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LOCALIZED STRUCTURES AND PATTERN FORMATION IN ACOUSTIC RESONATOR CONTAINING VISCOUS FLUID

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Abstract

We report on a new type of localized structure, an ultrasonic cavity soliton, supported by large aspect-ratio acoustic resonators containing viscous media. These states of the acoustic and thermal fields are robust structures, existing whenever a spatially uniform solution and a periodic pattern coexist. We derive the equations which describe the system including spatial effects. The spatio-temporal dynamics of the system is analyzed from the microscopic model, and the results are in good agreement with the numerical results, such as with previous experimental works.

Keywords: acoustic resonator, pattern formation, localized structures.

1 Introduction

Many systems in nature, when driven far from equilibrium, can self-organize giving rise to a large variety of patterns or structures. Although studied intensively for most of the last century, it has only been during the past thirty years that pattern formation has emerged as an own branch of science [1]. One of the most relevant features of pattern formation is its universality: systems with different microscopic descriptions frequently exhibit similar patterns on a macroscopic level. This universal character of pattern formation is evidenced when the microscopic models are reduced, under given assumptions, to simpler equations describing the evolution of a single variable, the so called order parameter [2]. They are often based on system symmetries and are independent of the microscopic differences among systems, providing a theoretical framework to understand the origins of non-equilibrium pattern formation [1]. This approach to pattern formation has been extensively applied to nonlinear optical cavities [3]-[5], such as lasers, optical parametric oscillators or Kerr (cubically nonlinear) resonators, where light in the transverse plane of the cavity has been shown to develop patterns with different symmetries (rolls, hexagons, and also quasi-patterns) as well as cavity solitons

(CS). The latter correspond to localized solutions often resulting from a bistability between two stationary, spatially extended states of the system, in the form of either self-trapped switching waves between two homogeneous states, or isolated pattern elements embedded in a homogeneous background. Optical CSs are particularly interesting objects because of their potential use as memory bits in optical information processing systems [6].

However, despite the existing analogies between optics and acoustics, acoustical resonators in the nonlinear regime have been much less explored than their optical counterparts. Furthermore, pattern formation studies in acoustics are almost lacking. The main reason lies in the weak dispersion of sound in common homogeneous media, which is responsible for the growth of higher harmonics, leading to wave distortion and shock formation. These effects are absent in optics, which is dispersive in nature. However, in some special cases it is possible to avoid the nonlinear distortion and recover the analogies [7]. It is, for example, the case of sound beams propagating in viscous media characterized by a strong absorption (e.g. glycerine), where sound velocity depends on fluid temperature, resulting in an additional nonlinearity mechanism of thermal origin. For the majority of fluids, temperature variations induced by an intense acoustic field result in a decrease of sound velocity, leading to a self-focusing of the beam. In the case of viscous fluids the characteristic length of self-focusing effects is much shorter than the corresponding to the development of shock waves [8]. Also, high frequency components are strongly absorbed in such media, so in practice the use of a quasimonochromatic description for wave propagation is justified.

2 Model

In a viscous medium sound propagates with a speed c that depends significantly on temperature, $c=c_0(1-\sigma T')$, where c_0 is the speed of sound at some equilibrium (ambient) temperature, T' denotes the variation of the medium temperature from that equilibrium due to the intense acoustic wave, and σ is the parameter of thermal nonlinearity. The propagation of sound in such a medium has been shown [8],[9] to be described in terms of two coupled equations for pressure, p', and temperature, T', deviations. These equations have been used to address problems such as self-focusing and self-transparency of sound [9]. They have been also the basis for the analysis of temporal dynamic phenomena in acoustic resonators [10]. In this case, a viscous fluid is bounded by two flat and parallel reflecting surfaces. One of the surfaces, vibrating at a frequency *f*, is an ultrasonic source providing the external forcing. Previous studies on this system [10,11] have reported, in the frame of the plane-wave approximation, bistability and complex temporal dynamics, in good agreement with the corresponding experiments. We have extended the previous model by considering the effects of sound diffraction and temperature diffusion in a large aperture resonator. These effects, which are responsible of the spatial coupling, can play an important role when the Fresnel number of the resonator $F = l^2/\lambda L >> 1$ (being l and L its transverse and longitudinal dimensions, respectively), see Fig. 1.

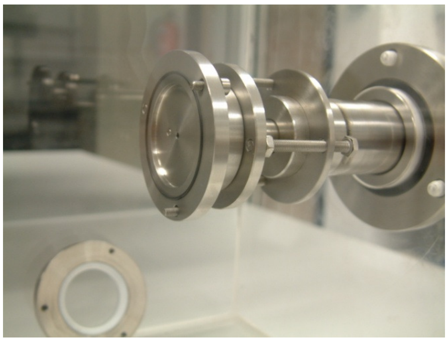


Fig. 1. Transverse profile of the transducer (wall of the resonator).

We model the resonator as usual [10,11]: the intracavity pressure field is decomposed into two counterpropagating travelling waves, $p' = p_+\exp\{i(\omega t-kz)\}+p\exp\{i(\omega t+kz)\}$+c.c., where t is time, z is the axial (propagation) coordinate, $\omega = 2\pi f$, and $k = \omega/c$, whose complex amplitudes p_+ and p_- are related through their reflections at the boundaries, and the temperature field is decomposed into a homogeneous component and a grating component, $T' = T_h+T_g\exp\{i2kz\}+ T_g*\exp\{i2kz\}$. All these amplitudes are slowly varying functions of space and time as the fast (acoustical) variations are explicitly taken into account through the complex exponentials. Under the assumption of highly reflecting walls we can adopt a mean field model, where the slowly varying amplitudes do not depend on the axial coordinate z and $p_+ = p \equiv p$, ending up with the following dimensionless equations [12]

$$\begin{aligned}
\tau_p \frac{\partial P}{\partial \tau} &= -P + P_{in} + i\nabla^2 P + i(H + G - \Delta)P \\
\frac{\partial H}{\partial \tau} &= -H + D\nabla^2 H + 2|P|^2 \\
\frac{\partial G}{\partial \tau} &= -\tau_g G + D\nabla^2 G + |P|^2
\end{aligned} \tag{1}$$

Where

$$\begin{aligned}
P &= p\sqrt{\frac{\sigma\omega t_p t_h \alpha_0}{2\rho_0^2 c_0 c_p}}, \\
H &= \omega t_p \sigma T_h,\ G = \omega t_g \sigma T_g
\end{aligned} \tag{2}$$

are new normalized variables, $\tau = t/t_h$ is time measured in units of the relaxation time t_h of the temperature field homogeneous component, and $\tau_p = t_p/t_h$ and $\tau_g = t_g/t_h$ are the normalized relaxation times of the intracavity pressure field and the temperature grating component, respectively. Their original values are given by $\tilde{t}_p^1 = c_0T/2L+c_0\alpha_0$, where T is the transmissivity of the plates and α_0 is the absorption coefficient of the medium, and $\tilde{t}_g^1 = 4k^2\chi$, where $\chi = \kappa/\rho_0 c_p$ is the coefficient of thermal diffusivity, ρ_0 is the equilibrium density of the medium and κ and c_p are the thermal conductivity and the specific heat of the fluid at constant pressure, respectively. Other parameters are the detuning $\Delta = (\omega_c-\omega)t_p$, with ω_c the cavity frequency that lies nearest to the driving frequency ω, and

$$P_{in} = \frac{c_0 p_{in}}{2L}\sqrt{\frac{\sigma\omega t_p t_h \alpha_0}{2\rho_0^2 c_0 c_p}}, \tag{3}$$

being p_{in} the injected pressure plane wave amplitude, which we take as real without loss of generality. Finally, in the transverse Laplacian operator the dimensionless transverse coordinates (x,y) are measured in units of the diffraction length $l_d = c_0\sqrt{(t_p/2\omega)}$, and the normalized diffusion coefficient $D = \chi t_h/l_d^2$. We note that when spatial derivatives are ignored and the complex fields are written in terms of moduli and phases, the model of [10] is retrieved.

The model parameters can be estimated for a typical experimental situation [10]. We consider a resonator with high quality plates (T=0.1), separated by L=5 cm, driven at a frequency f=2 MHz , and containing glycerine at 10ºC. Under these conditions the medium parameters are $c_0=2\times10^3$ m/s, $\alpha_0=10$ m^{-1}, $\rho_0=1.2\times10^3$ kg/m^3, $c_p= 4\times10^3$ J/kg K, σ = $^{10-2}$ K-1, and κ = 0.5 W/m^{-1}K^{-1} ($\chi=10^{-7}$m^{2}s$^{\tilde{1}}$). In this case $t_p=2\times10^{-5}$s, $t_g = 6\times^{10-2s}$, and our length unit l_d=2 mm. For a resonator with a large Fresnel number, the relaxation of the homogeneous component of the temperature is mainly due to the heat flux through the boundaries, and can be estimated from the Newton's cooling law as $t_h = 10^1$ s . (Remind that this is our time unit.) Then the diffusion constant D =10, and the normalized decay times $\tau_p = 10^{-6}$, and $\tau_g = 10^{-2}$ under usual conditions. We see that the problem is typically very stiff: $0 < \tau_p << \tau_g << 1$. In the following the results will be given to the lowest nontrivial order in these smallest decay times in order to not overburdening the expressions.

The spatially uniform steady state can be obtained by neglecting the derivatives in Eqs. (1). Introducing the notation $W = |P|^2$, $W_{in} = |P_{in}|^2$, one has $G = \tau_g W$, $H = 2W$, and

$$W_{in} = W + \left(\Delta - 2W^2\right)W \tag{4}$$

The characteristic curve W vs. W_{in} can display an S-shape, typical of the optical bistability of coherently driven optical Kerr cavities, as we show next. Note that bistability requires $\Delta > \sqrt{3}$.

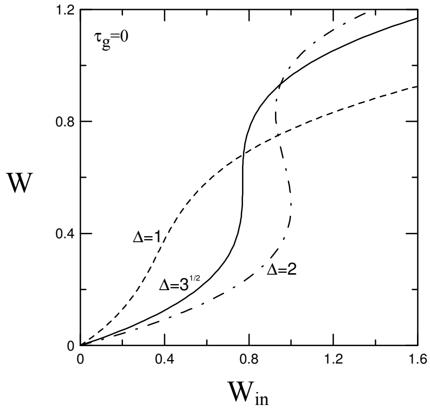


Fig.2. Characteristic curve [Eq. (4)] input vs. output intensity in the monostable, nascent bistability and bistable cases.

It is possible to demonstrate, by using standard multiple scale methods, that the mean field model, Eqs. (1), reduces, close to the nascent bistability point, to a modified Swift-Hohenberg order parameter equation [12].

The stability of the systems against spatial perturbation is analysed performing a linear stability of the model equations (1). It is obtained that the upper branch solution is unstable against spatial perturbations, giving rise to extended spatial patterns rolls). Nevertheless, in most of the multivalued domain, the extended patterns emerging at the modulational instability are unstable, being a transient state. As shown in Fig. 2, neighbour maxima collide and merge, the long-term evolution resulting in a number of localized structures or CSs, see Fig. 3.

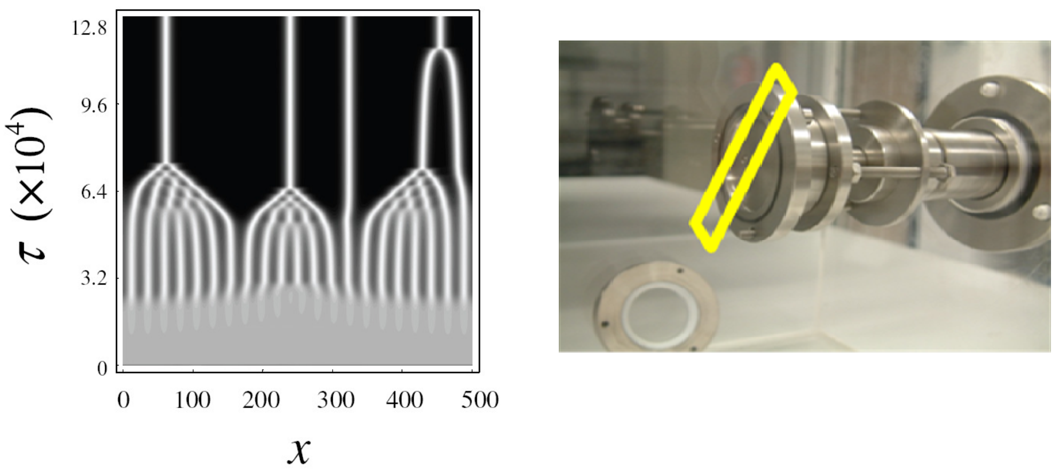
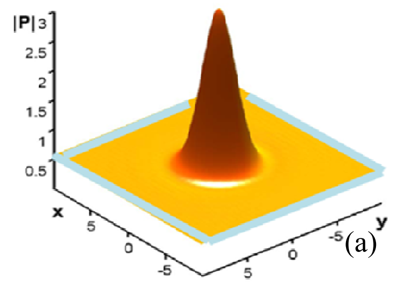
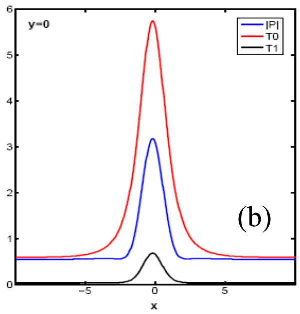


Fig. 3. Numerical simulation of pressure field in Eqs. (1). The modulational instability and the further collapse resulting in an array of CSs is shown. Time runs from bottom to top. The simulation corresponds to one-dimensional transducers, corresponding with the square marked in the right picture.

In order to verify the robustness of the result, we have numerically solved the full microscopic model given by Eqs. (1) for the realistic parameter values already evaluated in the case of the habitual twodimensional transducer case. The stable CS in shown in Fig. 4(a) and its profile, for the three different fields evolved, is shown in Fig 4(b). The squared area marked on the transducer surface in Fig. 4(c) corresponds to the surface where we are measuring.





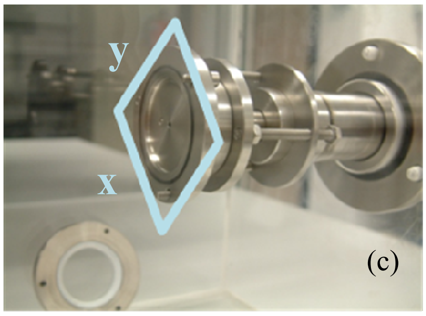
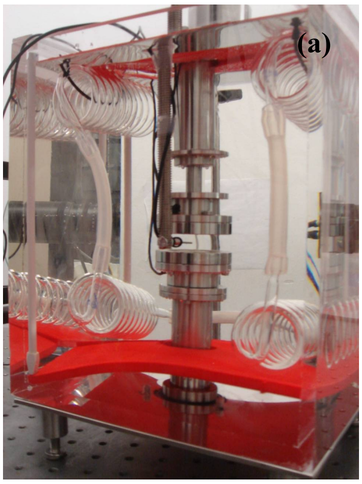


Fig. 4. (a) Spatial distribution of pressure field across the transverse dimension of the resonator. (b) Transverse distribution of pressure and temperature fields for y=0. (c) Transducer picture showing the transverse area scanned in (a).

In terms of the physical parameters, it is interesting to note that Fig. 3(b) corresponds to a driving intensity $I_{in}=p_{in}^2/\rho_0 c_0 \approx 1.5$W cm², and the characteristic size of the CS is about 6mm, on the order of the wavelength of the driving source. These values are in agreement with the experimental observations reported in [10].

3 Experimental setup.

The experimental setup consists in two transducers, one of them vibrates at frequency ω=1Mz, acting as external forcing, and the other one acting as a mirror. The viscous medium in the resonator is glycerin. The control of the temperature is critical in the system; and it is controlled by an external bath (Fig.5b). The measures are taken by using a needles hydrophone which scans the transeverse profile of the resonator (Fig. 5a).



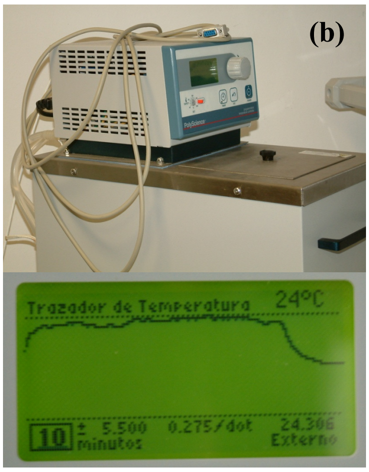


Fig. 5. (a) Thermoacoustic resonator and needle hydrophone. (b) Controlling temperature system.

The preliminary results show evidences of the existence of localized structures in the transverse profile of the resonator, see Fig.6. The picture corresponds to a scanned area of 15x15 cm^2, at a temperature of 25ºC and an input voltage of 18V.

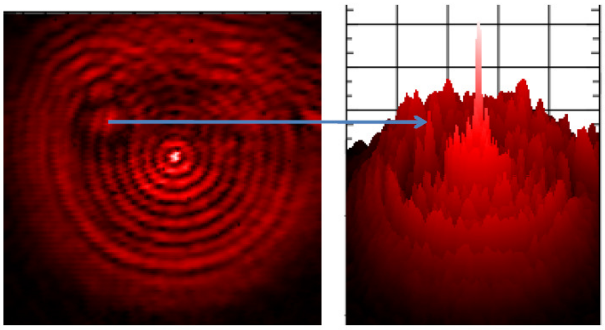


Fig. 6 Frontal and lateral view of the scanned transverse profile of the resonator.

4 Conclusions

In this work we present the model equations for an acoustic resonator containing a viscous fluid inside, considering the effect of spatial coupling, pressure diffraction and temperature diffusion. The numerical simulation of the model predicts the existence of extended spatial patterns and existence of localized structures (cavity solitons) in the system. The preliminary results seems to agree with the predictions.

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