ACOUSTIC BEHAVIOUR OF 3D PERIODIC STRUCTURES COMPOSED BY PERFORATED RESONATORS

PACS:43.50.Gf, 43.28.Js, 43.20.Fn

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ABSTRACT

The periodic arrangement of acoustic scatterers is known to exhibit peculiar attenuation effects in ranges of forbidden frequencies. On the other hand, the presence of perforated surfaces, eventually complemented by acoustic absorbent materials, has been extensively studied to reduce acoustic reflections by plane surfaces. In the present work, the implementation of an efficient Boundary Element model (BEM) is adopted to numerically reproduce the acoustic behaviour of combined acoustic structures, composed by perforated resonators periodically arranged in space.

Different numerical examples will be presented in this work enabling the analysis of the acoustic behaviour of different arrangements and compositions of such absorbent and sound insulation structured materials.

Keywords: Sound absorption; Sound reduction; Periodic structures; Perforated surfaces; Absorbing materials; 3D Periodic BEM

RESUMO

A disposição periódica de dispersores acústicos é conhecida por exibir efeitos de atenuação peculiares em intervalos de frequências proibidas. Por outro lado, a presença de superfícies perfuradas, eventualmente complementadas por materiais absorventes acústicos, tem sido amplamente estudada para reduzir as reflexões acústicas pelas superfícies planas. No presente trabalho, a implementação de um modelo de Elementos Fronteira (BEM) é adotada para reproduzir numericamente o comportamento acústico de estruturas acústicas combinadas, compostas por ressoadores perfurados dispostos periodicamente no espaço.

Serão apresentados diferentes exemplos numéricos neste trabalho, permitindo a análise do comportamento acústico de diferentes arranjos e composições de materiais estruturados isolantes absorvente e insonorizado.

Palavras-chave: Absorção sonora; Redução sonora; Estruturas periódicas; Superfícies perfuradas; Materiais absorventes; BEM 3D Periódico

INTRODUCTION

Periodic arrangements of acoustic scatterers are known to exhibit peculiar attenuation effects in ranges of forbidden frequencies. In fact, this is one of the successful approaches that has been observed in the search of acoustic metamaterials [1]. Romero-García et al. [2] studied an acoustic metamaterial made of a two-dimensional periodic array of scatterers exhibiting multi-resonant acoustic behaviour in the long wavelength regime. Sound propagation through periodic structures was also experimentally demonstrated in an array of Helmholtz resonators [3]. These concepts have also been explored by Claeys et al. [4]. They demonstrated the numerical design and performed an experimental validation of a periodic arrangement of cells with local resonant structures to introduce stop band behaviour, and then the authors tried to improve the acoustic behaviour of the 3D printed prototype. The acoustic stop bands with increased noise reduction were numerically predicted through unit cell modelling and then a finite element model was applied to predict the performance of the demonstrator [4].

On the other hand, the presence of perforated surfaces, eventually complemented by acoustic absorbent materials, has been extensively studied to reduce acoustic reflections by plane surfaces. Cameron [5] proposed a new concept for a roof system for a passenger car, in which the usual components where replaced by a sandwich panel with a perforated interior face sheet, to allow fluid interaction, and a structural foam layer, to provide sound absorption in the interior of the panel. Perforated surfaces were also explored by Ruiz et al. [6], making use of microperforated panels that dissipate energy of sound waves inside the perforations while replacing foams, used to generate acoustic absorption. Houston et al. [7] investigated the attenuation of pressure fluctuations by a group of multi-perforated panel absorbers and Helmholtz resonators by performing computational simulations and experimental works. Their results have demonstrated that Computational Fluid Dynamics can predict the noise attenuation from Helmholtz resonators with good accuracy.

In the present work, the implementation of an efficient 3D Boundary Element Model is adopted to numerically reproduce the acoustic behaviour of combined acoustic structures, composed by perforated resonators (incorporating absorbent fibrous/porous material) periodically arranged in space. A system composed by periodic arrangements of resonators was already presented by Boutin [8]. The aim was to define porous media microstructures in which inner resonance phenomena might occur. First, a periodic medium consisting in damped Helmholtz resonators embedded in a porous matrix was developed, then the features of acoustic wave propagation were determined. Richoux et al. [9] studied the propagation of high amplitude acoustic pulses in a lattice of Helmholtz resonators connected to a waveguide. A new method was developed to take into account both the nonlinear wave propagation and the different mechanisms of dissipation. A good agreement between numerical and experimental results was obtained. These studies showed the great importance of viscous losses in the generation of acoustic solitary waves in periodic locally resonant structures. The work by Sánchez-Dehesa et al. [10] can also be referred, since the authors studied the use of sonic crystal barriers made by perforated metallic shells filled with rubber crumb. The authors concluded that the combination of sound absorption by the porous cylinders and their periodic distribution lead to interesting attenuation properties in order to be used as protection devices in noisy environments.

Different numerical examples will be presented in this work, enabling the analysis of the acoustic behaviour of different arrangements and compositions of such absorbent and sound insulation structured materials. The governing equations and the numerical formulation will be briefly introduced, namely the 3D Boundary Element Method and its application to periodic structures which can be very efficient in terms of computational requirements. Numerical results were computed in the frequency domain, regarding the Insertion Loss determined by the periodic scatterers, being useful to analyse the influence of different geometric and acoustic parameters.

GOVERNING EQUATIONS

The partial differential equation referred as the Helmholtz equation can be used to mathematically describe, in the frequency domain ($\omega = 2\pi f$), the sound propagation in a 3D acoustic space,

$$\nabla^2 p(\mathrm{x},\omega)+\left(\frac{\omega}{c}\right)^2 p(\mathrm{x},\omega)=0\,, \tag{1}$$

with c corresponding to the sound propagation velocity, $p(\mathrm{x},\omega)$ representing the acoustic pressure in a point $\mathrm{x}\equiv(x,y,z)$ of the domain, and, in 3D problems, $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$.

When a harmonic point source, located at $\mathrm{x}_0\equiv(x_0,y_0,z_0)$, excites the acoustic propagation domain, the fundamental solution for the sound pressure (or Green's function), at a generic point $\mathrm{x}\equiv(x,y,z)$, can be written as:

$$G(\mathrm{x},\mathrm{x}_0)=\frac{\mathrm{e}^{-\mathrm{i}kr}}{4\pi r}\,,\quad \text{with } r=\sqrt{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2}\,. \tag{2}$$

In order to be able to address different acoustic problems, distinct boundary conditions can be imposed at the problem interfaces, namely, Dirichlet conditions, with known pressure values at the boundary ($p(\mathrm{x})=\bar{p}(\mathrm{x})$); Neumann conditions, when the normal velocity to the boundary is known ($v(\mathrm{x})=\bar{v}(\mathrm{x})$); and Robin conditions, when one can establish a relation between the pressure and the velocity, using the surface impedance of the boundary ($p(\mathrm{x})/v(\mathrm{x})=\bar{Z}(\mathrm{x})$). This condition enables a useful approach to model distinct absorbent solutions, namely, in the scope of the present work, a system composed by a perforated panel and located over an air gap, filled with an absorbing material. This boundary condition can be prescribed as a frequency-dependent impedance by:

$$\bar{Z}=Z_{panel}+Z_{filled\_air\_gap}=Z_{panel}-\mathrm{i}\,Z_{c_a}\cot(k_a D). \tag{3}$$

Here, Z_{panel} corresponds to the surface impedance of a perforated panel, which can be found in reference works ([11] [12]), and, for the porous material, Z_{c_a} and k_a characterize, respectively, the characteristic impedance and the wavenumber or propagation constant within the air gap. In this work, these characteristics are determined by standard models for porous materials, such as the one derived by Delany-Bazley [13]. This procedure represents only an approximation to the real behaviour of the system (since it considers a locally-reacting system instead of a bulk-reacting system), but in fact it allows very efficiently incorporating this behaviour within a more complex model.

NUMERICAL SOLUTION

Boundary Element Method - BEM

A 3D model based on the BEM [14] was implemented to numerically solve the previously mentioned Helmholtz differential equation (Eq. (1)), presenting clear advantages in the modelling of 3D exterior acoustic problems, in the presence of a generic obstacle and any reflecting or absorbing surfaces. After applying the reciprocity theorem to Eq. (1), the following classic boundary integral equation can be obtained,

$$C(\mathrm{x}_0)\,p(\mathrm{x}_0)=-\mathrm{i}\rho\omega\int_\Gamma G(\mathrm{x},\mathrm{x}_0)\,v_n(\mathrm{x})\mathrm{d}\Gamma-\int_\Gamma\frac{\partial G(\mathrm{x},\mathrm{x}_0)}{\partial n}\,p(\mathrm{x})\,\mathrm{d}\Gamma+p_{inc}(\mathrm{x}_\mathrm{s},\mathrm{x}_0)\,, \tag{4}$$

where Γ is the boundary surface; $p(\mathrm{x})$ and $v_n(\mathrm{x})$ represent the acoustic pressure and the normal component of the particle velocity, respectively; and $p_{inc}(\mathrm{x}_\mathrm{s},\mathrm{x}_0)$ is the incident field regarding the acoustic pressure generated by each real source placed at position $\mathrm{x}_\mathrm{s}\equiv(x_s,y_s,z_s)$.

The coefficient $C(\mathrm{x}_0)$ depends on the boundary geometry at the collocation point x_0, and equals ½ for a smooth boundary.

After ascribing the adequate boundary conditions, the boundary surface can be discretized into segments (boundary elements), and the appropriate interpolation functions within each segment are evaluated (in this case, constant interpolation functions were taken), leading to a linear system of equations. The solution of this system of equations allows computing the acoustic pressure and the normal velocity values at each boundary segment. Then, the sound pressure at any point of the domain can be evaluated by adequately using the previous boundary integral equation.

Periodic Boundary Element Method and Adaptive Cross Approximation

In the specific case of finite periodic structures, represented by equal scatterers, i.e. having the same geometry and properties, and being discretized with the same number of boundary elements, then a complete interaction matrix can be formed with the resulting system of equations. If periodicity exists within the spatially distribution of the scatterers, then some of the blocks of that matrix are exactly repeated and so they can be calculated only once. The features resulting from the spatial periodicity of the physical system have also been reported by Karimi et al. [15], when dealing with a periodic BEM formulation for regular phononic crystal structures, and allow immediate simplification of the problem if the structure to be analyzed is composed by a scatterer (or group of scatterers) that is repeated along one direction. However, note that this strategy is only valid when a completely regular periodic system is adopted, not incorporating any interior defects or spatial modifications.

An additional technique, the Adaptive-Cross-Approximation (ACA), has been applied in the scope of this work, considerably improving the efficiency of the BEM model, regarding CPU times and RAM memory requirements. The ACA technique is specifically applied to approximate all blocks of the BEM matrix, except those located along the diagonal; in that process, the calculation of only a much reduced number of rows and columns of those blocks is performed, thus consisting of a low-rank approximation of the original block. The solution of the resulting equation system requires an iterative solver to be used. Here, the Generalized Minimal Residual Method (GMRES) solver, together with a block-diagonal preconditioner, were applied.

The implemented model has been numerically verified by comparing its accuracy with the results of other established numerical models, having demonstrated to be very precise and remarkably competitive in what concerns to the required computational needs (CPU times and RAM memory).

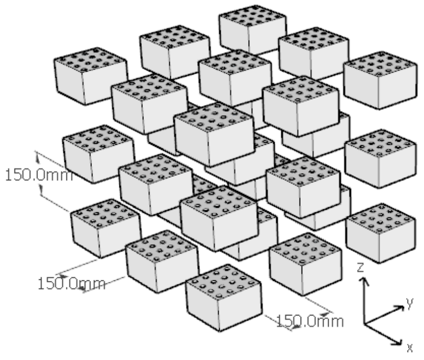
NUMERICAL APPLICATIONS

A numerical example has been selected to illustrate the proposed modelling strategy described in the previous sections. Therefore, consider a large set of discrete scatterers, periodically arranged in such a way to modify the sound pressure field emitted by a sound source and provide sound attenuation. The configuration of the system to be modelled will be described, and some results illustrating the computed acoustic field in the presence of a metamaterial composed by a periodic set of resonators will be presented. A preliminary parametric analysis of the acoustic behaviour of different sets of periodic resonators is also presented.

Periodic structure description

Consider a periodic set of individual resonators, spatially distributed in a square lattice arrangement, as illustrated in Figure 1a, in a partial view of just 27 scatterers (i.e. a regular distribution of resonators, with equal spaces of 150 mm between elements, along the three orthogonal directions). These elements represent individual resonators (with dimensions of 150*150*100 mm^3), formed by 4 lateral rigid and totally reflective walls, a rigid base and a top perforated panel, being filled with acoustically absorbent material in the air gap. This unitary perforated cell, which is periodically repeated in the final structure, can be observed in a cross section view in Figure 1b. For this first set of simulations, the perforated panel on the top of each resonator is characterized by a panel thickness of $L0 = 15\,\mathrm{mm}$ with a perforation diameter of

$2r = 2*5\,\mathrm{mm}$ and $\varepsilon = 5\%$ of total perforation rate. The absorbing material filling the air gap has a total thickness of $D = 25\,\mathrm{mm}$, presents an air flow resistivity value of $\sigma = 28000\,\mathrm{Ns/m^4}$, corresponding to a conventional mineral fibre (its sound absorption coefficient depends on the frequency being analysed, and a maximum value of almost 1.0 at the resonance frequency of about 470 Hz can be estimated as previously referred).



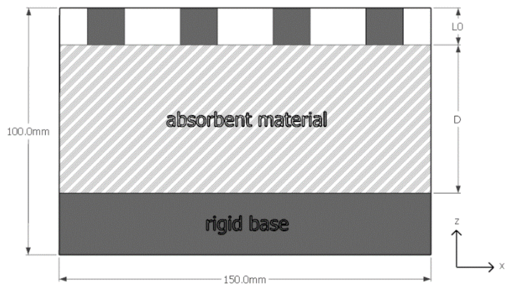


Figure 1: Configuration of the model of the 3D periodic structure: a) view of a small part of the structure; b) vertical cross section of the unitary perforated resonator with absorbing material in the air gap.

An array of 7*7*4 elements, spatially organized in a square lattice along the x, y and z directions and occupying an area of approximately 2*2 m² – in a horizontal cross section – is considered. This large set of organized resonators is excited by an almost plane wave, generated by a point sound source located at some distance above this structure, at point $(1\,\mathrm{m};\,1\,\mathrm{m};\,200\,\mathrm{m})$.

Sound Pressure Field – Numerical results

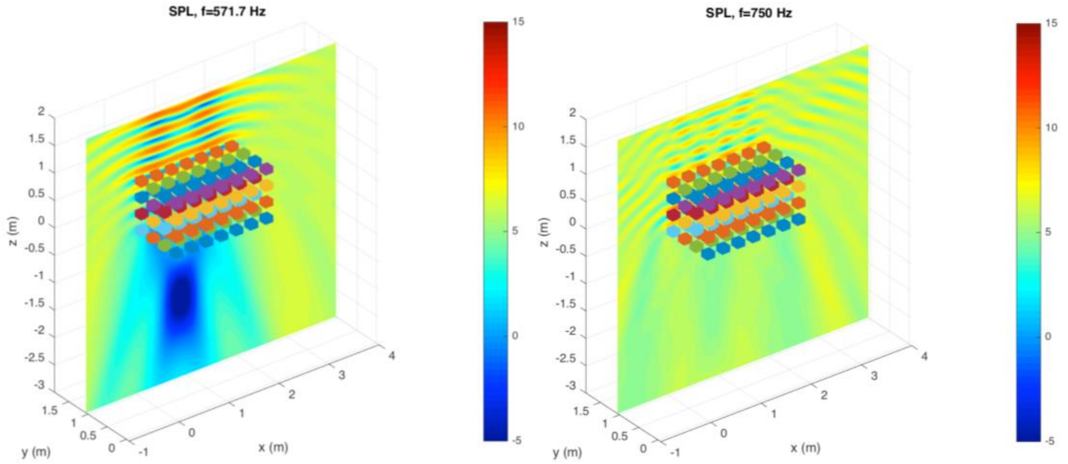
The described 3D periodic BEM formulation is firstly used to compute the sound pressure field near the periodic array of resonators, embedded in an unbounded acoustic medium. For an adequate discretization of each unitary cell, a set of 304 nodes and triangular boundary elements was adopted. When the periodic arrangement of resonators is considered, a large-scale problem arises, with a total number of almost 60000 discretization nodes. It must be referred that the use of an efficient numerical technique, like the periodic BEM with the ACA technique, enables modelling this large acoustic problem.

The Sound Pressure Levels (SPL) have been computed over a grid of numerical receivers, corresponding a vertical plane located at $y = 1.05\,\mathrm{m}$ (Figure 2). Two different exciting frequencies have been selected to present these results, namely at 571.7 Hz and 750 Hz, respectively in Figures 2a-2b. These frequency values have been chosen in order to illustrate the expected results, related to different acoustic behaviours of the periodic structure; specifically, the frequencies corresponding to: a) the estimated band gap for an interval between centres of the scatterers of 300 mm ($c/2a = 571.7\,\mathrm{Hz}$, being *a* the lattice constant between centers of the scatterers, as in a sonic crystal-type structure), and b) a frequency for which the impinging wave is not much attenuated by the periodic structure (respectively in Figures 2a-2b).

The results illustrated in these Figures 2a-2b demonstrate different acoustic patterns, namely the interaction between incident waves with the periodic structure. In Figure 2a, for the frequency of 571.7 Hz (where a stop band is expected to occur due to the sonic crystal-like geometry), a significant decrease in the SPL is clearly noticeable over a limited region behind the set of scatterers. On the other hand, at 750 Hz (Figure 2b), there is almost no SPL reduction on the sound pressure field between the upper and lower regions of the periodic structure, as expected.

Insertion Loss results – Parametric analysis

Additional numerical results were also determined so as to preliminarily analyze the behaviour of such a periodic structure in terms of sound attenuation over an enlarged range of exciting frequencies. In these parametric analyses, a range of frequencies from 30 to 1500 Hz is observed, and the acoustic Insertion Loss (IL) is evaluated at a grid of receivers located over a vertical plane (x-z plane, as in Figure 2) behind the first horizontal layer of elements, and by numerically comparing average SPL with and without the presence of the periodic structure.

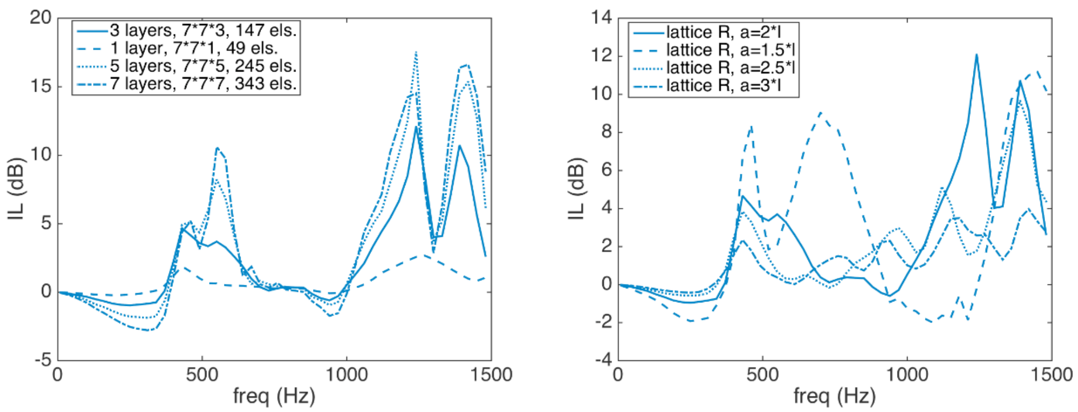


a) b)

Figure 2: Sound Pressure Levels (SPL, in dB) computed with the periodic BEM formulation in a vertical plane (y=1.05m), for two different exciting frequencies: a) 571.7Hz; b) 750Hz.

. Geometric parameters related to the periodic system

First, two parameters that geometrically define the periodic structure can be analyzed in Figure 3, namely the number of layers with 7*7 scatterers that are superposed along the vertical direction, from 1 to 7 layers (Figure 3a), and the distance defining the spatial periodicity of the resonators, known as lattice constant, a, in relation to the size of the resonator, l (Figure 3b). As expected, above 3 layers of scatterers it is possible to distinguish stop bands characteristic from periodic arrangement of elements (Figure 3a). In Figure 3b, the influence of the proximity between elements is visible, leading to stop band frequencies significantly distinct (from 384 Hz to 762 Hz).



a) b)

Figure 3: Insertion Loss (dB) curves for different geometric configurations of the periodic arrangement of resonator: a) varying number of layers with resonators, 7*7*1 to 7*7*7; b) varying lattice constants, a, in relation to the size of resonator, l.

. Parameters related to the acoustic resonator

Next, some parameters that acoustically determine the behaviour of each acoustic resonator are analysed, having in mind feasible ranges of values for each parameter. Since these parameters modify the sound absorption curve, its curve is also presented along with the Insertion Loss curve for each solution. Although other parameters were numerically assessed, only three of them are here illustrated, specifically, the perforation ratio and perforation diameter (Figures 4a-4b), the thickness of the absorbent fibrous material filling the air gap (Figures 4c-4d) and the thickness of the perforated panel on top of each resonator (Figures 4e-4f). Observing the IL curves on the right side of Figure 4, the main conclusion is that the system being studied has its acoustic behaviour affected by both the multiple scattering on the periodically arranged elements (as in sonic crystals) and also by acoustic absorption that occurs on the perforated resonator with absorbent material.

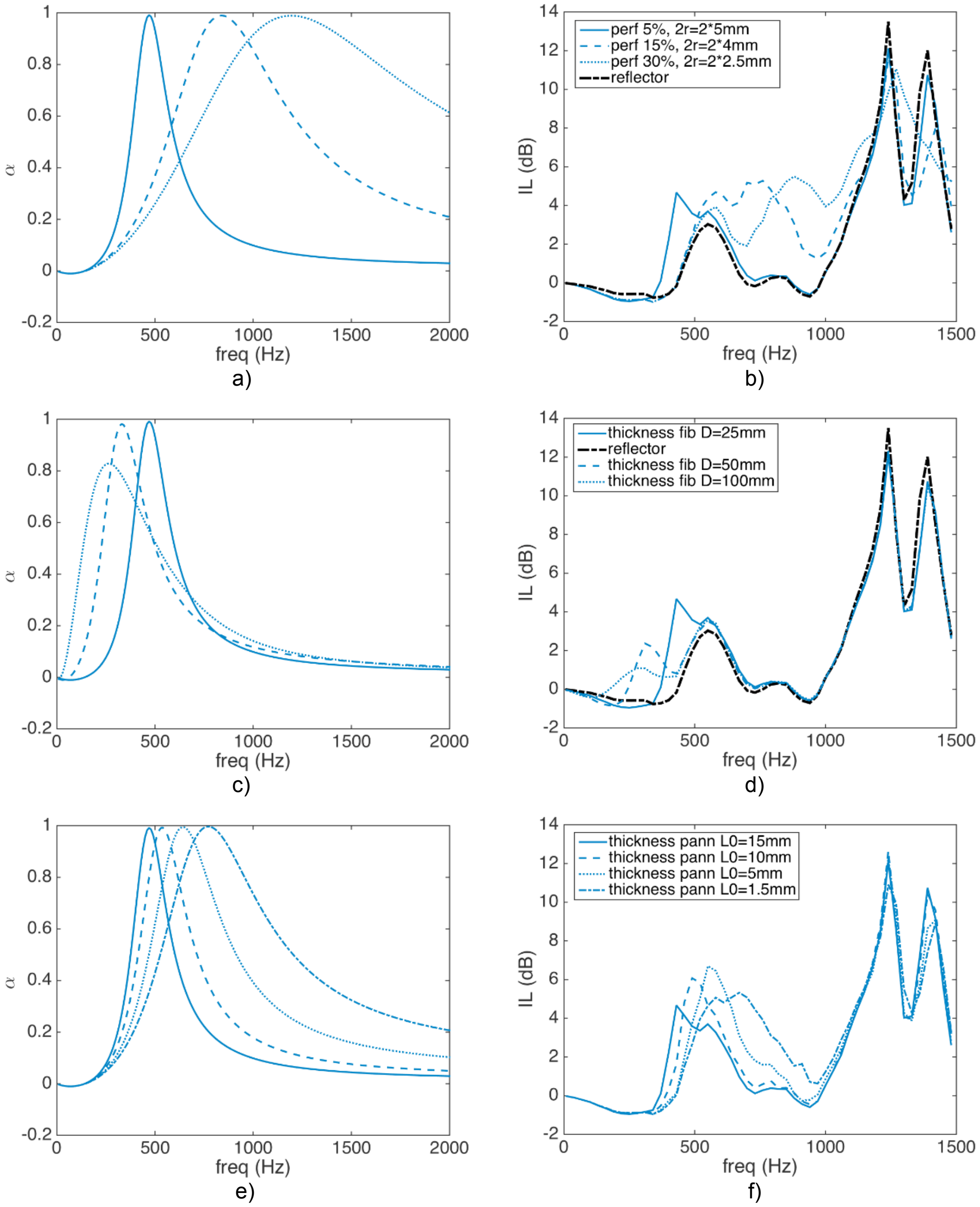


Figure 4: Sound absorption (on the left) and Insertion Loss (dB) (on the right) curves for different acoustic surfaces of the unitary resonator: a), b) varying perforation rate and perforation diameter; c), d) varying thickness of the absorbent material in the air gap; e), f) varying thickness of perforated panel.

CONCLUSIONS

In this work, a numerical model based in the Boundary Element Method (BEM) has been briefly described. An impressively efficient implementation of the model has been attained, making use of the periodicity of the physical system and incorporating an Adaptive Cross Approximation technique. An application example was here presented to illustrate the potentialities of this numerical tool in the modelling of the 3D sound pressure field scattered by a large number of resonators, periodically arranged in a metamaterial-type 3D structure. The results have shown the sound pressure field in the presence of an array of resonators at different exciting frequencies and the insertion loss provided for different configurations was also computed along a range of frequencies up to 1500 Hz. As it was expected, the characteristic frequency bandgaps have been identified, showing an increase of acoustic attenuation due different acoustic phenomena, related to periodic structures' and acoustic absorbing materials' concepts.

AKNOWLEDGEMENTS

This work has been framed within the POCI-01-0247-FEDER-017759 (SmartCore) Project, funded by the COMPETE 2020, Portugal 2020 and FEDER funds. This work was partly financed by FEDER funds through the Competitivity Factors Operational Programme - COMPETE and by national funds through FCT – Foundation for Science and Technology within the scope of the project POCI-01-0145-FEDER-007633. This work was also developed in the scope of COST (European Cooperation in Science and Technology) through the COST Action CA15125 – DENORMS: "Designs for Noise Reducing Materials and Structures".

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