INTERACTIONS OF (TWO) CLOSED-BOX LOUDSPEAKER SYSTEMS MOUNTED IN DIFFERENT CONFIGURATIONS

Pacs: 43.38.Hz

Adam Véronique
Laboratory of Electromagnetics and Acoustics (LEMA) - Swiss Federal Institute of Technology
Ecole Polytechnique Fédérale de Lausanne (EPFL) - STI – TOP – LEMA – Bâtiment B
CH – 1015 Lausanne
Switzerland
Tel: +41 021 693 46 30
Fax: +41 021 693 26 73
veronique.adam@epfl.ch

ABSTRACT

The aim of this paper is to predict whether the interactions between two closed-box loudspeakers have to be taken into account during their design phase. The study comprises a theoretical part involving modelling and calculation and a validation experimental part. Based on TS parameters, Rayleigh's surface integral and geometrical and unified theories of diffraction, the calculations enable modifications of loudspeaker behaviour in modulus and phase to be predicted. The effects of one loudspeaker on the other are studied as modifications of the system volume velocity (in-box) and input impedance. Finally, the measurements are compared with the calculations in order to validate the study.

INTRODUCTION

Within the framework of an outdoor active noise project, a need arose, namely to establish whether electrodynamic loudspeakers are likely to be affected by the primary noise or by the interactions between them. The aim of the study is to provide computation and measurement methods allowing to predict the interaction effects between two loudspeakers mounted in different configurations. The study comprises two main parts involving modelling, theory and calculations for the first part, and experimental validations in an anechoic chamber for the second one.

The equivalent circuit modelling of an electrodynamic loudspeaker shows that its behaviour can be completely determined from its input impedance [1]. The study starts with the reflection that the modifications in loudspeaker behaviour due to the presence of an impinging sound field applied on its radiating membrane could then also be analysed in the same way.

The chosen process then consists in analysing the loudspeaker modifications in behaviour, no longer as variations of its radiation impedance, but as variations of its electrical input impedance. Being located at the loudspeaker terminals, the electrical quantities (current and voltage) offer the advantage of being easily measurable. However, as this method is only effective for frequencies located close to the system resonance, it was decided to complete this study by volume velocity computations and measurements. The latter are obtained by measuring the sound pressure in the enclosure, as far as the box behaves like an acoustic compliance.

This paper is based on three previous works summarised and referred to as follows. In a preliminary work [5], the analysis and synthesis of closed-box loudspeakers were studied taking into account the quality of their phase responses. A second study consisted in measuring the modifications occurring on a closed-box loudspeaker subjected to an impinging sinusoidal sound field [6]. Finally, a third study analysed three different loudspeaker configurations, beginning with the simple case of two adjacent closed-box loudspeakers mounted in an IEC baffle, and ending with more realistic ones corresponding to the configurations of two adjacent and distant closed-box loudspeakers [7]. The present paper completes the study carried out in [7] by theoretical developments and experimental validation of the predictions carried out in the three cases.

THEORETICAL PREDICTIONS

When loudspeaker size is considered to be small compared to the wavelength in air, its membrane is assumed to behave like a flat rigid piston of surface S_d, radius a and uniform velocity $\underline{v}$. Mounted in an infinite baffle or in a closed box in order to separate backward and forward radiations, the loudspeaker is generally described by a lumped-constant circuit simplified in equivalent acoustical, mechanical or electrical circuits depending on the needs. Based on Thiele and Small parameters, these circuits enable the input impedance $\underline{Z}$ and volume velocity $\underline{q}$ to be readily calculated from input voltage as functions of the frequency. The sound pressure is then calculated without going through the radiated acoustic power, allowing the phase response of the system to be obtained.

In order to establish the necessary basis for calculating the interactions between two closed-box loudspeakers, let us at first treat the theory related to two closed-box pistons mounted in an infinite baffle. Due to the fact that the Green's function is known, the sound pressure radiated by one baffled piston at a distance d can be calculated according to the Rayleigh's surface integral, which offers the advantage of being an exact solution of the standard boundary integral equation method [1]:

$$\underline{p} = \frac{jZ_c k\underline{v}}{2\boldsymbol{p}} \int_{S_d} \frac{e^{-jkd}}{d} dS \qquad (1)$$

Without taking into account the interactions between the loudspeakers LSP_0 and LSP_1, the total sound pressure is calculated via the principle of superposition.

The medium reaction force applied on each loudspeaker is then calculated according to the sound pressure in the immediate vicinity of the driver membrane. This integral was solved by Lord Rayleigh in terms of Bessel and Struve functions [1]:

$$\underline{F}_{re} = \underline{q}\,\boldsymbol{r}c\left\{\left[1-\frac{J_1(2ka)}{ka}\right]+j\left[\frac{S_1(2ka)}{ka}\right]\right\} \qquad (2)$$

The radiation mass m_{ar} and radiation resistance R_{ar} are then deduced from relation (2).

When the interactions are taken into account, the behaviour of each loudspeaker is modified by the force exerted on its membrane by the other one [4]:

$$\underline{F}_{01,10} = \frac{jZ_c k\underline{v}_{1,0}}{2\boldsymbol{p}} \int_0^{a_{0,1}} \int_0^{2\boldsymbol{p}} \int_{-sin^{-1}(a_{1,0}/r_0)}^{sin^{-1}(a_{1,0}/r_0)} \int_{r_0 cos\,\boldsymbol{a}-\sqrt{a_{1,0}^2-r_0^2 sin^2\boldsymbol{a}}}^{r_0 cos\,\boldsymbol{a}+\sqrt{a_{1,0}^2-r_0^2 sin^2\boldsymbol{a}}} \boldsymbol{r}' e^{-jkr_s} d\boldsymbol{r}'\, d\boldsymbol{y} dr_s d\boldsymbol{a} \qquad (3)$$

Figure 1 shows the geometry necessary to the understanding of the analytical expression (3).

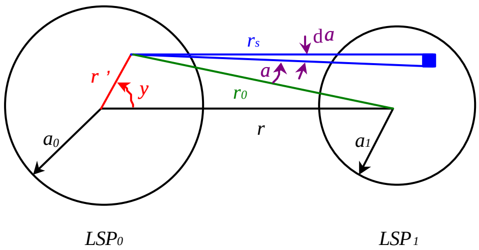

Figure 1 **-** Geometry necessary to the calculation of the analytical force (equ. 3)

As this integral has no main analytical solution, except in particular cases, the purpose in this paper is nevertheless to be able to compute it. That is why a discrete approach is proposed, which requires a double subdivision (in n and m elements) of both piston surfaces taking care to avoid element superpositions. The computation of the forces $\underline{F}_{01}$ and $\underline{F}_{10}$ enables then the radiation impedances to be calculated as: $\underline{Z}_{ar01} = \underline{F}_{01} /(S_{d0}\ \underline{q}_0)$ *and* $\underline{Z}_{ar10} = \underline{F}_{10} /(S_{d1}\ \underline{q}_1)$.

Subjected to the sound field of the other one, the modified volume velocity $\underline{q}'$ and input impedance $\underline{Z}'$ of each piston are calculated in replacing $\underline{Z}_{ar0}$ by $\underline{Z}'_{ar0} = \underline{Z}_{ar0} + \underline{Z}_{ar01}$ and $\underline{Z}_{ar1}$ by $\underline{Z}'_{ar1} = \underline{Z}_{ar1} + \underline{Z}_{ar10}$.

Whilst the assembly of a loudspeakers mounted in an infinite baffle is commonly considered to be the ideal theoretical one due to the absence of any corrupting phenomena, it is of small interest in practice. Normally, loudspeakers are mounted in closed or vented boxes which are designed on the basis of driver types and application requirements.

Whilst the enclosure does away with direct interferences resulting from front and rear driver radiations, the diffraction at the enclosure edges has now to be taken into account. According to Keller's Geometrical Theory of Diffraction (GTD) [2], the sound pressure can be directly calculated by adding direct and diffracted waves (fig. 2). The scope of the application related to this approximate theory is restricted to wavelengths much smaller than the source, due to the fact that propagation is studied here as a local phenomenon.

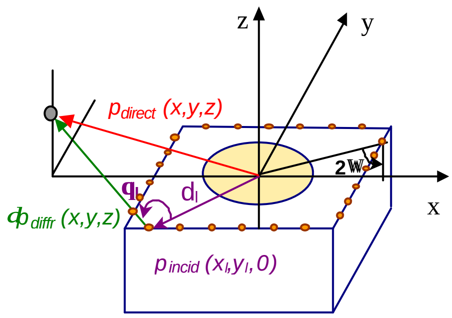

Figure 2 - Depiction of angles and distances related to the GTD calculation

The direct contribution corresponds to the radiation of a flat piston assumed to be mounted on an infinite baffle, and the diffracted contribution is calculated from incident waves interacting with the enclosure edges divided into l scattered elements. At each one of these points of diffraction, the sound pressure is calculated as:

$$\underline{p}_{incid}(x_l, y_l, 0) = \frac{jZ_c k}{4\pi} \sum_{i=1}^{n} \frac{\underline{q}_i}{d_{li}} e^{-jkd_{li}} \qquad (4)$$

The diffracted field is thus computed from l virtual secondary sources of $\underline{p}_{incid}$ excitation value and placed at the points of diffraction. The sound pressure diffracted by each boundary element of wedge angle 2Ω is given by:

$$\partial \underline{p}_{diffr}(x,y,z) = \underline{p}_{incid}(x_l, y_l, 0) \frac{e^{-jk\sqrt{(x-x_l)^2+(y-y_l)^2+z^2}}}{4\pi\sqrt{(x-x_l)^2+(y-y_l)^2+z^2}} F_{gtd}(x_l, y_l, \Omega) \qquad \text{with}$$

$$F_{gtd}(x_l, y_l, \Omega) = \frac{1}{\nu} \sin\frac{\pi}{\nu} \left[\frac{1}{\cos\frac{\pi}{\nu} - \cos\frac{\theta_l}{\nu}} + \frac{1}{\cos\frac{\pi}{\nu} + \cos\frac{2\pi - \theta_l - 2\Omega}{\nu}} \right] \text{and } \nu = (2\pi - 2\Omega)/\pi \quad (5)$$

As we can see, the diffraction amplitude becomes infinite close to the shadow boundary. Given that this phenomenon is not physical, Vanderkooy has limited his calculation method to observation angles q < 130° [2]. The total sound pressure is given by:

$$\underline{p}(x,y,z) = \underline{p}_{direct} + \sum_{i=1}^{l} \partial \underline{p}_{diffr} \qquad (6)$$

For observation points located close to shadow boundaries, the directivity function F_{gtd} has to be completed by a transition-region correction factor based on Fresnel integrals. This improved theory, called the Uniform Theory of Diffraction (UTD) will be used to calculate the interactions between two separate closed-box loudspeakers.

The $\underline{Z}$ and $\underline{q}$ modifications are then calculated according to the method used in the configuration of two closed-box loudspeakers mounted in an infinite baffle. These numerical computations have to be iterated several times in order to tend toward the solutions. The iteration number will depend on the tolerance margin chosen regarding to the required accuracy. In our case, the iteration number is considered to be acceptable when the difference between two consecutive iterations is bounded by $\pm 10^{-4}$ ($\pm$ 0.01%) for the modulus and $\pm$ 0.01 degree for the phase.

EXPERIMENTAL VALIDATIONS

The experimental validations are carried out in comparing computed and measured $\underline{Z}$ and $\underline{q}$ modifications of LSP_0 subjected to LSP_1 in the three configurations described in introduction. The numbers of surface subdivisions applied to both drivers are set to n = 400 and m = 36. The measurements are carried out using white noise excitations in a ratio $\underline{U}_1$ /$\underline{U}_0$ of 10. The calculation accuracy requires some preliminary measurements; then in addition to the Thiele and Small parameters of each driver, the measured function $\underline{U}_1$ /$\underline{U}_0$ is also introduced in calculation data (polynomial approximation of order 25).

According to the frequency, figure 3 shows LSP_0 computed and measured $\underline{Z}$ and $\underline{q}$ modifications in modulus and phase, in the configuration of two closed-box loudspeakers (LSP_0 and LSP_1) mounted side-by-side in the same IEC baffle ($r \approx 3.5\ a$).

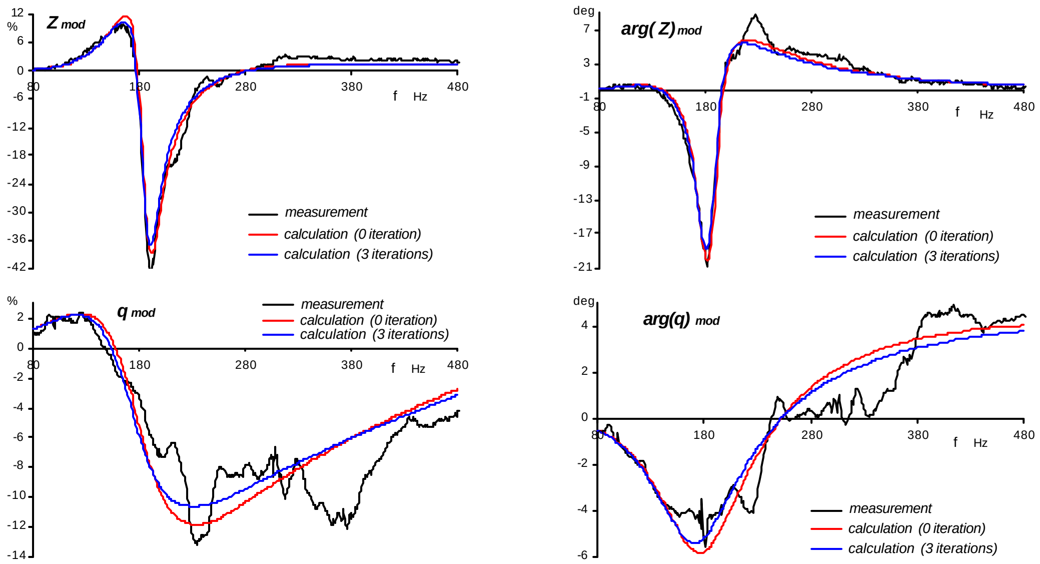

Figure 3 **-** Comparison between measured and calculated modifications of input impedance and volume velocity in modulus and phase – configuration of two closed-box loudspeakers mounted in an IEC baffle

In comparison with this previous configuration, the interaction calculations between two adjacent closed-box loudspeakers must be completed by the GTD developed in the theoretical part. As done in the baffled case, figure 4 shows LSP_0 computed and measured $\underline{Z}$ and $\underline{q}$ modifications in modulus and phase. The edge division number is set to 36 elements.

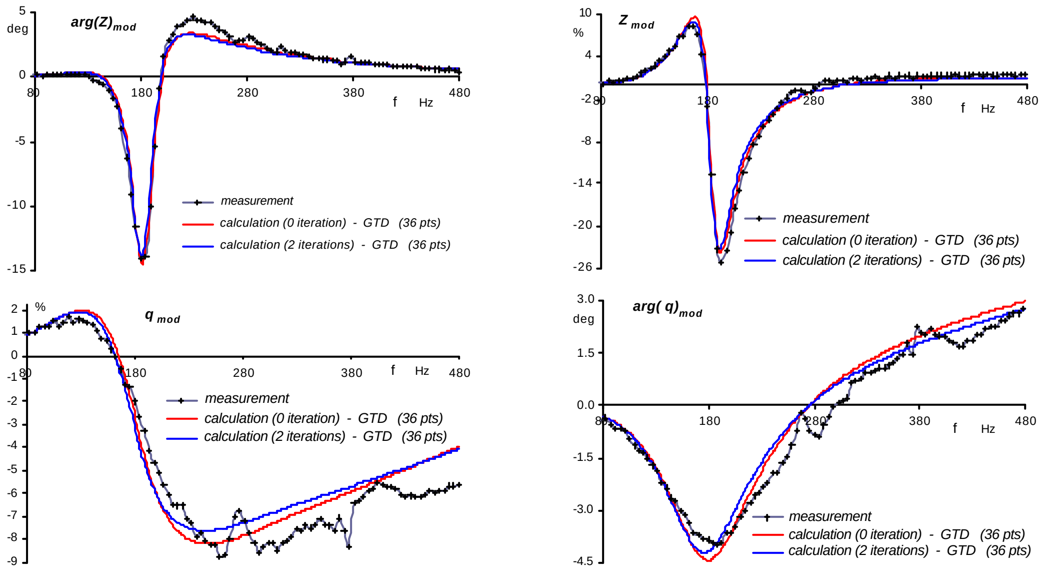

Figure 4 **-** Comparison between measured and calculated modifications of input impedance and volume velocity in modulus and phase – configuration of two adajcent closed-box loudspeakers

Representing a pseudo realistic case, the configuration of two separated closed-box loudspeakers makes up the outcome of this paper. The calculations are carried out in such a way as to compare the two diffraction methods GTD with q = 0° and UTD. Figure 5 shows the comparison between measured and calculated modifications in the case of two closed-box loudspeakers separated by 10 cm.

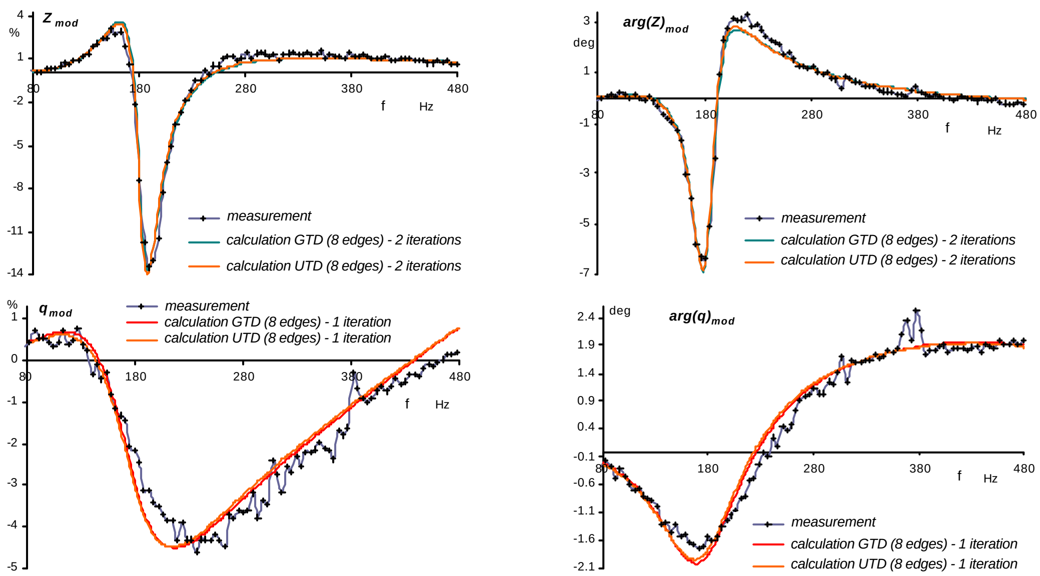

Figure 5 **-** Comparison between measured and calculated modifications of input impedance and volume velocity in modulus and phase – configuration of two distant closed-box loudspeakers

CONCLUSION

It is not overly presumptive to assert that the measurement results enable the theoretical predictions to be validated in all the three configurations studied in this paper.

The main conclusion lies in the fact that the effects of a closed-box loudspeaker on another loudspeaker can be analysed in terms of input impedance, volume velocity and therefore sound pressure. The orders of magnitude are such that the modifications can be calculated using numerical treatments. The latter turned out to be effective already after three iterations in the worst case (two loudspeakers mounted on an infinite baffle).

GLOSSARY

a	m	Radius of the projected surface S_d
c	m/s	Speed of sound
d	m	Distance between piston centre and observation point
f	Hz	Frequency
k	rad/m	Wave number
l	1	Number of edge divisions
m	1	Second piston surface division number
m_{ar}	kg/m^4	Acoustic radiation mass
n	1	First piston surface division number
$\underline{p}$	Pa	Sound pressure
$\underline{q}$	m^3/s	Piston volume velocity
r	m	Distance between pistons centres
$\underline{v}$	m/s	Piston membrane velocity
$\underline{F}_{01,10}$	N	$LSP_{1,0}$ force impinging on $LSP_{0,1}$ membrane
$\underline{F}_{re}$	N	Medium reaction force
R_{ar}	Ω_a	Acoustic radiation resistance
S_d	m^2	Effective projected surface area of the loudspeaker Diaphragm
U	V	Voltage at the loudspeaker terminals
$\underline{Z}$	Ω	Electrical input impedance
$\underline{Z}_{ar}$	Ω_a	Acoustic radiation impedance
ρ	kg/m^3	Air mass density
θ_l	rad	Observation angle at the diffracting edge element l
Ω	rad	Enclosure half wedge angle

ACKNOWLEDGEMENT

The work presented here was partly supported by the Swiss Federal Office for Education and Science (OFES). Program: Brite Euram, Project: Outdoors Active Noise Control Technologies "Silence light" BRPR-CT96-0232.

BIBLIOGRAPHICAL REFERENCES

1. M. Rossi *Acoustics and Electroacoustics*, Artech House, MA 02062, 1988
2. J. Vanderkooy "A Simple Theory of cabinet Edge Diffraction", *J.A.E.S.* 39(12), 923-933, December 1991.
3. W.L. Stutzman, G.A. Thiele *Antenna Theory and Design*, John Wiley and Sons, USA, 1981.
4. H.P. Neff "Mutual Impedance of Circular Pistons", *J.A.E.S.* 43(9):695-699, September 1995.
5. V. Adam "Amplitude and Phase Synthesis of Loudspeaker Systems", *108th AES*, Paris, France, number 5161, February 2000.
6. V. Adam "Comportement d'un Haut-parleur Soumis à un Champ Extérieur", *5th CFA*, Lausanne, Switzerland, 660-663, September 2000.
7. V. Adam "Interactions of (two) Closed-box Loudspeaker Systems", *17th International Congress on Acoustics,* Roma, Italy, number 4B.10.04, September 2001.