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PERFORMANCE OF SOUND DIFFUSERS IN THE LOW FREQUENCY RANGE

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ABSTRACT

The present work tries to study the relationship of the lowest frequency at which significant diffusion occurs with the dimensions, depth of the wells and total width, of a sound diffuser. In order to do so, we have implemented two different numerical schemes based on the Boundary Elements Method and on the Finite Difference Time Domain Method. Preliminary results reveal that the frequency range can be extended to low frequencies if a correct ratio between dimensions is chosen.

RESUMEN

El presente trabajo trata de estudiar la relación de la frecuencia más baja en la que se produce una difusión significativa con las dimensiones, la profundidad de los pozos y la anchura total, de un difusor de sonido. Para ello, hemos implementado dos esquemas numéricos diferentes basados en el Método de los Elementos de los Límites y el Dominio de las Diferencias Finitas. Los resultados preliminares revelan que el rango de frecuencia se puede extender a frecuencias bajas si se elige una proporción correcta entre las dimensiones.

1-INTRODUCTION

Sound diffusers are surfaces on which the sound is reflected in a non-specular way, that is, Snell's law is not satisfied. Such a device is frequently used in room acoustics to improve the diffuseness of the sound field and to reduce echoes and focalizations. The first ones were proposed by Schroeder [1] and consist of a set of wells with different depths that come to modify the phase of sound. Due to this, they are known as Schroeder or phase diffusers. When the variation of the depth of the wells is only in one direction, the resultant diffusers are called 1D Schroeder diffusers.

Among the different types of Schroeder diffusers, the most popular is the 7 wells QR (quadratic diffuser). In this study we will focus in this particular case. Figure 1 illustrates this sound diffuser.

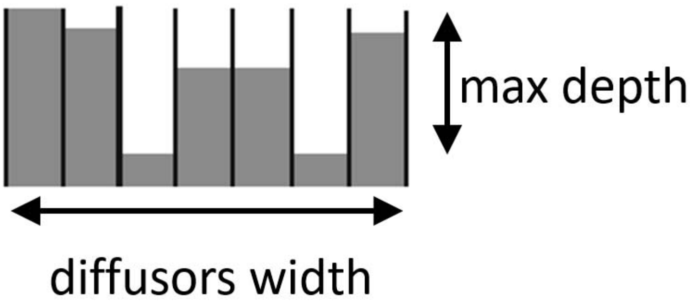

Figure 1. Section of a Quadratic Residue (QR) diffuser of 7 wells.

In order to quantify the performance of such a device, there are two different strategies that can be used, both standardized by ISO (International Organization for Standardization) [2-3]. In this work we will used the standard ISO 17497-2 2012 [3]. It is based on the measurement of the acoustic pressure of the reflected sound over a range of angles, between −90◦ to 90◦ in steps of 5◦ (37 measurements in total). For this purpose a microphone is sequentially positioned along a semi-circumference centered in the middle point of the test sample, which is composed by an array of three diffusers (with seven wells each one). The original signal has to be windowed in order to separate the reflected sound from the direct sound. The parameter measured using this technique is known as the diffusion coefficient:

$$d'_j = \frac{\left(\sum_{i=1}^{n} p_{ij}^2\right)^2 - \sum_{i=1}^{n}\left(p_{ij}^2\right)^2}{(n-1)\sum_{i=1}^{n}\left(p_{ij}^2\right)^2} \quad (1)$$

where d'_j is the diffusion coefficient for the j-th one-third octave band considered, p_{ij} is the reflected sound pressure for the j-th one-third octave band considered at the i-th measurement position, and n is the number of measurement positions (n = 37). This diffusion coefficient has to be averaged for different incidence angles (in our case 5). To normalize this diffusion coefficient from zero to one it is compared with a flat surface. The purpose of normalization is to remove edge diffraction scattering effects due to the limit size of the sample under analysis. The normalized diffusion coefficient, d_j, for the j-th one-third octave band considered, is defined as:

$$d_j = \frac{d'_j - d_{j,ref}}{1 - d_{j,ref}} \quad (2)$$

where $d_{j,ref}$ is the diffusion coefficient of a flat panel for the j-th one-third octave band considered. As a result, d_j is equal to zero for all frequencies in the case of a flat surface.

All these coefficients require tedious measurements in an anechoic chamber. To overcome this problem, numerical methods can be used. In particular FDTD is a well-established method that can be used for this purpose [4]. Figure 2 illustrates the simulation scheme used for this paper. Several techniques has been used in order to obtain the far field reflected by the sound diffuser (NFFFT: near field to far field transformation) and to remove the incident sound from the simulations (TFSF: total field – scattered field formulation). Further details of the simulations can be found at [4].

In a QR diffuser the depths of the wells are calculated by the following equation [5]:

$$S_n = n^2 \, mod N \quad (3)$$

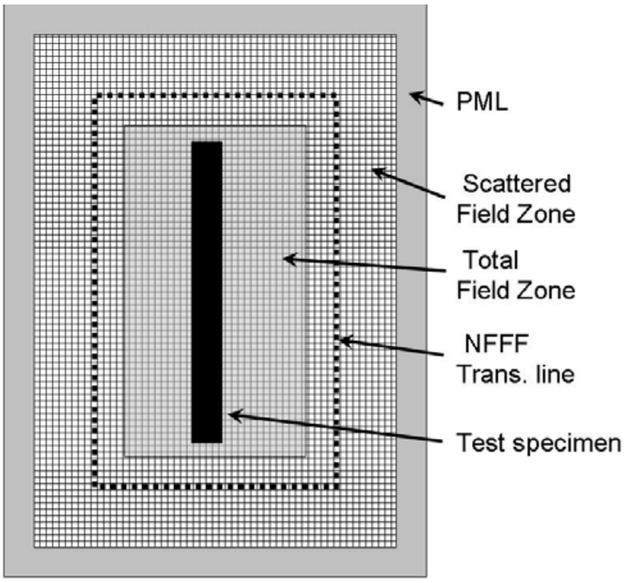

Figure 2. FDTD Simulation Scheme (reference flat panel). The figure shows the two simulation zones (total field zone and scattered field zone). It is as well illustrated the measurement points used to transform to far field (see text). The PML is an absorbing termination to simulate anechoic environment.

where N is a prime number on which the sequence is based (7 in this case) and n is the index to element Sn of the sequence. In our case (7 wells) Sn is [0 1 4 2 2 4 1] generated from n=[0 1 2 3 4 5 6]. The depths of each well is calculated from the sequence as follows.

$$D_n = \frac{S_n\, c}{2\, N\, f_0} \quad (4)$$

where D_n is the depth of the n-th well (we use here capital letters to avoid confusion with the diffusion coefficient), c is the speed of sound and f_0 is the so called design frequency. For frequencies below f_0 the diffuser tends to behave as a flat surface causing 0 diffusion. As a result, the lowest frequency where significative diffusion is achieved, $f_{\min D}$, (D stands for the effect of the depth of the diffuser) coincide with the design frequency, i.e.:

$$f_{\min D} = f_0 = \frac{S_n\, c}{2\, N\, D_n} \quad (5)$$

If the previous equation is particularized to the deepest well it reads as follows ($S_{n\,max}$=4)

$$f_{\min D} = f_0 = \frac{97.43}{D_{n\,max}} \quad (6)$$

where D_n max is the maximum depth of the wells, in other words, the diffusers depth.

Up to this point, the effect of the limited width of the diffusers has not been considered. In reference [6] this was considered for the first time. According to that paper, there is an additional limit that can increase f_{min}:

$$f_{\min L} = \frac{c}{\mathrm{L}} \quad (7)$$

where L is the total width of the diffuser.

In this paper we will consider the accuracy of equations 6 and 7 to evaluate the lowest frequency at which the diffuser is efficient. We will considerer that a value of the diffusion coefficient larger than 0.4 correspond to significative efficiency of the diffuser.

2-RESULTS

We have performed a systematic calculation of the effect of the depth and the width of the diffuser. For convenience the maximum depth considered has been 0.34 m (the wavelength at 1kHz) and the width has been limited to 1 m. Next figure illustrates the performance of two QR (7 wells) diffusers with different widths (1 m and 0.6 m) obtained with a FDTD simulation as a function of its maximum depth. Generally speaking, the thinner diffuser has lower values of the diffusion coefficient. However, the lowest frequency, f_{min}, seems to have very similar behaviour.

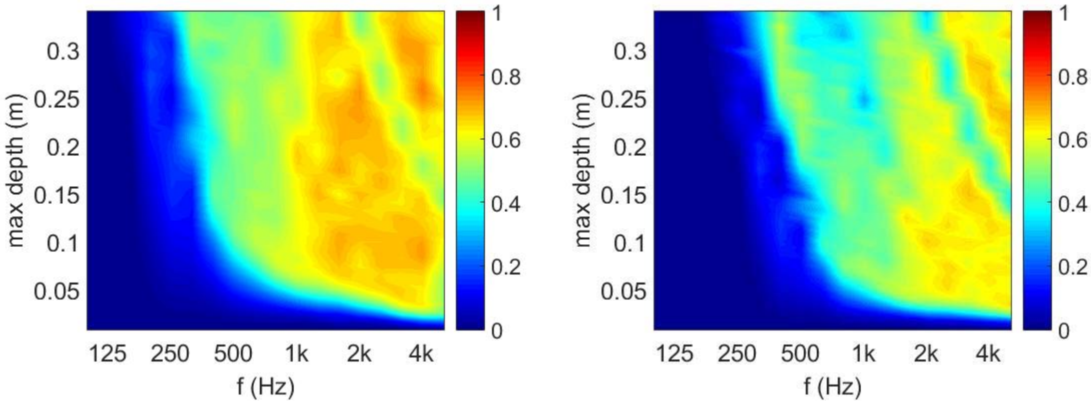

Figure 3. Normalized diffusion coefficient for two different 7 wells quadratic residue diffusers. X axis: frequency (Hz), Y axis: total depth of the diffuser. Left plot corresponds to a diffuser width of 1 meter and right plot to a 0.6 m one.

In order to better study the lowest frequency, figure 4 illustrates f_{min} for both cases. To obtain this plot we have found, for each possible depth between the limits (0 to 32 cm), the lowest frequency where significative diffusion is achieved. This implies to set an arbitrary limit for the diffusion coefficient to be considered as significative. Given than the maximum value of the diffusion coefficient is about 0.8, assuming a limit of 0.4 (i.e, 50% of the max) seems to make sense.

So, figure 4 illustrates f_{min} obtained from the simulations together with the theoretical limits commented above (equations 6 and 7). We can conclude that the limitation due to the width of the diffuser is no relevant and actually in both cases f_{min} follows the theoretical value due to the depth effect, i.e., $f_{min\ D}$. However this does not happen for shallow diffusers, when the maximum depth is lower than 0.2 m.

It is quite remarkable that there is a particular range of values of the maximum depth for which f_{min} is about one octave bellow $f_{min\ D}$ (see for instance the case of a 1m width diffuser for a maximum depth below 0.15 m). Actually, this was already pointed out by Schroeder in [7].

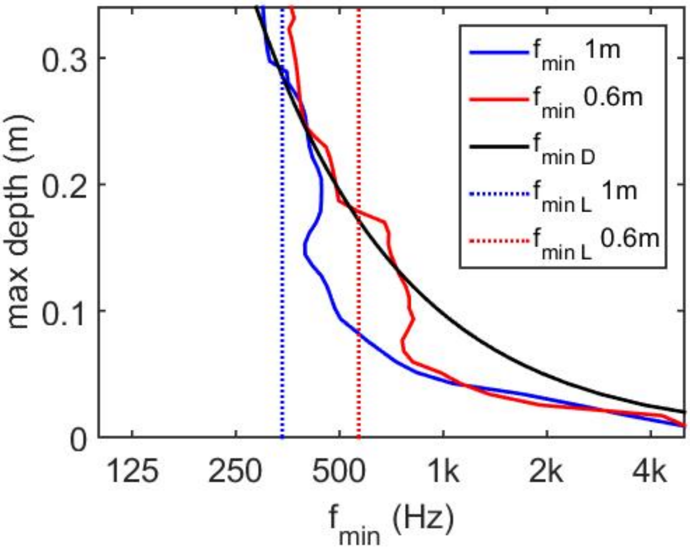

Figure 4. f_{min} for two different values of the width (1m (blue) and 0.6m (red)), Continuous lines . Theoretical f_{min} due to the limited depth of the diffuser (6), black line. Theoretical f_{min} due to the limited width of the diffusers (7), dotted lines.

Taking into account the differences between the numerical results for f_{min} and the theoretical predictions, we propose a simple expression to describe f_{min}, which describes reasonably well the observed numerical results, namely:

$$f_{\min F} = \frac{f_{\min D}}{1+e^{-\left(\frac{D_n max}{W}\right)^2}} = \frac{f_0}{1+e^{-\left(\frac{D_n max}{W}\right)^2}} = \frac{\frac{S_n c}{2 N D_n}}{1+e^{-\left(\frac{D_n max}{W}\right)^2}} \qquad (8)$$

where W is the width of each well, i.e., $W=L/N$. This fit is plotted in figure 5. As can be seen, f_{min} follows roughly this qualitative fit. However, the study has to be extended to more cases (different sequences, larger N, and so on), but this is beyond the scope of the present paper.

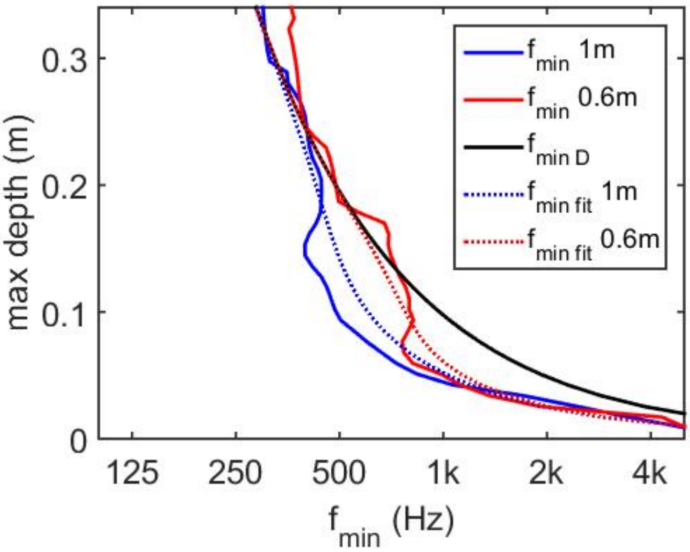

Figure 5. f_{min} for two different values of the width (1m (blue) and 0’6m (red)), Continuous lines . Theoretical f_{min} due to the limited depth of the diffuser (6), black line. Fits of f_{min} following equation (8), dotted lines.

3-OPTIMIZATION

In order to study if the low frequency limit can be overcome, we have perform an optimization. Given that such an optimization implies two parameters, f_{min} and $D_{n\ max}$, we have used a multi-objective algorithm, in particular a multi-objective evolutionary algorithm known as ev-MOGA [8]. Further details of the optimization algorithm can be found at [9].

The first step is to define a gene codification for the diffusers. The possible candidates are encoded by a set of seven genes that represent the normalized depth of each well, i.e.

$$\theta_n = D_n/0'34\ m \tag{9}$$

Being θ_n the n-th gene. Notice that 0'34 was fixed as the maximum depth. Next step is to define cost functions, in other words, the parameters that are to be minimized. As commented above the two parameters are f_{min} and $D_{n\ max}$. For convenience, in the optimization process, genes and cost functions were defined normalized to their maximum values.

A set of 1000 possible individuals generated randomly were introduced in the algorithm as starting point (initial population). Individuals are crossed randomly generating new individuals. New individuals are "measured" according to the cost functions. Eventually any individual can be removed, due to substitution by a new one, from the population if it does not belong to the so called Pareto front, defined as the set of points that are not dominated by any other individual of the population. Dominance refers to the fact that there is no any other individual with lower values of all the cost functions. After a few generations the Pareto front represent the "best" population that can be found.

It is important to highlight that it is necessary to have enough variability of each gene, in other words, the depth of the wells has to change in a tiny step. This is relatively hard to achieve with FDTD. Due to this we have used a BEM based algorithm instead. Since the classical geometry of a QR diffuser will be modelled, special care must be taken in order to apply the BEM to solve the problem. Indeed, the presence of the walls separating the diffuser's wells originates very thin surfaces which typically lead the direct BEM formulation to degenerate and lead to unstable equation systems. For this reason, a dual-BEM formulation is used here, in which the direct BEM integral equation (see equation (10)) is complemented by the so-called hypersingular BEM equation (see equation (11)). Details of this formulation can be found in [10], and thus only a general overview is here given regarding the BEM.

The classical boundary integral equation can be derived from the Helmholtz equation in the frequency domain by applying the reciprocity theorem, and in the case of rigid boundaries it can be written as:

$$C\ p(\mathbf{x}_0,\omega) = -\int_\Gamma H(\mathbf{x},\mathbf{x}_0,\omega,\mathbf{n})\ p(\mathbf{x},\omega) d\Gamma + p_{inc}(\mathbf{x}_0,\mathbf{x}_S,\omega) \tag{10}$$

where G represents the Green's function for the pressure defined before, and H is its first derivative with respect to the normal direction to the boundary Γ; similarly, p and q are the pressure and its first derivative in the normal direction to the boundary ($\mathbf{n}$), at point $\mathbf{x}$; $p_{inc}(\mathbf{x}_0,\mathbf{x}_s,\omega)$ represents the effect of a possible acoustic source located at point $\mathbf{x}_0$. The factor C equals 1/2 if $\mathbf{x}\in\Gamma$, and 1 for points not in the boundary but within the domain ($\mathbf{x}\in\Omega$,).

The hypersingular boundary integral equation can be derived by taking the first derivative of equation (10) with respect to the surface normal, and thus the required additional integral equation can be expressed as:

$$A\ p(\mathbf{x}_0,\omega) = -\int_\Gamma H'(\mathbf{x},\mathbf{x}_0,\omega,\mathbf{n},\mathbf{n_2})\ p(\mathbf{x},\omega) d\Gamma + p'_{inc}(\mathbf{x}_0,\mathbf{x}_S,\omega) \tag{11}$$

The Green's functions *G'* and *H'* can be seen as the derivatives of *G* and *H* with respect to the normal to the boundary at the loaded point, $\mathbf{n}_2$. In this equation, the factor A equals zero for piecewise straight boundary elements.

This formulation is used to analyse each configuration of the diffuser, allowing to compute the sound pressure scattered by the diffuser at any point of the acoustic domain. It is thus called multiple times from within the optimization algorithm, allowing the evaluation of the defined cost function for each individual.

Preliminary results of the optimization are summarized in figure 6. It can be seen that optimized diffusers can extend the low frequency range about one octave. In other words, the limit proposed by Schroeder is his first papers can be decreased above two octaves.

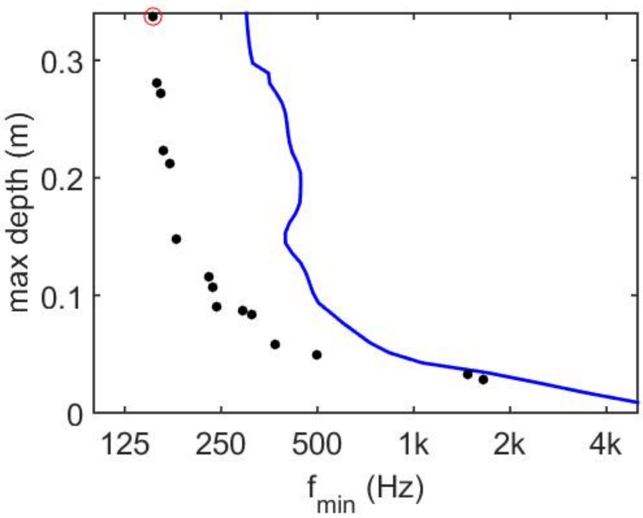

Figure 6. f_{min} for QR sound diffuser (width 1m) for different total depths (blue continuous line). Pareto front of the optimized diffusers (dots). The red circle correspond to the optimized diffuser illustrated in figure 7.

Finally, figure 7 illustrates one of the optimized diffusers, particularly the one with lower f_{min} (red circle in figure 6). It can be seen that f_{min} is nearly one octave lower for the optimized diffuser in comparison with the conventional diffuser. It is important to notice that, as a counterpart, there is a small decrease of the performance for frequencies one octave over f_{min}. In other words, the diffusion coefficient can be very uneven with frequency in the optimized diffusers. In future works we will include additional constrains in order to obtain optimized individuals with more robust behaviour.

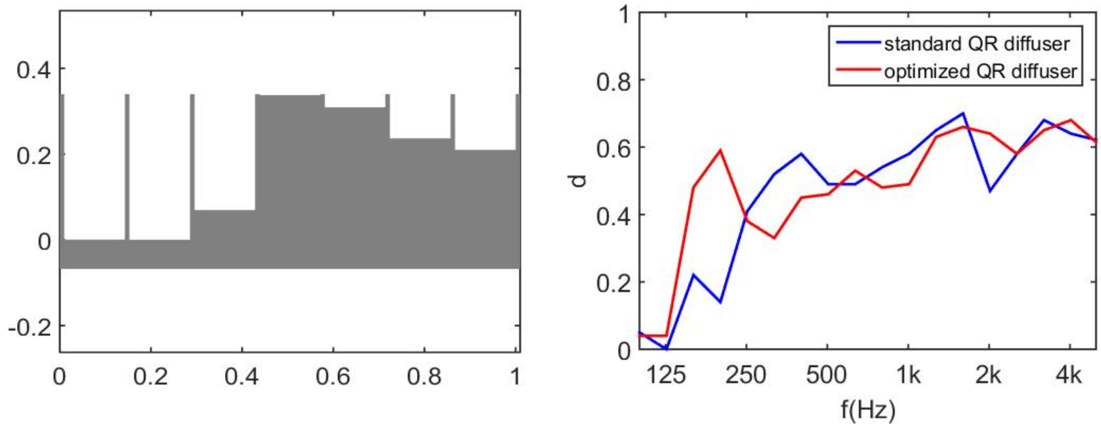

Figure 7. Left: Section of an optimized diffuser (red circle in figure 6). Right: Normalized diffusion coefficient versus frequency for a standard QR diffuser and the optimized diffuser illustrated on the left figure.

4-CONCLUSIONS

In conclusion, we have shown that the equations commonly used to predict the lower limit of the frequency range of a diffuser can overestimate it up to one octave in particular cases. Furthermore, we have proposed a new equation for the estimation of the lower limit with better agreement with the numerical results.

Additionally we have studied the possibility of extending the frequency range of a diffuser for low frequencies by means of a multi-objective genetic algorithm, showing that the low frequency limit can be extended about one octave in comparison with Schroeder diffusers.

In future works we will extend the analysis presented here to diffusers with larger number of wells and to other kinds of phase diffusers like primitive root diffusers.

5- ACKNOWLEDGEMENT

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6.-REFERENCES

[1] M.R. Schroeder, “Diffuse Sound Reflection by Maximum Length Sequences”, J. Acoust. Soc. Am. 57, No. 1, 149-150 (1975).

[2] ISO 17497-1:2004 “Acoustics -- Sound-scattering properties of surfaces -- Part 1: Measurement of the random-incidence scattering coefficient in a reverberation room”

[3] ISO 17497-2:2012 “Measurement of sound scattering properties of surfaces — Part 2: Measurement of the directional diffusion coefficient in a free field”

[4] Redondo J, Pico R, Roig B, Avis MR “Time domain simulation of sound diffusers using finite-difference schemes”. Acta Acustica & Acustica 93(4) (2007)

[5] Cox T. J., D'Antonio P., “Acoustic absorbers and diffusers. Theory, design and application”. Second Edition, 290, Taylor&Francis, London and New York (2009).

[6] Warner R. Th. ten Kate, “On the bandwidth of diffusors based upon the quadratic residue sequence” The Journal of the Acoustical Society of America 98, 2575 (1995)

[7] M. R. Schroeder. “Binaural dissimilarity and optimum ceilings for concert halls: More lateral sound diffusion” The Journal of the Acoustical Society of America 65, 958 (1979)

[8] Herrero JM, Reynoso-Meza G, Mart´ınez M, Blasco X, Sanchis J "A Smart-Distributed Pareto Front Using the ev-MOGA Evolutionary Algorithm". Int J Artif Intell Tools 23 (2) (2014)

[9] Herrero, J.; Blasco, X.; Sanchis, J.; Redondo, J. “Design of sound phase diffusers by means of multiobjective optimization approach using ev-MOGA evolutionary algorithm”. Struct. Multidiscip. Optim, 53, 861–879 (2016)

[10] Prieto Gajardo, C.; Godinho, L.; Amado-Mendes, P.; Barrigon Morillas, J. M. “Numerical analysis of acoustic barriers with a diffusive surface using a 2.5 D boundary element model”. Journal of Computational Acoustics, 23(03), 1550009 (2015).