ADVANCED ELECTRIC LUMPED MODEL OF A TRANSMISIÓN LINE LOUDSPEAKER SYSTEM

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ABSTRACT

Many models based on physical properties of damped pipes have been proposed to characterize the Transmission Line Loudspeaker Systems. Unfortunately, Thiele/Small parameters, usually employed in enclosure designs, are not useful as parameters of design for these loudspeakers. Derived from Locanthi horn model, an advanced circuit with lumped elements is presented. It can accommodate arbitrary flare shapes and damping. The influence of the main empirical parameters is tested on a prototype to validate the model. The main contribution of this paper consists on the model of the acoustic radiation impedance at the end of a long tube.

INTRODUCTION

A transmission Line Loudspeaker System is a sort of damped acoustic pipe consisting of an open-back loudspeaker cabinet, in which the diaphragm moves freely. This results in an extended bass response to low frequencies with a smoother transition into the mid-range.
The air in the line presents a physical impedance or damper to the motion of the driver, controlling both the excursion of the cone and the driver's natural resonance. Several theoretical models have been proposed to understand the system behaviour. First, Locanthi [1] and then Augsperger [2] presented an electric model based on lossy horn analogous circuit. A damped pipe or transmission line can be thought and analyse as an excited horn with certain losses. Nowadays, there is no accepted successful model that satisfy the designers expectations, as it is done with classic designs like Bass Reflex and Closed enclosures [3] y [4].

Despite this, the influence of the main parameters affecting the TL performance are known [5]: line length or quarter wave resonance frequency, cross sectional areas and stuffing densities. In this paper, an advanced electrical model is presented and simulated for different geometries and dampers. The output is a lumped electric impedance to model the effects of the air surrounding the end of the tube.

ANALOGOUS CIRCUIT MODEL

A modified and improved analogous circuit is presented in figure 1. It includes the typical mobility elements of the loudspeaker followed by an LCR ladder and a lumped impedance. This last represents the acoustic impedance seen by a plane circular piston in one end of a long tube, which is given by [6]:

$$Z_A = 4\delta^2 \frac{\rho_0 c}{\pi a^2} + j\delta \frac{\rho_0 \omega}{\pi a}$$

where δ=0.6133 is a constant obtained after some algebra. A suitable model to describe the air at the flush limit of the tube is that for the analogous circuit for a piston in a long tube. It has the same form as that for the piston in a infinite baffle, only the element values are different.

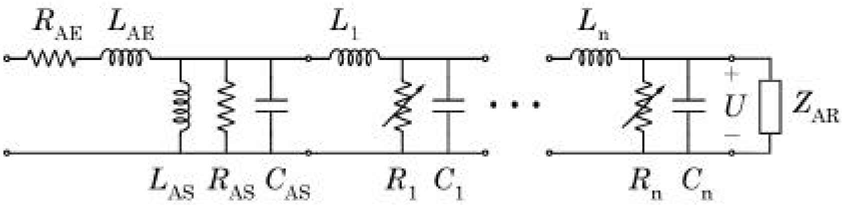

Figure 1. *Transmission line acoustical circuit including the impedance seen by the mouth*.

In this circuit, each LCR section is equivalent to an arbitrary shape of specific length and volume filled with stuffing material. Therefore, the coil represents the air compliance, the capacitor models the acoustic mass and the shunt resistance symbolizes the damping losses. The bandwidth in which the model gives satisfactory results is determined by the number of sections. As a rule of thumb, the maximum frequency is obtained multiplying the number of sections by 13, but a more conservative value would be 10.
The circuit elements comprising each *n*th section of the transmission line are derived from the basic definitions of acoustic compliance and mass.

$$L_n = \frac{S_0 K_s x}{\rho_0 c^2} = \frac{S_n x}{\rho_0 c^2} = \frac{V_n}{\rho_0 c^2} = C_A$$

$$C_n = \frac{\rho_0 x}{S_0 K_s} = \frac{\rho_0 x}{S_n} = M_A$$

where S_0 and S_n are throat and *n*th section areas, K_s is the ratio between these two areas S_0/S_n and x is the length of each section. Since pressure and volume velocity are the main parameters in the model, only acoustical values are considered.

In any electric ladder connexion, basic quadripoles provide a very efficient method of performing computer based calculations. It is possible then to relate input and output signals through a single matrix. Due to the features of ladder connexion, transmission quadripoles have been considered (figure 2).

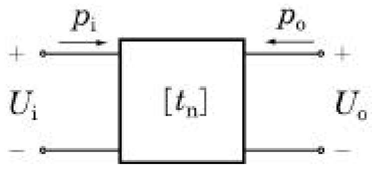

Figure 2. *Transmission quadripole modelling an arbitrary damped pipe. Input and output pressure and volume velocity define T parameters.*

Transmission parameters *A*, *B*, *C* and *D* are defined as follow:

$$A = \left.\frac{U_i}{U_o}\right|_{p_o=0} \qquad B = \left.\frac{U_i}{-p_o}\right|_{U_o=0} \qquad C = \left.\frac{p_i}{U_o}\right|_{p_o=0} \qquad D = \left.\frac{p_i}{-p_o}\right|_{U_o=0}$$

It is straightforward to use the above equations to show that $[t_n]$ can be written in the following form:

$$[t_n] = \begin{bmatrix} jw\dfrac{L_n}{R_n} - w^2 L_n C_n + 1 & jwL_n \\ \dfrac{1}{R_n} + jwC_n & 1 \end{bmatrix}$$

Figure 3 shows the complete transmission line acoustical circuit in which the LCR ladder is replaced by a single transmission matrix.

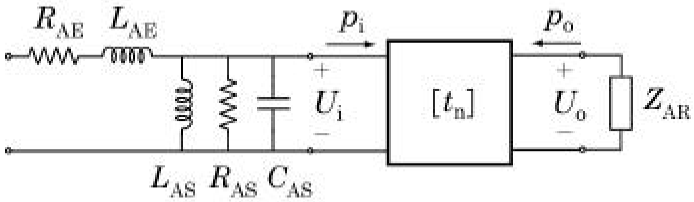

Figure 3. *Transmission line acoustical circuit including generated T-matrix.*

The acoustic power P_{AR} radiated to the front of the mouth is the power dissipated in the impedance Z_{AR} in figure 3. This power can be written in the form

$$P_{AR} = \frac{1}{2}|U_0|\mathrm{Re}[Z_{AR}(jw)]$$

The system electric impedance is that for the voice coil terminals, affected by both speaker and pipe behaviour. The latter can be expressed

$$Z_{ee} = R_E + jwL_E + \frac{(Bl)^2}{S_d^2\left(\dfrac{1}{jwL_{AS}} + jwC_{AS} + \dfrac{1}{R_{AS}} + Y_i\right)}$$

where Y_i is the electric admittance seen by the driver. The rear part of the diaphragm is charged with the air inside the pipe in the same way a closed box is affected by the compliance of the enclosure [4]. In figure 3, quadripole admittance is given by

$$Y_i = \left.\frac{U_i}{p_i}\right|_{p_0 = U_0 Z_{AR}}$$

The final analogous circuit is presented in figure 4.

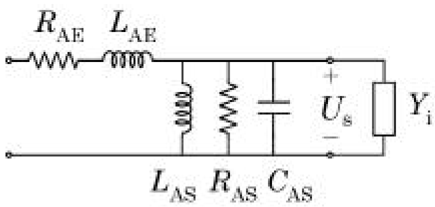

Figure 4. *Transmission line acoustical circuit with lumped admittance representing an arbitrary damped pipe.*

The on-axis sound pressure level is given by

$$SPL = 20\log\frac{|U_s Z_{AR}|}{2\cdot 10^{-5}} \quad \text{(dB)}$$

where the product of the rear cone volume velocity U_S and the acoustic impedance seen by a piston in the end of a long tube gives the complex pressure.

To solve for the rear radiation, volume velocity and pressure mouth conditions are given by relating the transmission parameters.

$$U_t = -U_s \qquad\qquad Y_i = \left.\frac{U_i}{p_i}\right|_{p_0 = U_0 Z_{AR}}$$

By using matrix properties, the pair of output values U_0 and p_0 is solved as follows:

$$\begin{pmatrix} U_0 \\ p_0 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}^{-1} \begin{pmatrix} U_t \\ p_t \end{pmatrix}$$

where output acoustic volume velocity is the required variable to solve for the output sound level pressure, as mentioned above.

SIMULATION AND TEST PROCEDURE

An algorithm to model the transmission line was programmed on Matlab, in which a 300 LCR sections was used to generate the required quadripole. This guarantees a usable bandwidth of, at least, 3 kHz together with few seconds computational time. To validate the model, a series of sound pressure and electrical impedance was performed on a modular transmission line prototype. With the ability to change physical features on the prototype, it is possible to simulate and test the influence of the three main parameters affecting the TL performance.

Pressure response measurements were carried out both in on-axis speaker and in TL mouth with a pair of GRAS 40AF condenser microphones. The system response is the complex sum of loudspeaker and pipe outputs. An Audio Precision System One Dual Domain analyser was configured to measure in non anechoic environment by using MLS technique. Impedance curves were also run on the AP analyser using built in voltage divider technique [7]. Before starting the experiments, an accurate test to determinate de Thiele/Small parameters of the chosen driver was made.

Figure 5 illustrates the pressure response of a 1.47 m effective length circular undamped pipe with section ratio S_0/S_D=1. A typical 5” low-mid driver was flush mounted in the centre of one end. It presents a resonance frequency of 108 Hz and a total quality factor of 0.61.

In the absence of damping material, the pipe resonates at odd numbers of quarter wavelength frequencies. Since the speaker is acoustically charged at these frequencies, the cone output is attenuated whereas mouth radiation is augmented. The nominal $\lambda/4$ frequency modes after including an end correction to the line length is 58.3 Hz. An unflanged exit boundary condition was applied to obtain a 1.47 m effective length. As seen in figure 5 a) in bold line, there is a slight change in the position of the first modes. Surprisingly, this low frequency behaviour is that for the pipe without end correction.

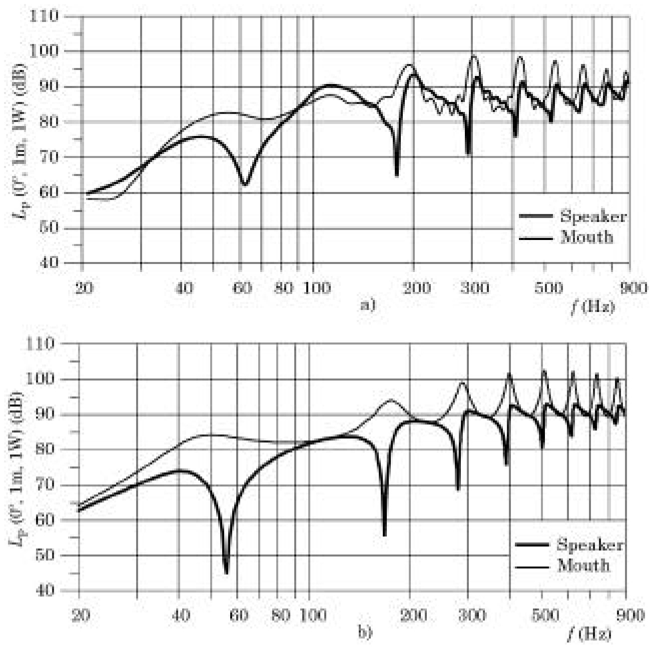

Figure 5. *a) Measured pressure response of undamped system, b) Response of analogous circuit.*

Analogous patterns are displayed in electric input impedance Z_{ee} for the same undamped pipe (see figure 6). Only the very first mode has been shifted 8 Hz approximately. Moreover, its impedance amplitude is fairly high in comparison with simulation results.

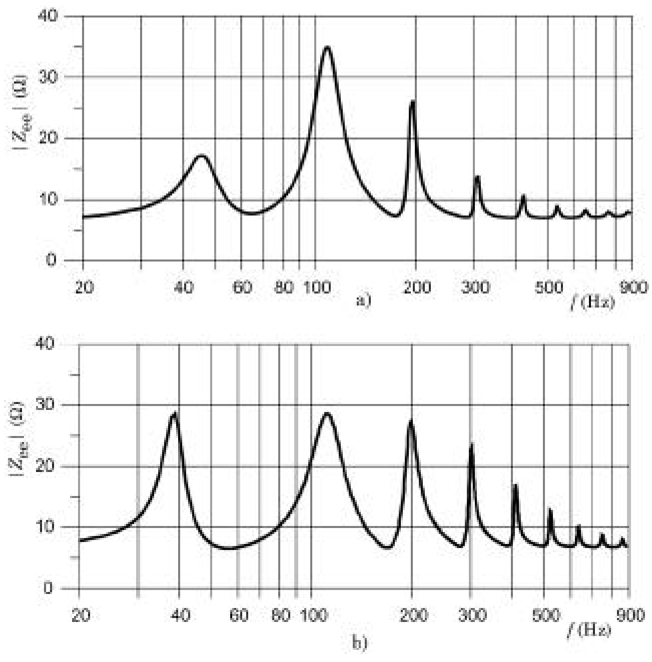

Figure 6. *a) Measured impedance response of undamped system, b) Response of analogous circuit.*

Finally, in figure 7, several simulated stuffing density behaviours together with a set of measurements are presented. The ability of damped material to smooth the curve is clearly shown. The latter also occurs in pressure response for any given cross sectional area and line length [5].

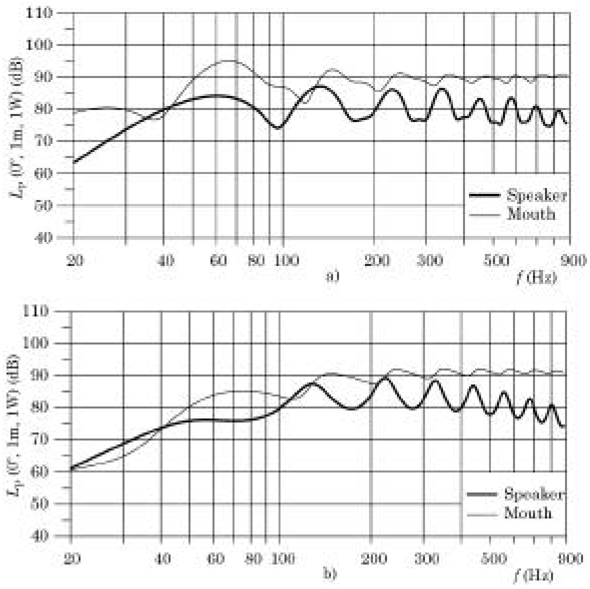

Figure 7. *a) Measured impedance response of a 5 kg/m^3 damped system, b) Response of analogous circuit.*

CONCLUSION

Transmission line loudspeaker is a sort of low frequency radiating system that can be modelled by using a ladder electrical network. It comprises *n* LCR meshes representing physical sections of the line. Since it is possible to change each section individually, all sort of geometrical modifications can be simulated prior to construction, for example, the so-called tapered line.

The main modification to Augsperger circuit is the addition of a lumped electrical impedance representing the impedance seen by a plane circular piston in one end of a long tube. By considering the real part of such impedance, an enhancement in the curve shape is achieved, especially in the quarter wavelength frequency peaks.

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