EXPERIMENTAL DETERMINATION OF THE ACOUSTICAL PARAMETERS OF RIGID AND LIMP MATERIALS USING DIRECT MEASUREMENTS AND ANALYTICAL SOLUTIONS

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ABSTRACT

In order to characterize sound propagation in porous media, it is necessary to determine a set of intrinsic macroscopic acoustical parameters such as resistivity, porosity, tortuosity, characteristic length, and thermal permeability. From the knowledge of resistivity, porosity, and density of the material, and the measurement of the complex dynamic density and bulk modulus, the missing parameters are found using analytical solutions. In this paper attention is focused on materials having a very low stiffness, and which can be considered as "limp". Experimental results obtained on materials of various resistivity and density are presented to show the reliability of this new method.

INTRODUCTION

As open-cell porous materials are utilized in many noise control applications, it is necessary to find their intrinsic properties regarding sound propagation. In a previous study, a characterization method for determining the tortuosity, characteristic lengths, and static thermal permeability of air-saturated porous media having a motionless frame was proposed [1,2]. It is based on the measurements of the dynamic density, bulk modulus, static airflow resistivity, and porosity of the material, and on analytical solutions derived from the Johnson et al's and Lafarge et al's models [3,4].

In this paper, the method is extended to limp materials, for which analytical solutions can also be found. Experimental results obtained on low density and low stiffness glass wools are presented.

1. THEORY

1.1. The Rigid And Limp Models

Under the homogenization hypothesis (wavelength large compared to the heterogeneities of the medium), a perfectly rigid-frame, as well as a limp porous medium, can be represented as a homogeneous dissipative medium characterized by an equivalent dynamic density ($\tilde{\rho}_{eq}$) and bulk modulus ($\tilde{K}_{eq}$). The tilde symbol denotes that these two functions are complex valued and frequency dependent. Then the macroscopic behavior of the material submitted to harmonic excitations ($e^{j\omega t}$) is given on the one hand by the wave equation, and on the other hand by the generalized Darcy law:

$$\Delta p + \omega^2 \frac{\tilde{\rho}_{eq}}{\tilde{K}_{eq}} p = 0 \qquad (1) \qquad\qquad \vec{v} = -\frac{1}{j\omega\tilde{\rho}_{eq}} \vec{\nabla} p \qquad (2)$$

where p and $\vec{v}$ are the macroscopic acoustical pressure and velocity, Δ and $\vec{\nabla}$ are respectively the Laplacian and gradient operators, and ω the angular frequency.

In the case of a rigid-frame (motionless skeleton) open-cell porous material, $\tilde{\rho}_{eq}$ is rewritten $\tilde{\rho}^{f}_{eq}$ to recall that the effective density of the homogeneous material only depends on the fluid phase of the material. $\tilde{\rho}^{f}_{eq}$ can be evaluated using the semi-phenomenological model introduced by Johnson *et al* [3] that only depends on the properties of saturating fluid : η, the kinematic viscosity, and ρ_0 the density ; and on macroscopic intrinsic geometrical parameters depending on the microstructure of the material : σ , the static air-flow resistivity, ϕ, the open porosity, α_∞, the tortuosity, and Λ the viscous characteristic length. Then according to the Johnson's *et al*'s model the equivalent density can be written:

$$\tilde{\rho}^{f}_{eq} = \frac{\alpha_\infty \rho_0}{\phi}\left[1 + \frac{\sigma\phi}{j\omega\rho_0\alpha_\infty}\left(1 + \frac{4j\alpha_\infty^2 \eta \rho_0 \omega}{\sigma^2 \Lambda^2 \phi^2}\right)^{1/2}\right] \qquad (3)$$

This description falls when the motion of the solid frame cannot be neglected anymore. Generalized poro-elastic models such as the Biot-Allard's models are then required to describe the behavior of the medium [5,6]. However, a simplified description can be obtained for limp materials, i.e. when the solid frame has no bulk stiffness [7]. Then the equivalent density of the homogeneous medium rewrites:

$$\tilde{\rho}^{\text{limp}}_{eq} = \frac{A\tilde{\rho}^{f}_{eq} + B}{\tilde{\rho}^{f}_{eq} + C} \qquad (4)$$

where the constants A, B and C are:

$$A = \rho_{mat} \qquad\qquad B = -\rho_0^{\,2} \qquad\qquad C = \rho_{mat} - 2\rho_0$$

and ρ_{mat} is the apparent density of the porous medium, $\rho_{mat} = (1-\phi)\rho_s + \phi\rho_0$ where ρ_s is the density of the solid part constituting the skeleton.

1.2. Analytical Inversion

In this part, the analytical expressions of α_∞ and Λ, derived from equations (3) and (4) are given, assuming the prior knowledge of $\tilde{\rho}_{eq}$, σ and ϕ for the rigid frame model, and adding ρ_{mat} for the limp model. For the sake of simplicity $\tilde{\rho}_{eq}$ rewrites:

$$\tilde{\rho}_{eq} = \tilde{X} + j\tilde{Y} \qquad (5)$$

1.2.1. Rigid model

Identifying real and imaginary parts of equation (3), and solving a second order equation leads to find on the one hand, the tortuosity:

$$\alpha_\infty = \frac{\phi}{\rho_0}\left(\tilde{X} - \sqrt{\tilde{Y}^2 - \frac{\sigma^2}{\omega^2}}\right) \qquad (6)$$

and on the other hand, the viscous characteristic length :

$$\Lambda = \frac{a_\infty}{f}\sqrt{\frac{2h r_0}{w\tilde{Y}\left(a_\infty r_0 / f - \tilde{X}\right)}} \tag{7}$$

Note that in the last equation Λ depends on a_∞, and must be calculated afterwards.

Theoretically, a_∞ and Λ don't depend on frequency, and the first step is to check this hypothesis when using these equations. Hence, two major reasons can explain an apparent frequency dependency of the parameters. The first one is linked to the fact that the Johnson's model fails to predict reality at low frequencies, i.e. below $w_{cv}(= s f / r_0 a_\infty)$ the viscous characteristic frequency of material. Then the method should be used in the middle frequency range according to this estimation. The second reason is related to a possible motion of the frame. In this case the limp inversion can be used for materials with very low stiffness, in a frequency domain where the stress in the skeleton can be neglected.

1.2.2. Limp model

Due to the form of equation (4), finding the parameters leads to slightly more complex expressions. Identifying again real and imaginary parts in equation (4) shows that the tortuosity is the solution of the following second order equation:

$$a a_\infty^2 + b a_\infty + c = 0 \tag{8}$$

where

$$a = \frac{r_0^2}{f^2}\left[E\left((X-A)^2 - Y^2\right) + 2FY(X-A)\right]$$

$$b = -2\frac{r_0}{f}\left[E\left((X-A)(B-CX) + CY^2\right) + FY(B-2CX+AC)\right]$$

$$c = s^2\left(\frac{E^2+F^2}{w^2}\right) + E\left((B-CX)^2 - Y^2C^2\right) - 2FYC(B-CX)$$

and

$$E = (X-A)^2 - Y^2$$

$$F = 2Y(X-A)$$

Numerical simulations show that the admissible root is given by : $a_\infty = (-b-(b^2-4ac)^{1/2}/2a$

The viscous characteristic length is given by:

$$\Lambda = \frac{a_\infty}{f}\sqrt{\frac{4h r_0 (E^2+F^2)}{w\Psi}} \tag{9}$$

where

$$\Psi = 2EY\left(C + r_0 a_\infty / f\right)\left(B - CX - r_0 a_\infty (X-A)/f\right)$$
$$+ F\left(\left(B - CX - f r_0 a_\infty (X-A)\right)^2 - Y^2\left(C + r_0 a_\infty / f\right)^2\right)$$

2. EXPERIMENTAL RESULTS

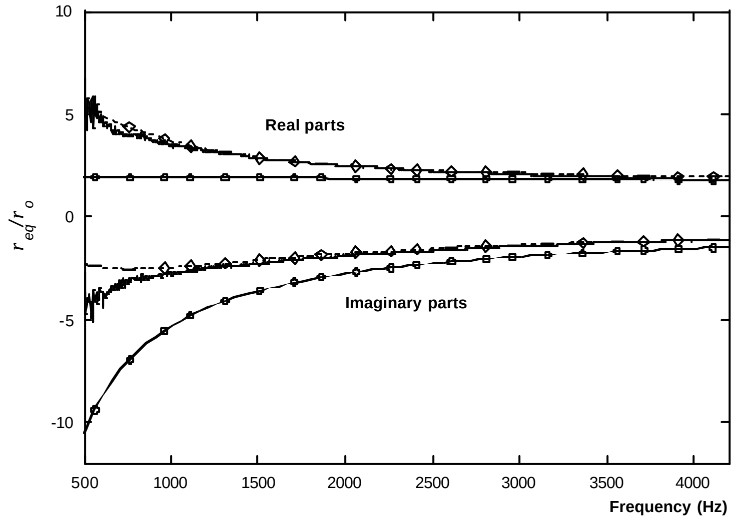
The reliability of the method has been tested on two low density glass wools having a very low stiffness. They have also been chosen for their relative high static flow resistivity in order to obtain limp behaviors. The very special properties of these two "cotton-like" materials, are due to the fact that the frame is made of very thin fibers, not usual regarding glass wools. As many mineral wools, they are anisotropic with a transverse isotropy. All the results presented in this part was obtained in the direction normal to the fiber planes. Firstly, s and f have been directly measured using classical techniques as described in [8]. The density of both materials have also been determined, and all these parameters being reported in Table 1. The equivalent densities $\tilde{r}_{eq}$ was obtained using a Kundt tube based method [9]. This technique allows determining the characteristic impedance ($Z_c = (\tilde{\rho}_{eq} \tilde{K}_{eq})^{1/2}$), and the wave number ($k_c = \omega(\tilde{\rho}_{eq} / \tilde{K}_{eq})^{1/2}$) from impedance measurements, and then to deduce $\tilde{\rho}_{eq}$ and $\tilde{K}_{eq}$.

The tortuosity and viscous length has been determined in a sufficiently high frequency range (between [2500,3800] Hz for mat. 1, and [2800,4000] Hz for mat. 2), to avoid problems linked to the limitations of the Johnson et al's model, or elastic behaviors of the materials. It appears that the identification of the parameters cannot be performed using the analytical solutions of the rigid model, but only using those of the limp model.

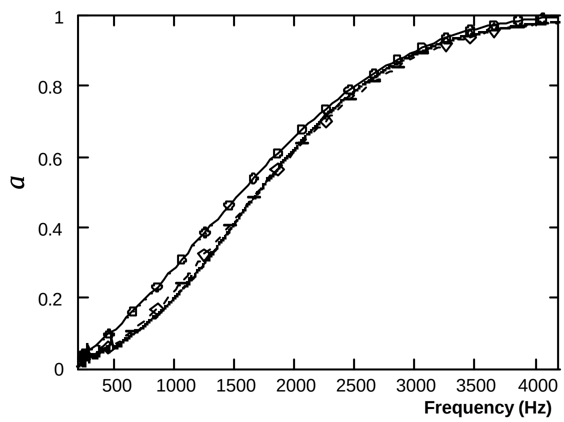
	σ (Nm^{-4}s)	ϕ	α_∞	Λ (µm)	Λ' (µm)	k_0' (10^{-10} m²)	ρ_{mat} (kgm^{-3})
Mat.1	**38200** *(± 301)*	**0.995** *(± 0.003)*	**1.00** *(± 0.03)*	**33** *(± 2)*	**105** *(± 4)*	**57.5** *(± 8.1)*	**8.5** *(± 0.1)*
Mat. 2	**151180** *(± 2095)*	**0.991** *(± 0.004)*	**1.03** *(± 0.15)*	**11** *(± 2)*	**42** *(± 3)*	**20** *(± 7.7)*	**15.1** *(± 0.15)*

Table 1 : Parameters of Mat. 1 and 2, obtained from direct (σ, ϕ, and ρ_{mat}) measurements, and using analytical solutions of the limp model (with Johnson et al's model) (α_∞, Λ), and the Lafarge et al's model (Λ', k_0'). The italic numbers are the standard deviations obtained with each techniques.

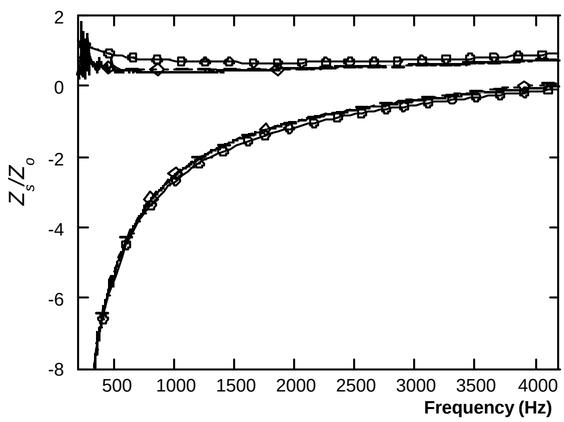
In order to predict the absorption coefficient and surface impedance, the thermal parameters (Λ', the thermal characteristic length, and k_0' the thermal permeability) used in the Lafarge et al's model [4], have also been determined with a very similar analytical inversion technique [1,2].



(a)



(b)



(c)

Figure 1 : Comparison between normalized equivalent densities (a), absorption coefficients (b) and normalized impedances (c) (rigid backing) , given by the rigid (—o—) and limp models (– ·), and measured (▬), for Mat.1.The thickness of the material is d=15.8 mm. Z_0 is the specific impedance of air.

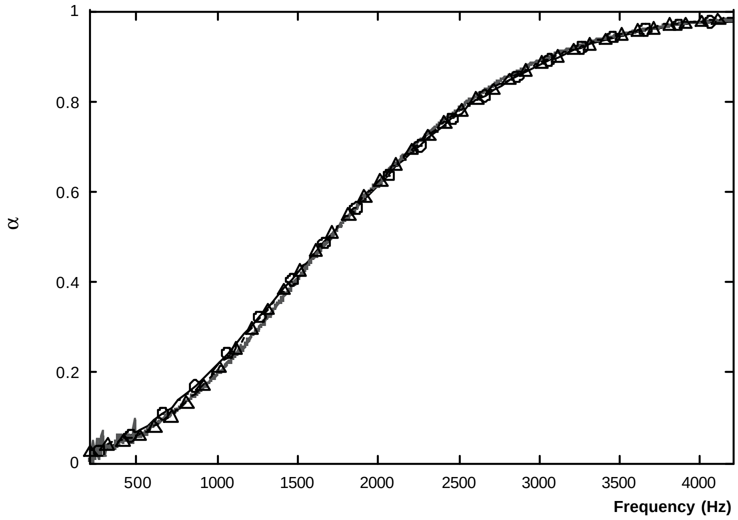


Figure 2 : Comparison between absorption coefficients (rigid backing) given by the limp model (—o—), the Biot-Allard model (···Δ–), and measured (▬), for Mat.1. The thickness of the material is d=15.8 mm. The elastic parameters of the Biot-Allard's model have been manually set to : E (Young's modulus)=7 kPa, n (Poisson's ratio) =0, and m (loss factor)=0.05.

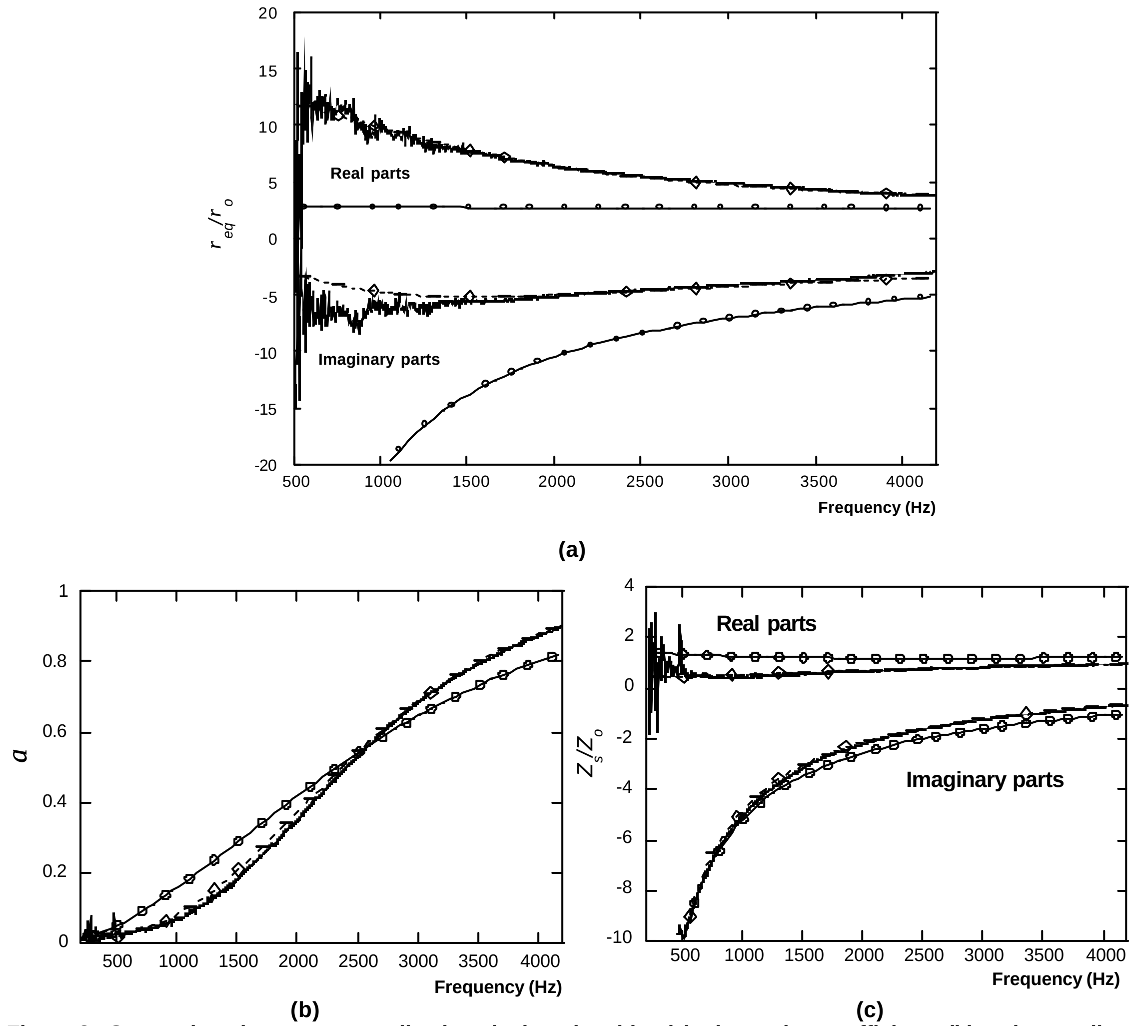


Figure 3 : Comparison between normalized equivalent densities (a), absorption coefficients (b) and normalized impedances (c) (rigid backing) , given by the rigid (—o—) and limp models (–), and measured (▬), for Mat.2. The thickness of the material is d=7.7 mm.

It appears that, for both materials, the analytical solutions using the limp model allows the identification of the equivalent density with a very good precision above 1000 Hz (Figures 1 and 3). The agreement with the absorption coefficient and surface impedance is also excellent. The experimental results have also been compared to the rigid model, using the same acoustical parameters. It clearly shows that the rigid approximation cannot be used for these materials in the frequency range of measurement. Some discrepancies appear at lower frequencies with the limp model. These are probably due to an elastic (and not only inertial) behavior of the frame. Figure 2 shows that a better prediction of the measured absorption coefficient can be obtained using the Biot-Allard's model (for semi-infinite materials) [6].

3. CONCLUSION

The proposed characterization method shows that it is possible to take into account the limp behavior of low stiffness porous materials, for determining their intrinsic acoustical parameters. This middle-frequency method only requires classical Kundt tube measurements, and the prior knowledge of the static air-flow resistivity, porosity, and apparent density. One interest of the method is the possibility to check the validity of the basic hypothesis (limp, rigid frame) inspecting the "frequency dependency" of the parameters. Comparison between experimental results, obtained on two glass wools, and the limp model, using the so determined parameters, confirm the applicability of the method.

4. ACKNOWLEDGMENTS

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