HACES SONOROS EN CRISTALES DE SONIDO FINITOS

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ABSTRACT

Physical properties of 2D finite periodic arrays are observed using acoustic beams with finite spatial width. Analytical methods are used to study the differences between the approximation of infinite periodic medium and the finite case. However, in the most experimental cases the physical properties of the system are constrained by both the size of the sample and the width of the acoustic beam. In this work, beam propagation properties through finite sonic crystals are investigated.

RESUMEN

Los cristales de sonido son estructuras constituidas por elementos dispersores ordenados de forma periódica. Diversos métodos analíticos utilizados para estudiar la propagación de ondas acústicas en este tipo de estructuras se han desarrollado para cristales infinitos. Sin embargo, en las aplicaciones reales con cristales de sonido, éstos siempre tienen un tamaño finito y las fuentes generan haces con tamaños finitos. En este estudio se analizan por medio de simulación numéricas, las características de la propagación de haces sonoros en cristales de tamaño finito.

1 INTRODUCTION

A Sonic Crystal is a periodic arrangement of cylindrical inclusions embedded in a homogeneous host material. The host material may be solid, the term Phononic Crystal is used in this case. For many of the applications usually these scatterers can be considered as infinitely rigid and the propagation inside them is not possible. Sonic Crystals are designed to guide the acoustic energy in an appropriate way and present to show particular effects as focalization, auto-collimation or filtering [1].

The propagation of acoustic waves in a 2D periodic media can be described from its dispersion relation and the isofrequency contours. Band gaps and propagation curves describe the propagation features for a specific direction in an infinite crystal [2]. In the most experimental cases the finite size of the crystal must be considered. The theoretical approach for a 2D finite period array is more complicated. If the beam is wide enough to be comparable to the size of the crystal at the front interface, the edge effects may become important and they cannot be neglected. It is the same case at opposite case: extremely narrow beams (comparable to the period of the crystal) do not propagate in the crystal. In the field acoustics, little attention has been paid to investigate how the finitude of the beam and the size of the sample may affect wave propagation through and outside the crystal.

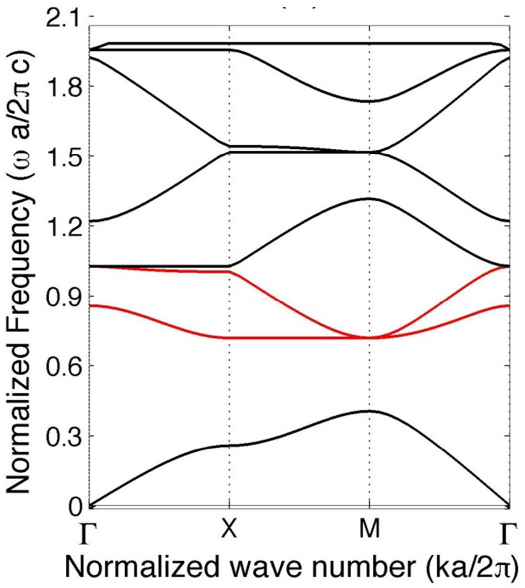

Fig. 1. Acoustic Band Structure of the Sonic Crystal used for simulations with square periodicity (r=1m, a=0.4545) obtained by the Plane Wave Expansion method

2 STATEMENT OF THE PROBLEM

Special phenomena like focalization induced by propagation in a periodic medium may be significantly altered if the size of the beam is considered. The aim of the work consists of evaluating the influence of the beam size and the finite size of the Sonic Crystal in its propagation properties. The acoustic beam is described by the pressure distribution:

$$P_0\left(x, y\right) = e^{-j \cdot k \cdot x} \cdot e^{-\frac{y^2}{\sigma^2}} \tag{1}$$

and corresponds to a Gaussian-weigthed beam in the transverse direction and travelling through the positive direction of x. The variable σ^2 is the variance and is related with the width of the beam.

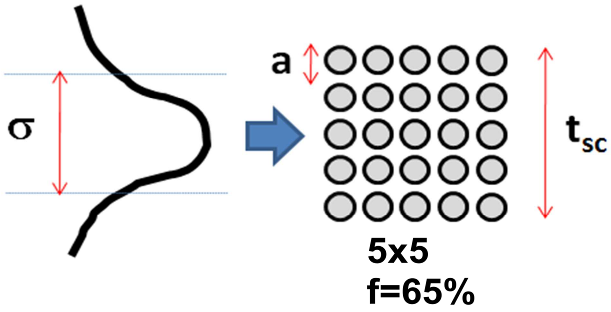

Fig. 2. Parameters used for simulations related to the beam and the Sonic Crystal

A distribution of 5x5 infinite straight cylinders in a square lattice with radius a=0.4545 and separated r=1 normalized units). The filling fraction of the structure is f=65%. Three types of beams can be defined by comparing the size of the beam (σ), the transverse size of the Sonic Crystal (a) and the transverse width of the structure (t_{SC}):

- Plane wave $\mathbf{s/t_{sc}>>1}$ **and** $\mathbf{s/a>>1}$
- Broad beam $\mathbf{t_{sc} > s > a}$
- Narrow beam $\mathbf{t_{sc}> a > s}$

Plane Wave Expansion is used to get the dispersion relation of the crystal is obtained assuming a monochromatic wave and finding the eigenvalues (see Fig. 1). The Helmholtz equation is converted the into an infinite matrix eigenvalue problem, which is then truncated and solved numerically [3]. By solving this eigenvalue problem the frequencies corresponding to each Bloch wave can be obtained, providing the dispersion relationship and band structure of the periodic medium.

$$\frac{\omega^2}{B(\mathbf{r})}p(\mathbf{r})+\nabla\left(\frac{1}{\rho(\mathbf{r})}\nabla p(\mathbf{r})\right)=0 \quad \longrightarrow \quad \omega = \omega(\mathbf{k}) = \omega\left(k_{\parallel}, k_{\perp}\right) \tag{2}$$

Different phenomena like attenuation, collimation and focalization of sonic beams
Multiple Scattering Theory (MST) is used to solve the scattering problem of a finite width beam produced by the array of scatterers [4]. The distribution of the N cylinders is parallel to the z-axis and they are located at (r_i, θ_i) with i=1, 2, …, N to form a regular square array perpendicular to the x-y plane. The total pressure in the point (x,y) is

$$P(x,y)=e^{ikx}e^{\frac{-y}{\sigma}}+\sum_{l=1}^{N}\sum_{s=-\infty}^{\infty}A_{ls}H_s(kr_l)e^{is\theta_l} \tag{3}$$

where A_{ls} are determined by the solution of the system of equations obtained by MST, r_l and θ_l are the polar coordinates of the measuring point respect to the l–th scatterer. The first term on the right hand side of equation (3) corresponds to the incident wave, and the second to the scattered wave.

3 ATENUATION

The transmission properties of the crystal are evaluated using MST. For a frequency lying in the Band Gap Figures 3a,3b and 3c show the transmission of beams with different sizes impinging at the left side of the crystal. The attenuation is optimized if the width of the beam is similar to the size of the sample.

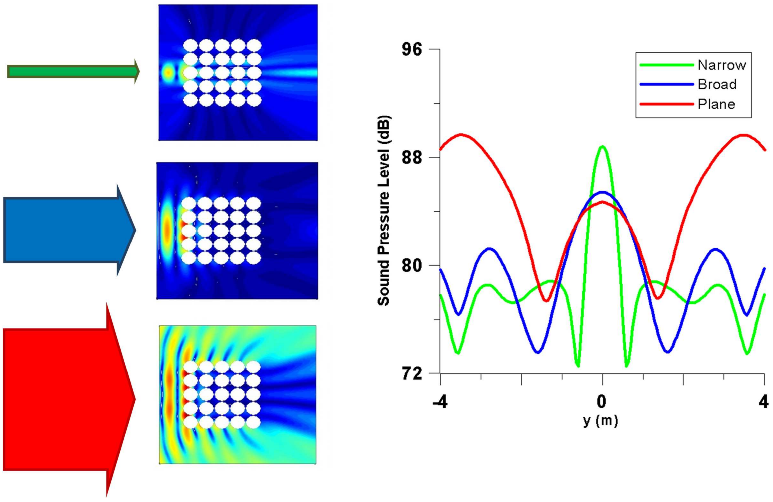

Fig. 3. Beam propagation behind the crystal for (a) a narrow beam (□=0.5), (b) a broad beam (□=2) and a (c) plane wave (□ =1000). Sound Pressure Level transmitted though the crystal in a transverse section at a distance x=4a

For wider beams (Figure 3c), the diffraction effect at the edges becomes important. The propagation of waves around the sample increases the total sound field behind the crystal. In the case of very narrow beams (Figure 1a), the propagation through the crystal is like in a homogeneous medium. In this case, the beam is not impinging the scatterers, and despite the frequency corresponds to the bandgap, it propagates through the crystal with a very low attenuation.

4 FOCALIZACIÓN AND COLLIMATION

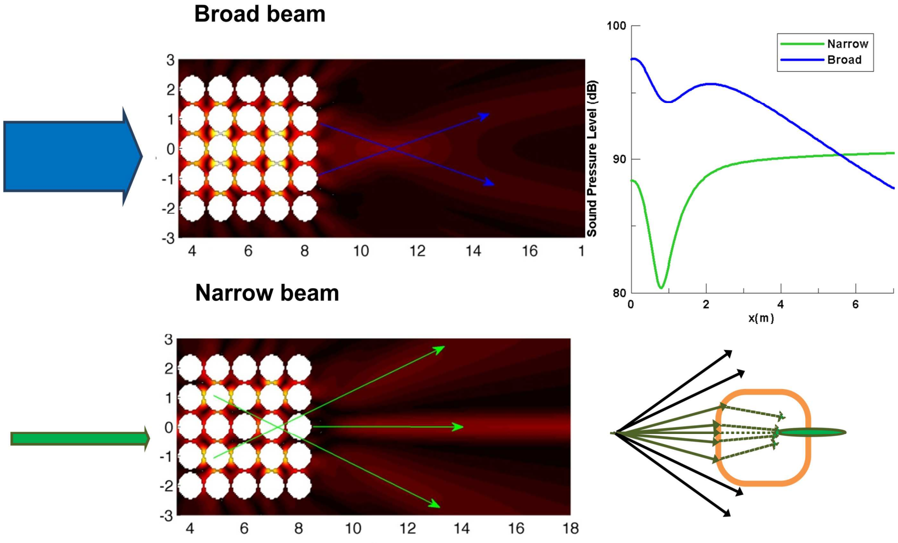

Fig. 4. SC as in Figure 1. The curves correspond to f=270Hz (second and third bands, in red in Figure 1A). Blue and green lines represent the projection on the curves of the spatial components of a wide and narrow beam respectively. Beam propagation behind the crystal

Special phenomena like focalization induced by propagation in a periodic medium may be significantly altered if the size of the beam is considered. For very narrow beams (Figure 4a) the Sonic Crystal in the second and third bands splits into three beams. Besides, for a beam as wide as the sample (Figure 4b) the focalization is produced some periods behind the crystal.

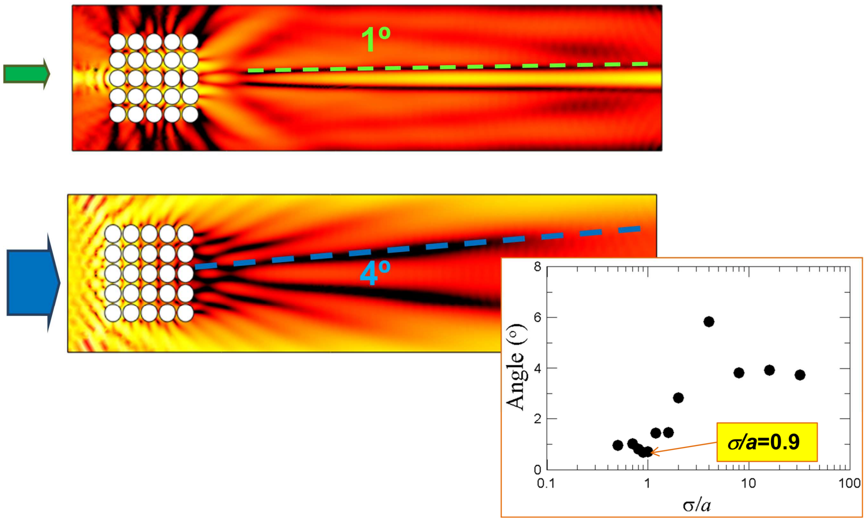

Fig. 5. Collimation effect for a (a) narrow beam and (b) a broad beam. In (c) the tilt angle of the beam propagating out of the crystal is represented for different widths

Wave beams diverge when they propagate in homogeneous materials due to diffraction. Nevertheless, for some particular conditions a particular regime where diffraction spreading vanishes is possible in periodic materials, the so-called self-collimation [5]. Inside a Sonic Crystal, the dispersion relations for propagation Bloch modes are modified, and the envelopes of acoustic waves can propagate without diffraction broadening. As it can be seen in Figs. 5a and 5b, this phenomenon depends strongly on the relative size of the beam. The tilt of the beam is different for different sizes of the width and its minimum value (collimation) is obtained for beams with a similar width compared to the transverse period of the periodic structure (see Fig. 5c).

5 CONCLUSIONES

The physical properties of a finite SC depend on its relative size, periodicity and the width of the beam. The attenuation is optimized if the beam is as large as the transverse dimension of the crystal. Focalization and collimation by transmission and reflection depend strongly on the width of the beam. For future works, the design of an experimental set-up to control the relative width of the acoustic beam must be considered.

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