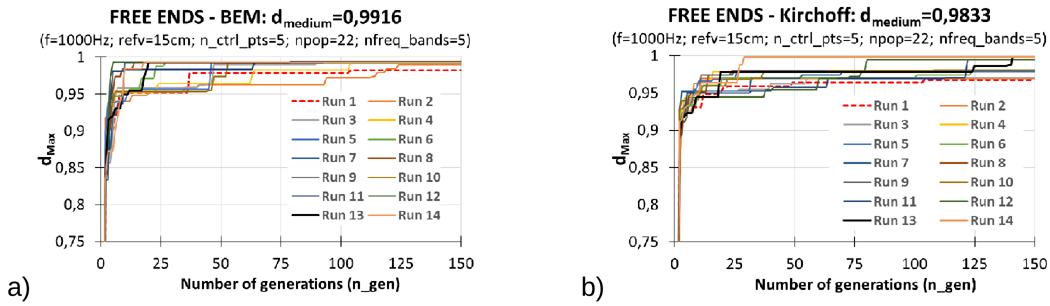


Figure 2: Several test runs (with the same input data): a) BEM analysis; b) Kirchoff's approximation.

In order to compare the results obtained with the two numerical approaches, the Figure 3 shows two solutions whose maximum diffusion coefficient resulting from the optimization process had a very close values, d_{Max}≈0.984, for the frequency band of 1000 Hz (obtained after 150 iterations, with 10 control points, ***refv***=0.15 m and with the "***compatible***" endpoints). It is worth mentioning that, in the case of Kirchoff's method, although the optimization is performed using that approximation, at the end the diffusion coefficient is calculated using the BEM, Figure 3e), to allow a correct assessment of the real performance of the diffuser.

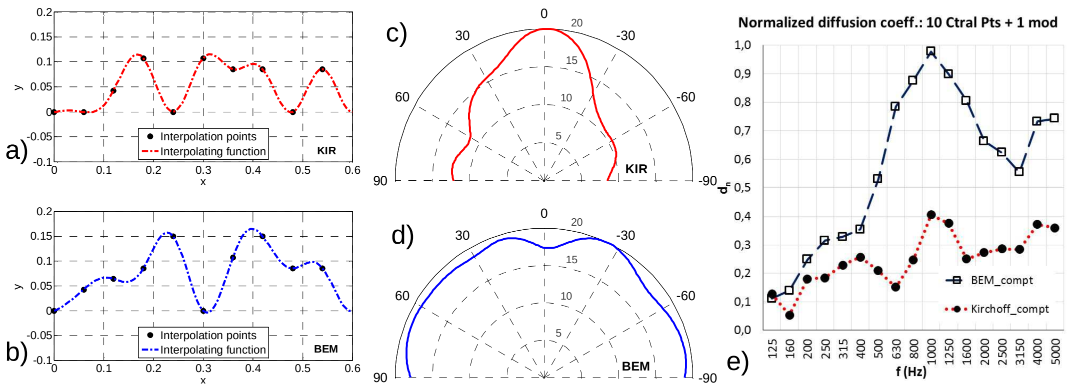


Figure 3: Optimized shapes. Geometric configuration: a) Kirchoff's approximation; b) BEM analysis; Polar at 1000 Hz: c) Kirchoff's approximation; d) BEM analysis; e) Normalized diffusion coefficient calculated with BEM analysis for the two configurations.

The conclusion is quite obvious: the Kirchoff's approximation, although allowing to obtain a similar maximum value in the frequency band of optimization, its diffusion coefficient value is quite low. This can happen because the interaction between the curved surfaces is not negligible and the Kirchoff's approximation may not be valid anymore.

3.3 EVALUATION OF THE INFLUENCE OF THE (MAXIMUM) HEIGHT OF THE CONTROL POINTS, *refv*

The Figure 4 shows a parametric study, using BEM analysis and the geometric constrain "***compatible***", to evaluate the influence of the maximum height of the control points of the RBFs (***refv***) on the maximum value of the optimized diffusion coefficient, for three different frequency bands: 500 Hz, 1000 Hz and 2000 Hz. From this figure, it can be seen that for higher optimization frequencies, lower diffusion coefficients are achieved (***d***$_{500Hz}$=0.994, ***d***$_{1000Hz}$=0.984 and ***d***$_{2000Hz}$=0.952), although the differences are small and occur for different ***refv*** values, as expected (***refv***$_{500Hz}$=0.27 m; ***refv***$_{1000Hz}$=0.12 m and ***refv***$_{2000Hz}$=0.18 m).

More difficult is the interpretation of the results in the case of the optimization for the frequency of 2000 Hz, whose maximum occurs for relatively high height (***refv***≈λ_{2000Hz}), higher than 1000 Hz case. However, this does not mean that the heights of the control points take the maximum value, since it may have 8 levels (3 bits) between 0 and ***refv***. To analyze this, among the different "runs", the ***refv*** configuration was chosen for which the diffusion coefficient was maximum and the result closest to the mean value, and the geometric configuration for each case is illustrated in Figure 5.

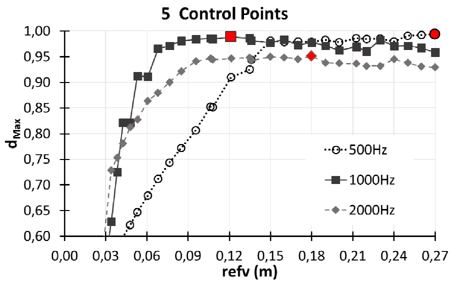


Figure 4: Average value of the maximum diffusion coefficient as a function of the maximum height of the control points – the red dots correspond to their respective maximums.

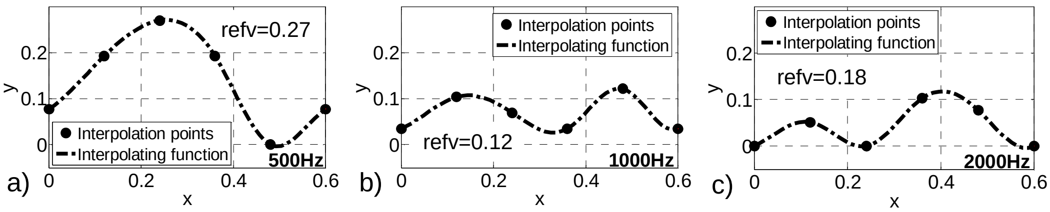
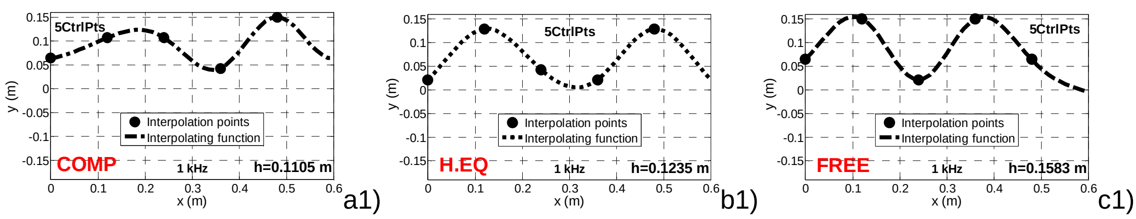


Figure 5: Geometric configuration for the maximum of average value of the diffusion coefficient: a) f=500 Hz, d_{max}=0.996; b) f=1000 Hz, d_{max}=0.984; b) f=2000 Hz, d_{max}=0.953.

The above Figure 5 shows that the curves have similar shapes, in particular for f=1000 Hz and f=2000 Hz. In the latter case, it is also verified that, although ***refv***=0.18 m (for d_{max}), the highest control point height is y_4=0.10 m, with Δy=0.121 m being the difference between the highest and the lowest point of the curve (depth of the diffuser) – for f=500 Hz, y_3=0.27 m and Δy=0.275 m and for f=1000 Hz, y_5=0.12 m and Δy=0.096 m.

3.4 EVALUATION OF THE INFLUENCE OF THE GEOMETRIC CONSTRAINTS

In order to analyze the influence of the geometric constraints of the extreme points of the surface to be optimized, 9 solutions are compared whose diffusion coefficient is very similar, d_{max}≈0.984 ± 0.2%. The resulting configurations are shown in Figure 6 and the normalized diffusion coefficients are presented in Figure 7. The geometric constraints do not appear to affect the overall shape of the diffusion curve in Figure 7, within the same number of control points, but the shape changes a lot with the increase of the number of control points.



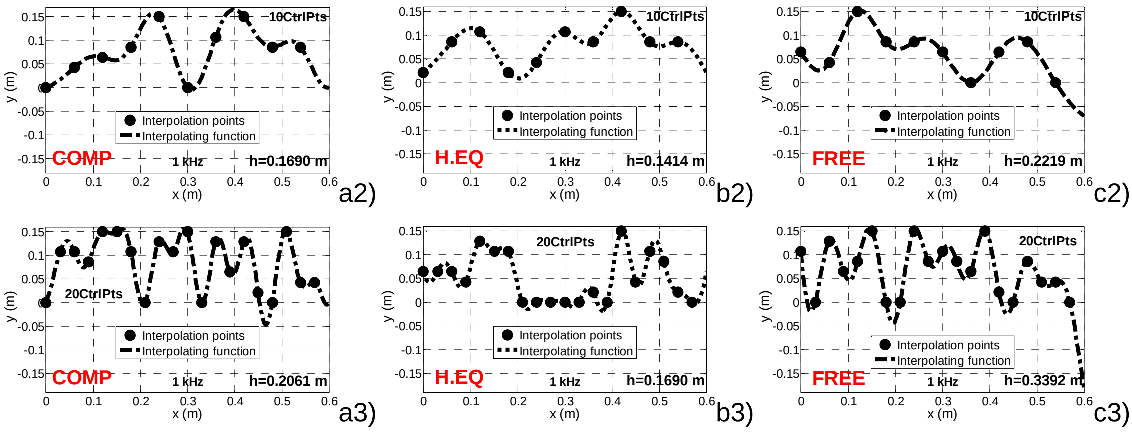


Figure 6: Optimized forms, resulting from the BEM analysis for f=1000Hz (*refv*=0.15m) whose diffusion coefficient is very similar (d_{max}≈0.984): #1) 5 control points; #2) 10 control points; #3) 20 control points; a#) Compatible endpoints; b#) Equal height endpoints; c#) Free endpoints.

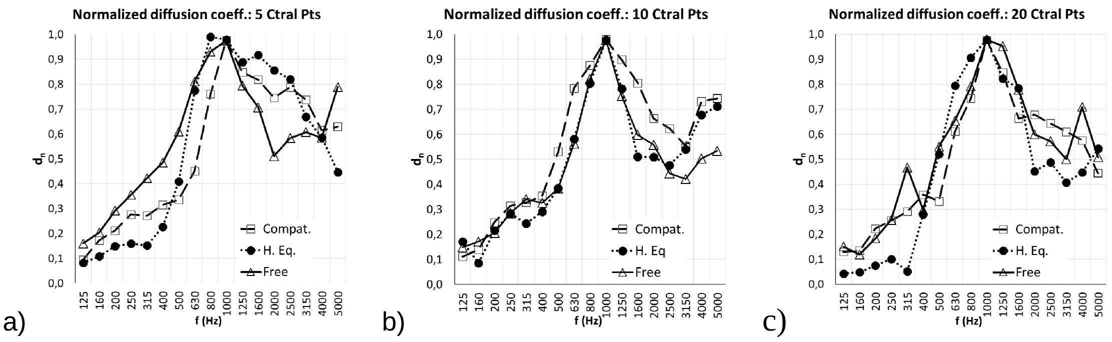


Figure 7: Normalized sound diffusion coefficient of optimized shapes from Figure 6.

3.5 EVALUATION OF THE INFLUENCE OF THE NUMBER OF CONTROL POINTS

For BEM analysis and ***refv***=0.15 m and f_{opt}=1000 Hz, the Figure 8 shows the influence of the number of control points on the (average) value of the maximum diffusion coefficient, on the dispersion of optimized values relative to the mean and also shows the influence on the depth of the curvilinear surface (understood as the difference between the highest and the lowest point of the curve).

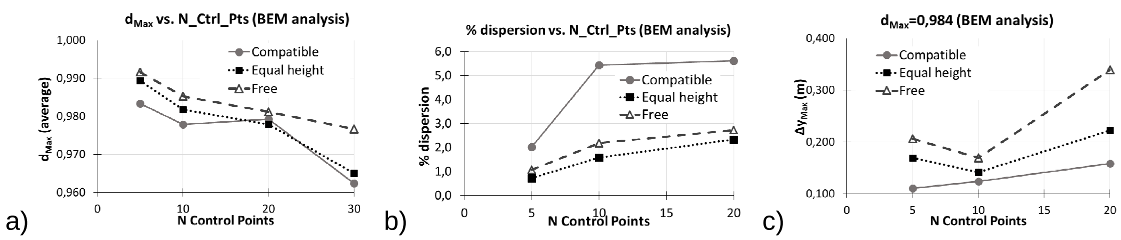


Figure 8: a) Average value of the d_{max} as a function of the number of control points; b) Relative dispersion of optimized values relative to the mean value for d_{max} (after 150 iterations) as a function of the number of control points; c) Variation the depth of diffuser with the number of control points, for a constant diffusion coefficient, d_{max}=0.984.

In general, it is possible to observe that the average value of the diffusion coefficient decreases with increasing RBF control points. On the other hand, the dispersion of results increases with increasing number of control points. It can also be observed that the number of control points also influences the maximum depth of the optimized surface. In fact, since the width of the diffuser is fixed (L = 0.60m), it was expected that the control points and the type of constraints imposed on the endpoints influenced the maximum depth of the diffuser - this is a very important aspect to take into account in the development of acoustic diffusers, either due to constructive practical issues or due to architectural impositions. Moreover, with the increase in the number of control points, the surface becomes more wrinkled (see Figure 6), which makes it more fragile and difficult to produce future prototypes. Thus, having the results obtained in mind, in the case of a BEM analysis, a small number of control points should be used in the optimization process.

3.6 THE INFLUENCE OF THE NUMBER OF MODULES

To evaluate the influence of the number of modules put side by side, the surfaces presented in Section 3.4 (*f*=1000Hz; ***refv***=0,15m; $\boldsymbol{d_{max}}$≈0.984) were simulated (BEM analysis). To illustrate the results, Figure 9 shows those obtained for the case where the endpoints have the same height, ("***h_equal***").

It can be observed that, as expected, the diffusion coefficient decreases as the number of modules increases, although it maintains the same appearance. However, the frequency at which the maximum diffusion occurs increases (slightly) with the number of modules.

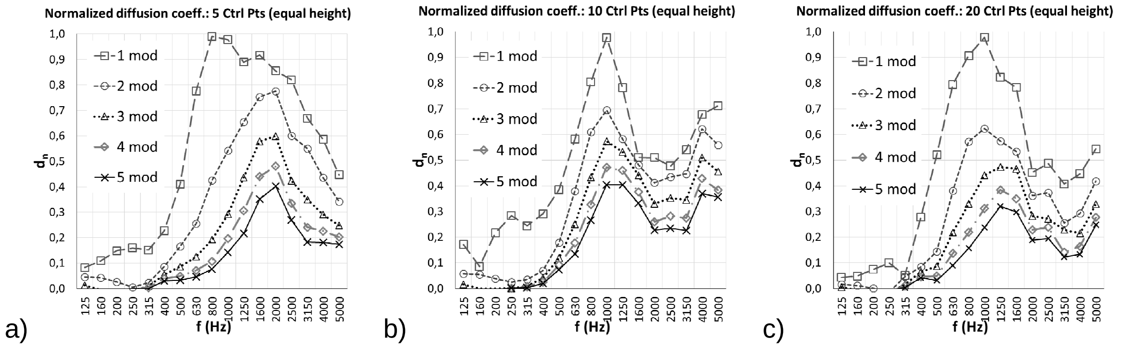


Figure 9: The normalized diffusion coefficient for various sets modules: a) 5 control points; b) 10 control points; c) 20 control points.

As can be seen in Figure 10, increasing the number of modules raises more lobes in certain directions, thus decreasing the diffusion coefficient.

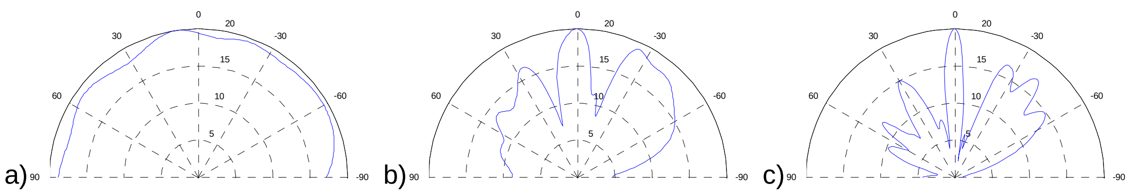


Figure 10: a) Effect of number of modules on the polar responses for the case of 10 control points and equal height endpoints: a) 1 module; b) 3 modules; c) 5 modules.

4 COMPARISON OF RESULTS BETWEEN BEM AND FDTD ANALYSIS

A comparison analysis was also performed in this work, trying to assess if the results provided by the BEM were in line with those computed independently by a totally different numerical method. For that, a FDTD model was used, based in the works of Redondo et al [5].

As for the simulations with the Finite Difference Time Domain technique (FDTD), only a small area is simulated around the test domain. The calculations have been carried out in a rectangular grid comprising about 400 by 700 elements, each about 1 cm in size. In order to operate with a Courant number as close to 1 as possible, the sampling frequency is 10 kHz. This numerical scheme is excited by a linear source placed on the right side of the integration area in which the specimen to be studied is placed. The impulsive responses corresponding to the pressure and velocity of the near-field particles on the far-field transformation line are recorded in order to obtain the sound pressure in the 37 far-field virtual microphones at a distance of 50 meters from the diffuser with angles between 90 and -90 degrees and with a pitch of 5 degrees. Ricker wavelets with central frequencies of 250 Hz and 2 KHz have been used, which covers all bands of frequencies of interest in the characterization of the diffuser. Figures 11 and 12 show a Ricker wavelets reflected from the two optimized surfaces of the Figure 6, a1) and a2), at different time instants. Further details of the FDTD simulation can be found in [5]. In some of the particular cases developed for the present work, the size of the mesh has been modified to better reproduce the geometries under investigation.

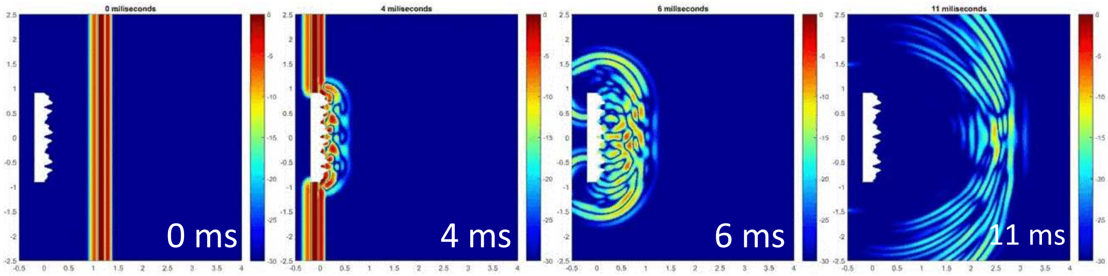


Figure 11: Plane wave reflected from 3 modules of the diffuser a1 (5 control points) using an FDTD model. The numbers indicate the time instant and frame order of the snapshots.

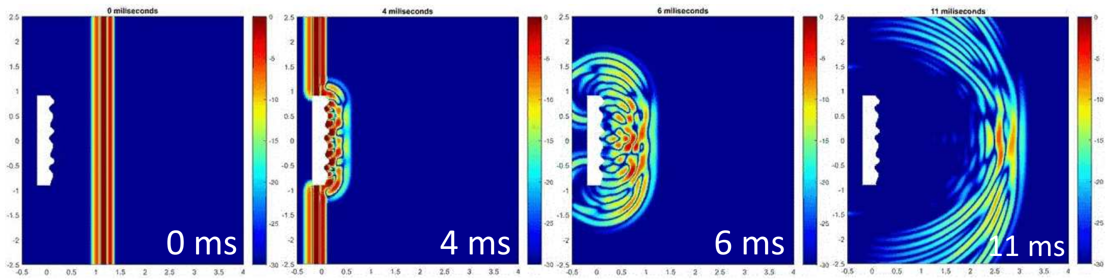


Figure 12: Plane wave reflected from a 3 modules of the diffuser a2 (10 control points) using an FDTD model. The numbers indicate the time instant and frame order of the snapshots.

Figure 13 presents the results (normalized diffusion coefficient) obtained by the FDTD and BEM analysis for two surfaces of Figures 6a1) and 6a2). Although the results show a significant difference between 160 Hz and 400 Hz, the methods of analysis have some similarity. The authors believe that these preliminary results show that further modelling work has to be done in order to adjust both implemented models, and take into account, for example, the use of the same number of individual frequencies per frequency band and the same geometric discretization of the surface.

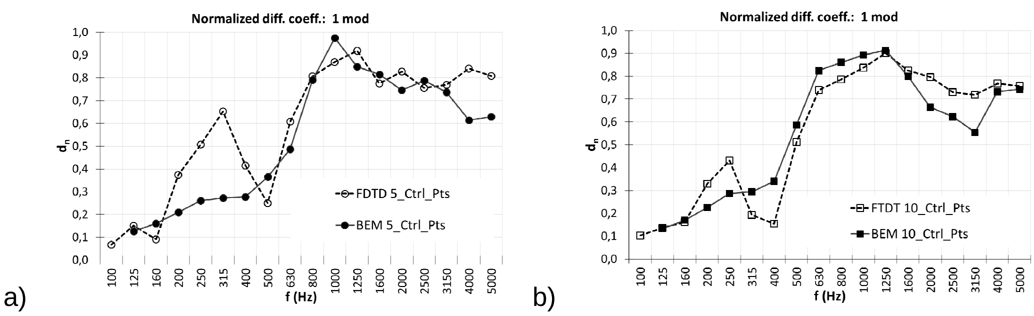


Figure 13: The normalized diffusion coefficient for the surface with BEM and FDTD analyses, for surface optimized for *f*=1000Hz, ***refv***=0.15m and endpoints "***compatible***": a) 5 control points; b) 10 control points.

5 CONCLUSIONS

The present paper presented an extensive study performed on the optimization of sound diffusers with curvilinear, organic forms. Both Kirchoff's integral approximation and the BEM were used and compared, allowing to conclude that Kirchoff's approximation results may not be reliable in the case of the analysed surfaces. A more detailed analysis was thus performed with BEM, also including geometric constraints at the two ends of the diffuser; this analysis allowed concluding that constraints related to the height and slope of the diffuser's end do not originate a significant degradation of its performance. The generation of the diffusive surface was also studied using different numbers of control points to define its geometry, and the authors concluded that a small number of points can be sufficient to provide good diffusion, while generating adequate surfaces for real production. Finally, a comparison with an FDTD code was performed, and good correlation was observed between the two methods for medium and high frequencies.

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