Acústica 2008
20 - 22 de Outubro , Coimbra, Portugal
Universidade de Coimbra

HIGH-AMPLITUDE ACOUSTIC FIELD IN A DISC-SHAPED RESONATOR

M. Cervenka, R. Balek. M. Bednarik

CTU in Prague, FEE, Technicka 2, 166 27 Prague 6, Czech Republic
e-mail: cervenm3@feld.cvut.cz

Abstract

This paper is concerned with theoretical study of high-amplitude shock-free acoustic field in a gas-filled piston-driven cylindrical resonator where the diameter exceeds its height in a substantial way. Due to this geometry, only radial modes are driven. Since these modes are non-equidistant, resonance conditions are not fulfilled for higher harmonics of the acoustic field resulting from nonlinear effects, they are suppressed, the acoustic saturation effect is avoided and *Q*-factor of the resonator increases. Such a resonant cavity behaves similarly as a variable-cross-section resonator, but the geometry is much simpler which gives rise to its greater utilization e.g. in ultrasonic processing applications. For the purpose of numerical simulation of the problem, set of Navier-Stokes equations formulated in cylindrical coordinates is used. The equations are integrated by using of a central semi-discrete difference scheme, characteristic boundary conditions are implemented. Numerical results confirm excitation of shock-free high-amplitude waveform in the resonant cavity.

Keywords: acoustic resonator, Navier-Stokes equations, numerical integration

1 Introduction

Nowadays, we can observe rapid development of nonlinear acoustics that is the branch of acoustics concerned with acoustic waves, whose amplitudes cannot be treated as infinitesimally small ones. This situation is caused by wide application area of these waves and also by fast development of powerful computer technology allowing their theoretical study.

One of the application areas is connected with utilization of high-amplitude acoustic standing waves generated in resonators driven by means of vibrating piston or inertial force. When a standing wave is driven into high amplitude, nonlinear effects (convection, nonlinear relation between density and pressure) cause distortion of originally harmonic wave. As the thermo-viscous attenuation is proportional to the square of frequency, dissipation of acoustic energy increases in a substantial way due to excitation of higher harmonics and acoustic pressure amplitude is no longer proportional to amplitude of driving.

A few techniques have been proposed for suppression of nonlinear effects and thus improvement of the *Q*-factor of acoustic resonators. One of them utilizes shaped acoustic resonators, where higher eigenfrequencies are not integer multiples of the eigenfrequency fundamental, see e.g. [1-3]. Due to this fact, resonance conditions are not fulfilled for the higher harmonics and their driving is thus ineffective. Different approach is based on the technique of multifrequency driving, see e.g. [4, 5]. It is shown there that utilization of suitably chosen second (or even higher) harmonics of driving signal can result in suppression of cascade processes of higher harmonics generation.

Philosophy of this work issues from the firstly mentioned approach (shaped resonators), whereas it overcomes one its disadvantage - complicatedness of the acoustic resonator cavity. Here a thin, piston-driven cylindrical resonator is used, nevertheless, transversal mode is driven instead of a mode longitudinal. As higher eigenfrequencies are not integer multiples of the eigenfrequency fundamental for cylindrical waves, resonance conditions are not fulfilled for higher harmonics and thus appearance of shock wave and intense nonlinear dissipation can be avoided. Geometry of the problem is shown in Figure 1.

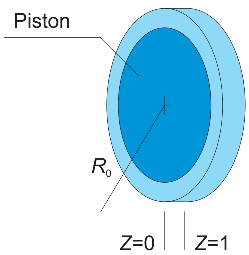

Figure 1 – Geometry of the problem formulated using dimensionless coordinates.

2 Model equations

Model equations of nonlinear acoustics are represented by nonlinear partial differential equations. For the sake of simulation of the above mentioned problem, the appropriate model equation must be a multidimensional one, some of them have recently been proposed in literature, see e.g. [6-8]. Here, set of dimensionless equations issuing from Navier-Stokes equations is used. With the dimensionless quantities

$$(Z,R)=\frac{(z,r)}{l},\ T=\omega t,\ \Lambda=\frac{\rho}{\rho_0},\ (U,V)=\frac{(u,v)}{\pi c_0},\ (P,E)=\frac{(p,e)}{\rho_0\pi^2 c_0^2},$$

where z, r are axial and radial coordinates, l is length of the resonant cavity, t is time, ω is driving frequency, ρ, ρ_0 are the total and ambient fluid density, u, v are the axial and radial components of acoustic velocity, c_0 is the small-signal speed of sound and p, e are the acoustic pressure and energy respectively, the set model equations reads

$$\begin{aligned}
&\frac{\partial\Lambda}{\partial T}+\frac{\partial}{\partial Z}\left(\frac{\Lambda U}{\Omega}\right)+\frac{\partial}{\partial R}\left(\frac{\Lambda V}{\Omega}\right)=-\frac{\Lambda V}{\Omega R},\\
&\frac{\partial(\Lambda U)}{\partial T}+\frac{\partial}{\partial Z}\left(\frac{P+\Lambda U^2}{\Omega}\right)+\frac{\partial}{\partial R}\left(\frac{\Lambda UV}{\Omega}\right)=-\frac{\Lambda UV}{\Omega R}+\frac{G}{\pi^2\Omega}\left(\frac{\partial^2 U}{\partial Z^2}+\frac{\partial^2 U}{\partial R^2}+\frac{1}{R}\frac{\partial U}{\partial R}\right),\\
&\frac{\partial(\Lambda V)}{\partial T}+\frac{\partial}{\partial Z}\left(\frac{\Lambda UV}{\Omega}\right)+\frac{\partial}{\partial Z}\left(\frac{P+\Lambda V^2}{\Omega}\right)=-\frac{\Lambda V^2}{\Omega R}+\frac{G}{\pi^2\Omega}\left(\frac{\partial^2 V}{\partial Z^2}+\frac{\partial^2 V}{\partial R^2}+\frac{1}{R}\frac{\partial V}{\partial R}-\frac{V}{R^2}\right),\\
&\frac{\partial E}{\partial T}+\frac{\partial}{\partial Z}\left[\frac{(E+P)U}{\Omega}\right]+\frac{\partial}{\partial R}\left[\frac{(E+P)V}{\Omega}\right]=-\frac{(E+P)V}{\Omega R}.
\end{aligned} \tag{1}$$

The equations are supplemented by the pressure-energy relation $P=(\gamma-1)[E-\Lambda(U^2+V^2)/2]$, where γ is the ratio of specific heats. Here $\Omega=\omega/\omega_0$ is ratio of the driving frequency and frequency of the fundamental longitudinal mode $\omega_0=\pi c_0/l$ and $G=b\omega_0/\rho_0 c_0^2$ is the attenuation parameter where b is the diffusity of sound.

Left sides of the equations (1) are formulated in the conservative form, the right sides represent geometrical sources (due to formulation of the problem with use of curvilinear coordinates) and dissipative terms. Model of the acoustic energy dissipation process is in the equations (1) somewhat simplified, it is assumed that the acoustic field is non-vortical (the terms containing rotation of velocity are omitted) and the small third-order terms in the energy transfer equation are dropped. All the dissipation mechanisms are included in heightened value of the parameter G.

For the purpose of numerical integration of the set (1), high-resolution second-order central semi-discrete difference scheme [9] was used. The characteristic boundary conditions were implemented as the adiabatic free-slip ones [10] (hard resonator's walls at $R=R_0$, $Z=1$), time-varying position of vibrating piston (boundary at $Z=0$) was implemented to avoid nonphysical matter flux through the boundary. For $R=0$, symmetry of the problem was utilized.

3 Eigenfrequencies of a cylindrical resonator

In order to obtain the first insight into the problem, the equations (1) can be linearized and solved analytically. However, it is simpler and more straightforward to issue from linearized wave equation that can be written in dimensionless form

$$\frac{\partial^2\Phi}{\partial R^2}+\frac{1}{R}\frac{\partial\Phi}{\partial R}+\frac{\partial^2\Phi}{\partial Z^2}+\pi^2\Omega^2\left(1-\mathrm{i}\Omega G\right)\Phi=0, \tag{2}$$

where $\Phi(R,Z,T)=\Phi(R,Z)\exp(\mathrm{i}T)=\varphi/\pi c_0 l$ is dimensionless velocity potential and $\mathrm{i}=\sqrt{-1}$. Solution of the Eq. (2) can be easily found using the Fourier series. Using homogeneous Neumann boundary conditions at the walls and

$$\frac{\partial\Phi}{\partial Z}=U_p(R)$$

at the piston wall ($Z=0$), where $U_p(R)$ is distribution of the vibration velocity, the solution can be written in the form

$$\Phi(R,Z)=\sum_{k=0}^{\infty}\frac{\alpha_k}{\beta_k\sin\beta_k}\mathrm{J}_0\left(\nu_k R/R_0\right)\cos\left[\beta_k(1-Z)\right];\ Z\in\left\langle 0,1\right\rangle, R\in\left\langle 0,R_0\right\rangle, \tag{3}$$

where $\mathrm{J}_0()$ is the first-kind zero-th order Bessel function, ν_k represents the *k*-th root of equation $\mathrm{J}_1(\nu)=0$, $\beta_k^2=\pi^2\Omega^2(1-\mathrm{i}\Omega G)-\nu_k^2/R_0^2$ and α_k are coefficients of the Bessel-Fourier decomposition of U_P, Individual eigenfrequencies can be found from the condition $\pi^2\Omega^2-\nu_k^2/R_0^2=n^2\pi^2$, where n is an integer, in form

$$\Omega_{n,k}=\sqrt{n^2+\nu_k^2/\pi^2 R_0^2}. \tag{4}$$

This equation shows that in the case of the pure longitudinal modes $\Omega_{n,0}=1,2,3,\ldots$, which means that the higher eigenfrequencies are the integer multiples of the eigenfrequency fundamental. For driving of these modes, it must be fulfilled that $\alpha_0\neq 0$. In the case of the pure transversal modes, Eq. (4)

shows that $\Omega_{0,k} = \nu_k / \pi R_0$, $k > 0$, which means that the higher eigenfrequencies are not integer multiples of the eigenfrequency fundamental. For example, if $R_0 = 6$ (radius of the resonator is six times greater than its length), we obtain

$$\Omega_{0,k} = 0.203278,\ 0.372188,\ 0.539719,\ 0.706844, ...$$

For driving of the *k*-th transversal mode, it must be fulfilled that $\alpha_k \neq 0$. It should also be noted that for reaching high-amplitude acoustic field in the *k*-th transversal mode, value of the coefficient α_k must be maximized. For example, if the velocity of the left resonator's wall (piston) is given as

$$U_p(R) = A_p \mathrm{H}(pR_0 - R); \quad p \in \langle 0,1 \rangle, \tag{5}$$

where A_p, p are constants, H is the Heaviside function and the 1st transversal mode is driven, the maximum value for α_1 is obtained for

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\int_0^x \mathrm{J}_0(\nu_1 R / R_0) R \mathrm{d}R \right)_{x = pR_0} \overset{!}{=} 0 \ \Rightarrow\ p = \frac{\mu_1}{\nu_1} \approx 0.6276,$$

where μ_1 is the 1st root of equation $\mathrm{J}_0(\mu) = 0$.

4 Numerical results

Some numerical results obtained by means of numerical integration of Eqs. (1) are presented in this section. For all the cases, hard-walled piston-driven cylindrical resonator filled with air at ambient room conditions is assumed. Parameter G accounting for all the attenuation processes was set to $G = 10^{-3}$. Two geometrical configurations are compared here. Firstly, it is a disc-shaped resonator with $R_0 = 6$, where the 1st transversal mode is driven using piston velocity distribution $U_p(R,T) = A_p \mathrm{H}(0.625 R_0 - R) \sin T$, which is close to "the optimal driving". And secondly, it is an elongated thin resonator with $R_0 = 1/6$, where the 1st longitudinal mode is driven using piston velocity distribution $U_p(R,T) = A_p \sin T$. Acoustic pressures presented in the following figures are "measured" at the center of vibrating piston ($R = 0, Z = 0$).

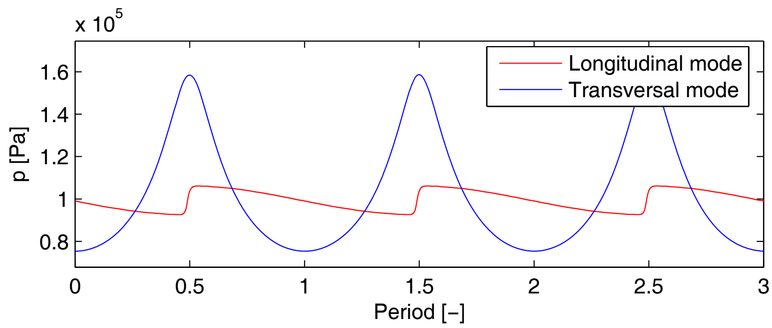

Figure 2 – Acoustic pressure in the case of longitudinal and transversal mode driving.

Figure 2 shows comparison of time courses of longitudinal ($\Omega = 1$) and transversal ($\Omega = 0.2064$) mode pressures for the same driving amplitude $A_p = 5 \times 10^{-4}$ at the resonance conditions. It can be seen that the transversal mode attains substantially higher amplitude compared to the highly distorted longitudinal mode, where shock wave appears.

Figure 3 shows comparison of amplitude characteristics (the 1st harmonics of acoustic pressure) in the case of longitudinal and transversal mode driving at resonance conditions. As it is common in nonlinear systems, the pressure amplitudes are not proportional to amplitude of driving. This situation is caused by excitation of higher harmonics that undergo stronger the nonlinear attenuation that is proportional to the square of frequency.

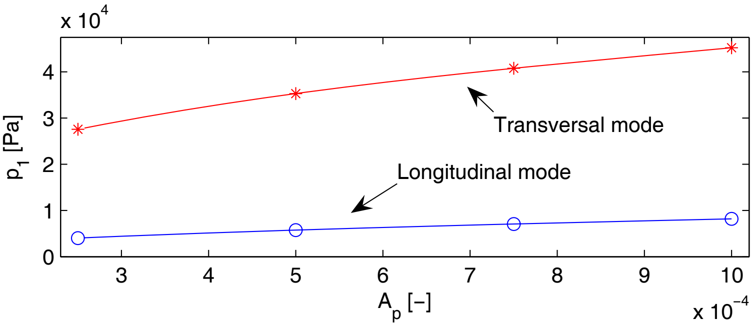

Figure 3 – Amplitude of the 1st harmonics of pressure with respect to driving velocity in the case of longitudinal and transversal mode driving.

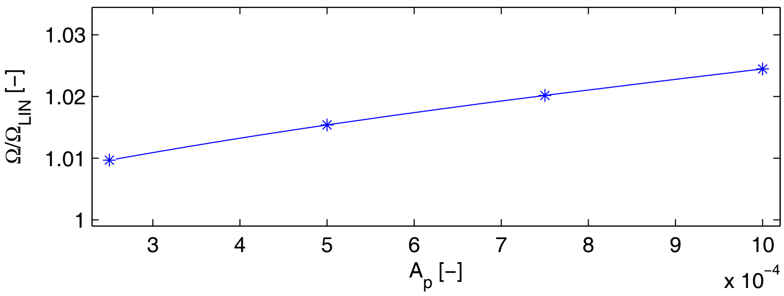

Figure 4 – Shift of resonance frequency as a function of driving velocity amplitude (transversal mode).

Figure 4 shows dependence of transverse-mode resonance frequency on amplitude of driving. The linear theory predicts this resonance frequency as $\Omega = \Omega_{LIN} \approx 0.2033$. It can be seen from the figure that the *nonlinear* resonance frequency increases with amplitude of driving. Such a behavior is due to the mechanical analogy called the *hardening-spring* resonance frequency shift. This resonance frequency shift is not observed for the case of longitudinal mode driving.

5 Conclusions

It was shown in the paper that driving of transversal modes in a disc-shaped resonator can be used for generation of high-amplitude acoustic fields without shock-wave appearance. These acoustic fields are substantially stronger than the fields obtained in constant-cross-section resonators with longitudinal modes driven. The idea of this approach is similar to *resonance macrosonic synthesis*, where non-equidistant eigenfrequencies distribution is attained by using variable-cross-section cavities. However, geometry of the problem is substantially simpler in this case that may be interesting fact in the case of possible application.

Acknowledgements

This work was supported by grant MSM 6840770015 and GACR grant project No. 202/06/0576.

References

[1] Atchley, A. A.; Gaitan, D. F. *Finite amplitude standing waves in harmonic and anharmonic tubes*. J. Acoust. Soc. Am. **93**, 1993, pp. 2489-2495

[2] Illinski, Yu. A.; Lipkens, B.; Lucas, T. S.; Van Doren, T. W.; Zabolotskaya, E.. *Nonlinear standing waves in an acoustical resonator*. J. Acoust. Soc. Am. **104**, 1998, pp. 2664--2674

[3] Luo, C.; Huang, X. Y.; Nguyen, N. T. *Generation of shock-free pressure waves in shaped resonators by boundary driving*, J. Acoust. Soc. Am. 121, 2007, pp. 2515

[4] Huang, P. T.; Brisson, J. G.. *Active control of finite amplitude acoustic waves in a confined geometry*. J. Acoust. Soc. Am **102**, 1997, pp. 3256-3268

[5] Gusev, V. E.; Bailliet, H.; Lotton, P.; Job, S.; Bruneau, M. *Enhancement of the Q of a nonlinear acoustic resonator by active suppression of harmonics*. J. Acoust. Soc. Am. **103**, 1998, pp. 3717-3720

[6] Vanhille, C.; Campos-Pozuelo, C. *Numerical and experimental analysis of strongly nonlinear standing acoustic waves in axisymetric cavities*, Ultrasonics 43, 2005, pp. 652

[7] Cervenka, M.; Bednařík, M. *Nonlinear standing waves in 2-D acoustic resonators*. Ultrasonics. 2006, vol. 44, Suppl. 1, pp. 773-776

[8] Cervenka, M.: *Numerical Simulation of Two-Dimensional High-Amplitude Acoustic Field in Cylindrical Resonators Using Kuznetsov's Equation*. 19th International Congress on Acoustics [CD-ROM]. Madrid: Sociedad Espaňola de Acústica, 2007

[9] Kurganov, A.; Tadmor, E. *High-Resolution Central Schemes for Nonlinear Conservation Laws and Convection-Diffusion Equations,* J. Comput. Phys.160, 2000, pp. 241

[10] Poinsot, T. J.; Lele, S. K. *Boundary conditions for direct simulations of compressible viscous flows*, J. Comput. Phys., vol.101, 1992, pp. 104-124