



# Combining Adaptive Filtering and Wavelet Techniques for Vibration Signal Enhancement

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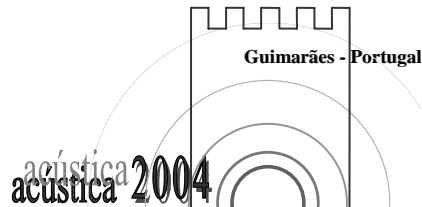
**ABSTRACT:** The condition monitoring and fault diagnosis of equipment is accomplished through the analysis and interpretation of signals acquired from sensors and transducers. However, any structure-borne vibration propagated through the neighboring structures will produce a background vibration in which the needed vibration signals for diagnosis are often submerged, in particular during the early stage of failure development. If the background noise level is too high or when the vibration signature level is too low, traditional techniques such as wavelet de-noising analysis could be ineffective in canceling the noise of such signals. In this article the combination of an adaptive signal enhancement and the wavelet transform for de-noising a vibration signal measured on a rolling contact bearing is presented. Two algorithms were used as the adaptive weight-control mechanism. The final aim of the adaptive filter was the minimization of the mean-square value of the error signal which implies the maximization of the output signal-to-noise ratio of the system. The results showed that the combination of the adaptive vibration signal enhancement and the wavelet transform yielded the best signal-to-noise ratio. Therefore, the result can reveal hidden signal structures that are directly associated with a bearing's internal defect.

## 1. INTRODUCTION

The main purpose of condition monitoring is to detect the presence of faults and damage in machinery during operation. It encompasses both diagnosis and prognosis in order to determine the remaining safe operating life of a machine before a breakdown or failure occurs [1, 2].

One of the most important components in rotating machinery are the bearings, where the rolling contact types are more commonly used. They consist of rolling elements contained between the inner and outer raceways. The rolling elements are normally kept from touching each other by a cage. Because of the metal to metal contact, this bearing provides very little vibration damping. Therefore, rolling element bearings provide a very good signal transmission path from the vibration source to the outer bearing housing. Each component of the bearing will generate specific frequencies as defects initiate and become more prevalent.

The condition monitoring and fault diagnosis in machinery is accomplished through the analysis and interpretation of signals acquired from sensors and transducers. The impacts have a wide-band energy which often sets off some modes of resonance of the bearing elements. This process adds additional impulsive components into the vibration and results in vibration signals of a non-stationary nature. Wavelet analysis has been proved as a promising tool to overcome this problem. Several wavelet-based techniques have been presented for the



feature enhancement and feature extraction of transient and non-stationary signals [3]. These techniques are much more effective than traditional techniques and have been successfully used in the condition monitoring and fault diagnosis of mechanical systems [4].

However, any structure-borne vibration propagated through the neighboring structures will produce a background vibration in which the needed vibration components for diagnosis are often submerged, in particular during the early stage of failure development. If the background noise level is too high or when the bearing vibration signature level is too low, traditional techniques such as wavelet de-noising analysis are often ineffective in canceling the noise of such signals. Therefore, enhancing the signals before de-noising to extract the frequency components is a very important task in detecting the defect.

Moreover, for model-based identification methods in frequency domain it has been established that its performance can be affected by errors and the results could not be accurate. This means that a cause of error for the identification methods can also be the presence of noise in the vibration signals or errors in the order analysis [5].

In this work, an adaptive close-loop feedback control was applied to a bearing vibration signal buried in a background vibration noise in order to enhance this signal before apply a wavelet de-noising method to identify the frequency components of the rolling contact bearing.

## 2. ADAPTIVE FILTERING

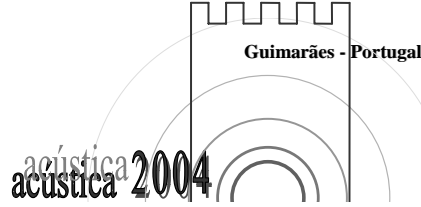
If accurate information of the signals to be processed is available, the designer can easily choose the most appropriate algorithm to process the signal. When dealing with signals whose statistical properties are unknown, fixed algorithms do not process these signals efficiently. The solution is to use an adaptive filter that automatically changes its characteristics by optimising the internal parameters.

The most widely used adaptive FIR filter structure is the transversal filter, also called tapped delay line, that implements an all zero transfer function. For this realization, the output of the filter  $y[n]$  is a linear combination of the filter coefficients, that yields a quadratic mean error ( $MSE=E\{|e[n]|^2\}$ , where  $E\{\}$  denotes the statistical expectation operator) function with a unique optimal solution.

The usual method of estimating a signal corrupted by additive noise is to pass the composite signal through a filter that tends to suppress the noise while leaving the signal relatively unchanged. The design of such filters is the domain of optimal filtering, which originated with the pioneering work of Wiener [6].

Noise cancelling is a variation of optimal filtering that is highly advantageous in many applications. It uses an auxiliary or reference input derived from one or more sensors located at points in the noise field where the signal is weak or undetectable. This input is filtered and subtracted from a primary input containing both signal and noise. As a result, the primary noise is attenuated or eliminated by cancellation.

If filtering and subtraction are controlled by an appropriate adaptive process, noise reduction can, in many cases, be accomplished with little risk of distorting the signal or increasing the output noise level [7].



In the signal enhancement application the reference signal consists of a desired signal  $x[n]$  that is corrupted by an additive noise  $N_1[n]$ . The input signal of the adaptive filter is a noise signal  $N_2[n]$  that is correlated with the interference signal  $N_1[n]$ , but uncorrelated with  $x[n]$ .

### 2.1. Normalized Least-Mean-Square

The least mean-square (LMS) is a search algorithm in which a simplification of the gradient vector computation is made possible by appropriately modifying the objective function. The convergence characteristics of the LMS can be shown for stationary environment and the convergence speed is dependent on the eigenvalue spread of the input signal correlation matrix [8]. Other features of the LMS are unbiased convergence in the mean to the Wiener solution and stable behaviour when implemented with finite-precision arithmetic.

The NLMS algorithm can be summarized by the following two equations:

$$e[n] = d[n] - \mathbf{w}^H[n] \mathbf{u}[n], \quad \text{and} \quad (1)$$

$$\mathbf{w}[n+1] = \mathbf{w}[n] + \frac{\mu}{\alpha + \|\mathbf{u}[n]\|^2} \mathbf{u}[n] e^*[n], \quad (2)$$

where  $n=1, 2, \dots$ ,  $e[n]$  is the error,  $d[n]$  is the desired signal,  $\mathbf{w}[n]$  is the vector of filter coefficients (taps),  $\mathbf{u}[n]$  is the vector of input signal,  $\alpha$  and  $\mu$  are positive constants, the superscript  $H$  denotes Hermitian transposition, the asterisk denotes complex conjugation, and the norm in Eq. (2) corresponds to the Euclidean norm. A sufficient condition for the NLMS algorithm to be convergent in mean square is that  $0 < \mu < 2$  [9]. If no previous information for the values of  $\mathbf{w}$  is available, then it is usual to initialise the algorithm with  $\mathbf{w}[0]=\mathbf{0}$ .

### 2.2. Recursive Least-Squares

Least-squares algorithms aim at the minimization of the sum of the squares of the difference between the desired signal and the model filter output. When new samples of the incoming signals are received at every iteration, the solution for the least-squares problem can be computed in recursive form resulting in the recursive least-squares (RLS) algorithm. This algorithm has excellent performance when working in time-varying environments. All these advantages come with the cost of increased computational complexity and some stability problems, which are not as critical in the LMS-based algorithms.

The RLS algorithm, for  $n=1,2,\dots$ , can be summarized by the following equations:

$$\mathbf{K}[n] = \frac{\mathbf{P}[n-1] \mathbf{u}[n]}{\lambda + \mathbf{u}^H[n] \mathbf{P}[n-1] \mathbf{u}[n]}, \quad (3)$$

$$\xi[n] = d[n] - \mathbf{w}^H[n-1] \mathbf{u}[n], \quad (4)$$

$$\mathbf{w}[n] = \mathbf{w}[n-1] + \mathbf{K}[n] \xi^*[n], \quad \text{and} \quad (5)$$

$$\mathbf{P}[n] = \frac{\lambda}{\mathbf{P}[n-1] - \mathbf{K}[n] \mathbf{u}^H[n] \mathbf{P}[n-1]}, \quad (6)$$



where  $\lambda$  is a constant, called the forgetting factor, which is close to but less than 1 and  $\xi[n]$  is called the *a posteriori* error. The initialisation for the algorithm is  $\mathbf{w}[0]=\mathbf{0}$  and for the matrix  $\mathbf{P}[0]=\delta^{-1}\mathbf{I}$ , where  $\mathbf{I}$  is the identity matrix and  $\delta \ll 1$ .

### 2.3. Wavelet de-noising

In several research fields, a common problem consists of recovering a true signal from incomplete, indirect or noisy data. The development of fast computers has allowed the practical implementation of wavelets that help in solving this problem, through a technique called wavelet shrinkage and thresholding methods [10, 11].

Decomposing a data set using discrete wavelet transform (DWT) is analogous to use filters that act as averaging filters and others that produce details. Some details in the data set correspond to the resulting wavelet coefficients. These coefficients can be used later in an inverse wavelet transformation to reconstruct the data set. If the details in the data set are small, they can be omitted without substantially affecting the main features of the data set. Therefore, thresholding means to set to zero all the coefficients that are less than a particular threshold. This process generally gives a low-pass and smoother version of the original noisy signal. The objective of wavelet de-noising is to suppress the additive noise  $N_1[n]$  from a signal  $s[n]$ , where  $s[n]=x[n]+N_1[n]$ . The signal  $s[n]$  is first decomposed into  $L$ -level of wavelet transform. Then, for noise suppression, the thresholding of the resultant wavelet coefficients is performed. The thresholding is based on a value  $\delta$  that is used to compare with all the detailed coefficients. Two types of thresholding are more popular:

- 1) Hard thresholding, which is the usual process of setting to zero the coefficients whose absolute values are lower than the threshold, and
- 2) Soft thresholding, which is an extension of hard thresholding by first setting to zero the coefficients whose absolute values are lower than the threshold and then shrinking the nonzero coefficients toward zero.

Then, in summary, the technique of wavelet de-noising consists in transforming, thresholding and inverse-transforming the signal [12].

## 3. RESULTS

An experiment was conducted to acquire real signals for testing an adaptive rolling contact bearing vibration signal enhancement system. The experimental scheme is shown in Fig. 1, where some of the blocks indicate the filtering process. The isolation of the rolling bearing to be analysed and the isolation of other parts of the machine were removed in order to increase the effect of the structure-borne noise vibration which contaminated the signal of the bearing. The vibration produced by the bearing was sensed by means of an ICP accelerometer mounted on the bearing cover. A second ICP accelerometer was attached to a selected point on the test rig base in order to measure the vibration reference signal. Therefore, the accelerometer in the bearing sensed the real signal  $x[n]$  plus the contaminating noise  $N_1[n]$ , and the second accelerometer sensed a reference signal  $N_2[n]$ . Both signals were digitally recorded simultaneously, using a sampling frequency of 1,024 Hz during 10 seconds, so the number of samples collected for each channel was 10,024. A multi-channel digital data acquisition system was used for this purpose. To avoid aliasing, the signals were processed

through low-pass filters with cut-off frequency of 500 Hz. The signals were then saved as data files in order to be processed later in a workstation lab. The nominal rotational speed of the axis was 780 rpm (13 Hz).

As can be seen from Fig. 1, the error signal for this application is given by

$$e[n] = x[n] + N_1[n] - y[n] = x[n] + N_1[n] - \sum_{k=1}^M w_k N_2[n-k], \quad (7)$$

where  $M$  is the number of filter coefficients (taps). Then, the resulting  $MSE$  (quadratic mean error for real values), is

$$MSE = E\{e^2[n]\} = E\{x^2[n]\} + E\{(N_1[n] - y[n])^2\}, \quad (8)$$

where it has been assumed that  $x[n]$  is uncorrelated with  $N_1[n]$  and  $N_2[n]$ . Equation (8) shows that if the adaptive filter, having  $N_2[n]$  as the input signal, is able to perfectly predict the signal  $N_1[n]$ , then the minimum value of  $MSE$  is given by  $\eta_{\min} = E\{x^2[n]\}$ , where the error signal, in this situation, is the desired signal. The effectiveness of the signal enhancement scheme will depend on the correlation between  $N_1[n]$  and  $N_2[n]$ .

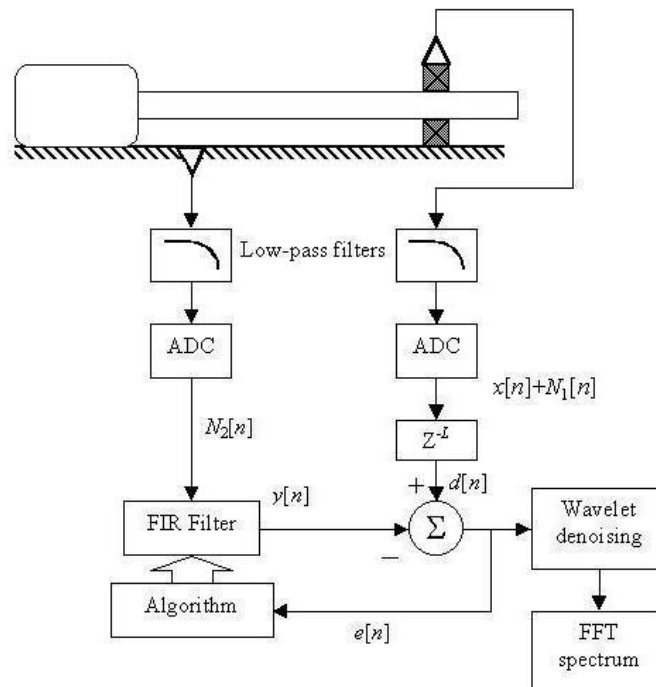


Figure 1 – *Experimental set-up*

From Fig. 1 it can be seen that a delay  $Z^{-L}$  is applied to the input. In some applications, it is recommended to include this delay of  $L$  samples in the reference signal or in the input signal,

such that their relative delay yields a maximum cross-correlation between  $y[n]$  and  $N_1[n]$ , reducing the  $MSE$  [8]. This delay provides a kind of synchronization between the signals involved. The NLMS and RLS algorithms presented in Eqs. (1)-(6) were implemented in a computer code to test the performance with the real data.

The following results were obtained with a 10-tap implementation of the adaptive FIR filter shown in Fig. 1. The NLMS and RLS algorithms were used as the adaptive weight-control mechanism.

Figure 2 displays the results of the time signatures at the input and at the output of the FIR filter using both NLMS and RLS adaptive algorithms.

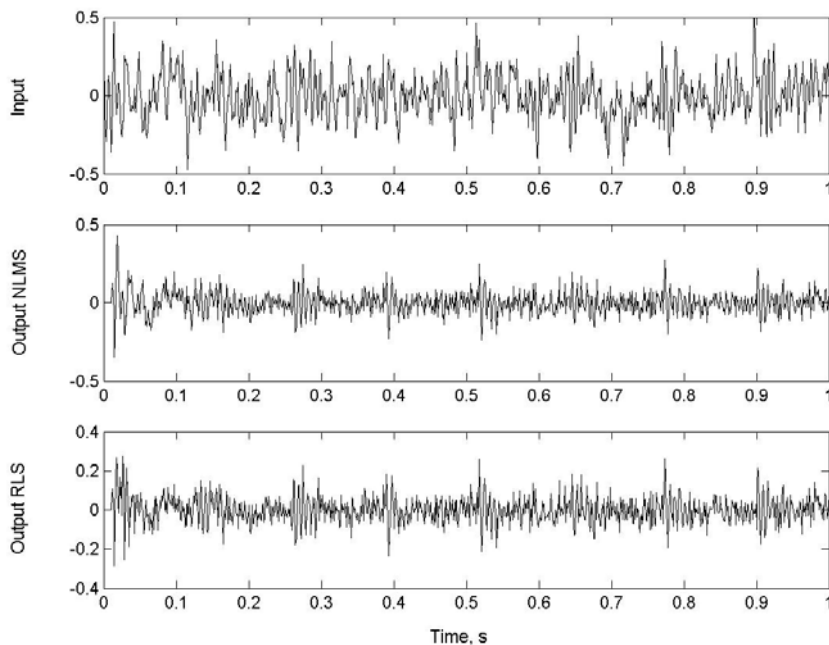


Figure 2 – Time signals at the input and at the output of the FIR filter using the NLMS and RLS adaptive algorithms

The optimal results for the NLMS algorithm were obtained using  $\mu=0.09$  and a delay  $L=5$ . For the RLS algorithm the optimal results were achieved using  $\lambda=0.999$  and a delay of  $L=5$ . It was impossible to achieve convergence at the filter's output without using a delay at the input.

It is seen that the RLS algorithm shows faster convergence than the NLMS, as usual for stationary signals [9]. However, other authors have reported the opposite when dealing with non-stationary signals [13].

In addition, wavelet transforms were used for de-noising the data. The data was transformed into an orthogonal wavelet basis. Thresholding was applied to shrink the noisy wavelet coefficients and then the modified wavelet coefficients were used to reconstruct the signal by the inverse wavelet transform. Several de-noising schemes were applied, using several wavelets and thresholding methods. For the de-noising process, the best results were obtained from the combination of db4 (Daubechies 4) wavelet [14] and hard thresholding with an interval dependent threshold setting.

In Fig. 5 the results of FFT spectra are presented. Figure 5(A) shows the results without using any kind of noise cancellation. Figure 5(B) shows the results using wavelet transform for de-noising the measured signal without using the adaptive FIR filtering. Finally, Fig. 5(C) presents the results when wavelet de-noising is applied after the signal has been enhanced using the adaptive FIR filter. Clearly, the best results are achieved when both adaptive filtering and wavelet de-noising techniques are applied, so the signal-to-noise ratio at all frequency regions is increased significantly. The upper end of the high-frequency range remained relatively clean due to the low levels of extraneous signal components in this region. In Fig. 5(C), it can be distinguished the presence of a spike at the very low frequency region, which corresponds to the shaft rotational speed (modulation frequency). Also, a spectral peak around 150 Hz and side-bands with a spacing of 8 Hz are observed. This components can be indicating some defect or resonance in the bearing. A method to study this effect is to change the rotational velocity of the shaft. Certainly, the resonances should disappear and only the forcing frequencies would remain.

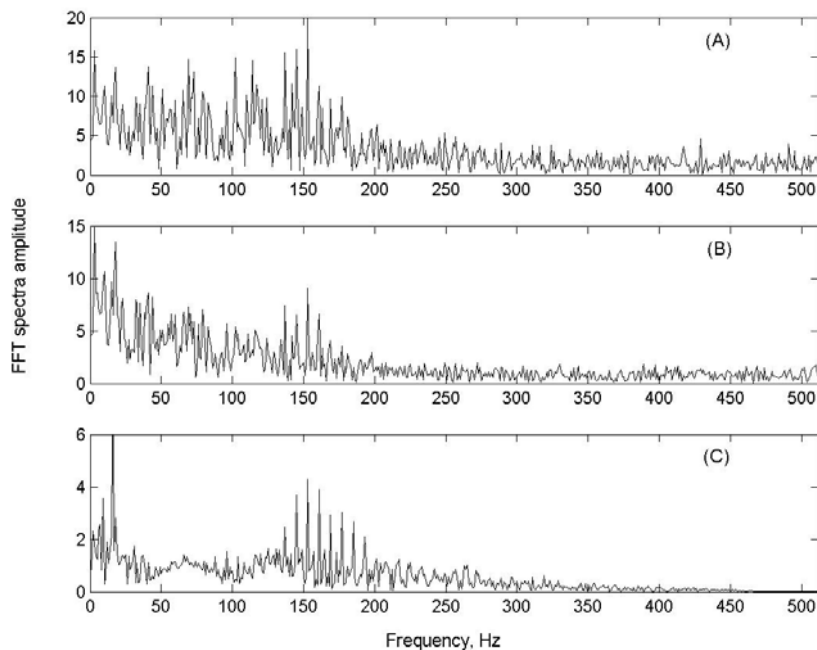


Figure 2 – Results of FFT spectra: (A) without noise cancellation; (B) with wavelet de-noising; (C) with wavelet de-noising combined with adaptive FIR filter.

## 4. CONCLUSION

The proposed technique constitutes a successful application of the adaptive filtering combined with wavelet thresholding for vibration signal enhancement when the useful vibration signatures become submerged within the noise and interference from external signals. For this particular application, it can be concluded that the rate of convergence of the RLS algorithm is faster than that of the NLMS, but this is attained at the expense of a large increase in



computational complexity. In addition, the correct selection of the analysing wavelet with different properties is of critical importance for enhancing the fault features in the wavelet analysis.

When just using wavelet de-noising, significant gains in the signal-to-noise ratio are evident compared to the direct FFT analysis of the noisy measured signal. However, more modest improvements are achieved when compared to the application of a previous adaptive signal enhancement of the measured signal. Used in conjunction with a wavelet de-noising analysis, the technique provides promising enhanced diagnostic capabilities. The extracted signal also provides a sensitive and accurate basis from which the severity of localised rolling-element bearing faults can be analysed.

Further research can investigate the improvement of the adaptive signal enhancement system by using several vibration reference sensors to be mounted at various locations on the machine.

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