NUMERICAL ANALYSIS OF AN AIR JET: TOWARD UNDERSTANDING SOUNDING OF AIR-JET DRIVEN INSTRUMENTS

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ABSTRACT

An air-jet driven instrument can make sounds in various oscillation modes according to the air pressure supplied to it. The goal of this research series is to understand the sounding mechanism to the extent that the transition among the oscillation modes can be simulated. To this end, the motion of an air jet coming out from a flue slit should be properly modeled. As the jet travels through the pipe mouth, it is deflected by the sound field. Our intention is to model the jet deflection directly from the first principle of fluid dynamics. As a preliminary research effort, the jet deflection was simulated numerically. This paper presents the results of this simulation.

INTRODUCTION

The sounding of wind instruments can be considered a self-excited oscillation. For air-jet driven instruments such as flutes, recorders and organ flue pipes, the following two processes constitute a feedback loop:

1) An air jet coming out from a flue slit impinges the edge opposite to the slit. A part of the flow split by the edge enters the pipe and excites the pipe acoustically.

2) The sound field generated at the pipe mouth deflects the jet.

Process 1) has been well understood by means of flow and momentum conservations applied to the air entering the pipe[1]. Process 2), however, is a difficult aerodynamical question concerning how the air jet travels under the influence of the sound field.

The theory of fluid instability[2] can be used for analyzing the motion of a jet unless it is deflected by the sound field (free jet). In this theory, an infinitesimal velocity disturbance is imposed on the free jet having a particular velocity profile. It is then possible to examine how the disturbance develops in space and in time. Drazin and Howard[3] and Mattingly and Criminale[4] examined the instability of a jet having the sech^2 velocity profile called the Bickley profile[5]. These examinations shows that the free jet is unstable, or that the infinitesimal disturbance increases exponentially. They also show that the growth factor μ depends on the angular frequency ω, and that the disturbance propagates with a phase velocity that is about half the value of the jet velocity.

Unfortunately, the theory of instability cannot be directly applied to the analysis of jet deflection. This is because the theory imposes several conditions: that the jet has no boundary condition, that the fluid is inviscid, that the disturbance is infinitesimal, that the external sound field is absent, etc. Conversely, the jet coming out from the flue slit has the following properties: it is constrained to stay at the flue slit, it diffuses (and decelerates) due to the viscosity, and it is deflected with large amplitude by the sound field generated in the pipe.

Due to the difficulty described above, jet deflection has usually been examined with hypothetical models. Fletcher and Thwaites developed a jet deflection model[6] using the idea of "negative acoustic displacement," which allows the model to satisfy the boundary condition at the flue slit. The results of the free jet analysis could then be applied to jet deflection. Yoshikawa and Saneyoshi developed another jet deflection model with an assumption that the pressure gradient perpendicular to the jet traveling direction drives the jet[7]. Adachi examined the Fletcher and Thwaites model from the dynamical point of view and improved the Yoshikawa and Saneyoshi model by introducing a driving force caused by the jet instability[8]. These models do not, however, take the effect of jet deceleration into account, except for regarding the 'mean' velocity from the flue slit to the edge

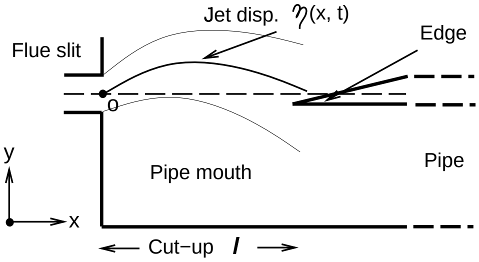
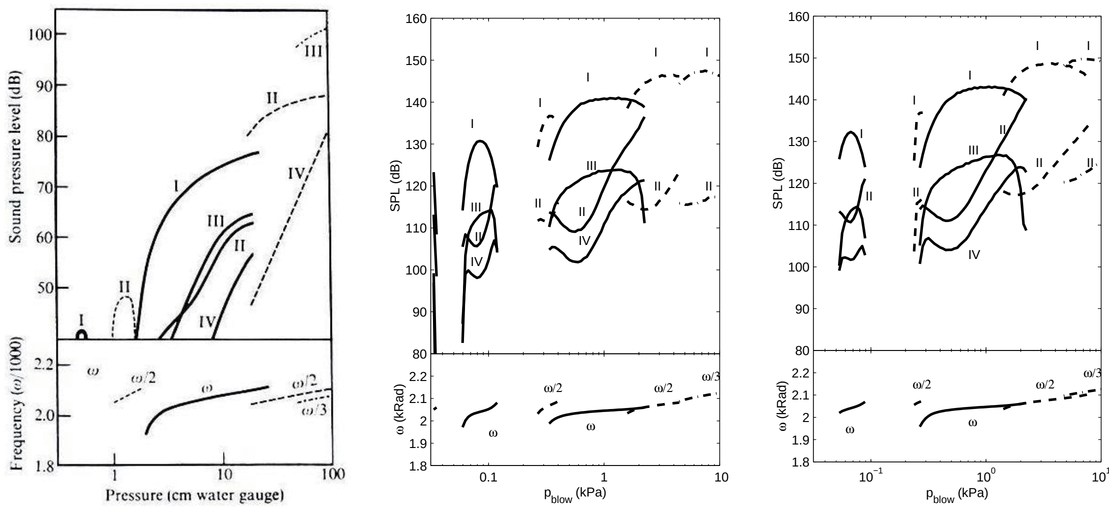


Fig. 1: Schematic diagram of flue slit, mouth and edge of an organ flue pipe



(a) (b) (c)

Fig. 2: Mode transition behavior of an E4 flue pipe: (a) Measured by experiment (quoted from Fletcher and Rossing [11]). (b) Simulated with acoustic displacement model. (c) Simulated with modified pressure gradient model.

as the jet velocity. The models also extend the assumption of the infinitesimal disturbance to the jet deflection with large amplitude. Consequently, the transition among the oscillation modes has not been explained by these hypothetical models.

This research aims to develop a jet deflection model directly from the Navier-Stokes equation, which governs fluid dynamics. Our goal is to appropriately simulate the mode transition. As a first step, it was confirmed by CFD (computational fluid dynamics) that the jet emerging from a slit is deflected by the sound field. This paper presents the results of this simulation.

There has been little research on air-jet driven instruments using CFD. Skordos and Gussman tried to generate sound directly from the Navier-Stokes equation without modeling anything[9]. A CFD simulation of the edge tone was also done recently[10], and it was reported that the simulated sound frequency was in good agreement with that obtained by experiment.

MODE TRANSITION

As described in the previous section, the sounding of air-jet driven instruments is a self-excited oscillation. The sounding occurs and stops according to parameters of the sounding system. One of the major parameters is the jet velocity, which is controlled by air pressure supplied to the instrument. Figure 2 (a) depicts a diagram of the mode transition that was obtained in an experiment with an E4 (330 Hz) organ flue pipe[11]. The horizontal axis represents the supplied pressure. In the lower pane, the vertical axis represents the sound frequency. In the upper pane, it represents the sound pressure level of each harmonic component. The figure indicates that sounding occurs on the first resonance mode (solid line) when the supplied pressure is in a moderate range around 1 kPa. As the pressure is increased, the second mode (dash line) and the third mode (dash-dot line) are excited. These are called overblows. The first and the second modes are also excited when the supplied pressure is low. These are called underblows.

Figures 2 (b) and (c) depict the transition diagrams simulated with the Fletcher and Thwaites model and with the modified pressure gradient model, respectively. In the experiment shown in figure 2 (a), the six oscillation regimes and the mode transition among them are simulated properly. An increase in the sound frequency with increasing supplied pressure within an oscillation regime is also obtained. There are, however, several points in which the simulation results are not in good agreement with the experiment. The pressure ranges of the simulated underblows are higher than those of the actual underblows. They are also wider. The simulated sound pressure levels are 10 to 20 dB higher than the actual levels*. The level difference is larger for underblows.

For sounding, it is crucial at which moment the air jet comes into the pipe. According to the theory of the jet-drive mechanism[1], sounding occurs under the condition that the volume flow entering the pipe increases when the sound pressure at the pipe mouth is positive. In this case, the jet can give energy to the oscillating air column in the pipe. This condition means that the jet should be deflected toward the instrument when the sound pressure is positive.

The inconsistency between the simulation and the experiment comes from the fact that the models do not provide the correct phase difference between the sound field and the jet deflection. The excessively high sound pressure level results from the excessively large amplitude of the jet deflection provided by the models. It may also come from neglecting the dissipation effect of the vortex shedding at the edge[12]†.

APPROACH

Several research steps are considered to simulate the mode transition from the Navier-Stokes equation:

1) Simulating the motion of a jet emerging from a slit to the semi-infinite space.

2) Confirming that the jet is deflected by the external sound field imposed perpendicularly to the jet traveling direction.

3) By changing the jet velocity, the jet deflection is simulated. From the amplitude and the phase of the deflection to the sound field obtained in the simulation, a jet deflection model is constructed.

4) From this model, the mode transition is estimated with the physical modeling simulation.

5) To examine the effect of the edge on the jet movement, step 3) is done with the edge added to the semi-infinite space.

6) With the deflection model obtained from step 5), step 4) is done.

7) The results of steps 4) and 6) are compared with that obtained by experiment.

This paper presents the results of steps 1) and 2).

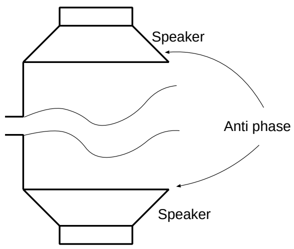
An assumption of the incompressible fluid is imposed in the calculation. The region in which the jet travels is considerably smaller than the wave length of the sound we deal with. The jet velocity is also much smaller than the sound velocity. In these situations, it is appropriate to impose this assumption. For space discretization, the finite element method is used. This brings an advantage that any geometric shape such as an edge can easily be represented in the calculation domain.

The external sound field imposed on the calculation domain can be represented by lump air oscillating perpendicularly to the jet traveling direction. This can be realized as shown in figure 3, where a pair of speakers is placed and driven anti-phase. The same situation can be obtained numerically by giving a boundary condition on the upper and lower boundaries such that velocity oscillates sinusoidally in-phase.

The edge tone is a sounding similar to that of air-jet driven instruments. It is generated on the system of a slit and an edge, having no resonator. There is no external sound field except for the

*Figures 2 (b) and (c) shows the sound pressure level at the pipe mouth, whereas figure 2 (a) shows the sound pressure at a position 1 m away from the instrument. Therefore, the difference is much larger than 20 dB. There is, however, still a difference of 10 to 20 dB when comparing these levels at the same position after correction.

†This effect may be included in the theory of the jet-drive mechanism. A further examination is needed.



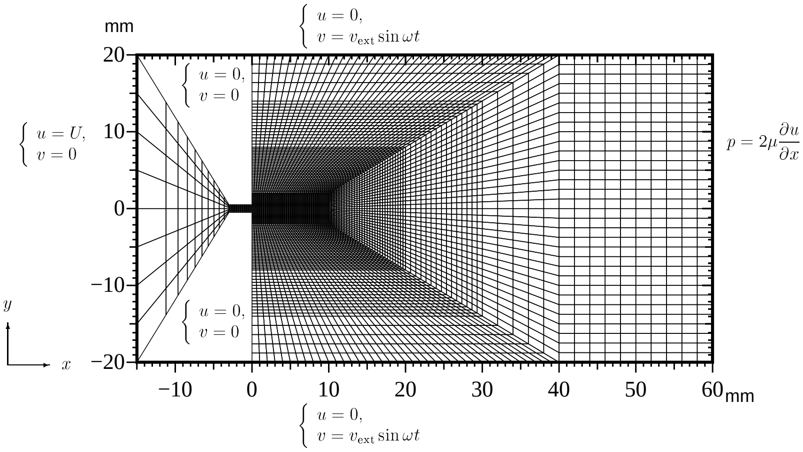


Fig. 3: (left) A free jet perturbed by the sound field generated by a pair of speakers.

Fig. 4: (right) Shape, finite element mesh and boundary conditions of the calculation domain.

sound field generated by the jet itself. For the edge tone, the interaction between the edge and the jet appears to be essential. As Coltman[13] pointed out, the sounding of air-jet driven instruments is different from the edge tone‡. However, it is assumed that the edge-tone is generated at the attack, that is, the very beginning of a sound[14].

COMPUTATIONAL FLUID DYNAMICS

The Navier-Stokes equation for the incompressible flow and the continuity equation are

$$\dot{u}_r = F_r(u) + G_r(p), \tag{1}$$
$$\partial_s u_s = 0, \tag{2}$$

where r and s are spatial indices, u_r is velocity, and p is pressure. The dot on the left-hand side of eq. (1) indicates the time derivative. $F_r(u)$ and $G_r(p)$ in eq. (1) denote

$$F_s(u) = -u_r\partial_r u_s + \nu\Delta u_s, \tag{3}$$
$$G_s(p) = -\frac{1}{\rho}\partial_s p, \tag{4}$$

where ν is kinematic viscosity and ρ is air density. Ren and Utnes[15] proposed using a centered time scheme for time discretization as well as a velocity correction method. This method represents velocity u_r as the sum of an intermediate velocity u_r^* and its correction δu_r, and calculates each term as follows:

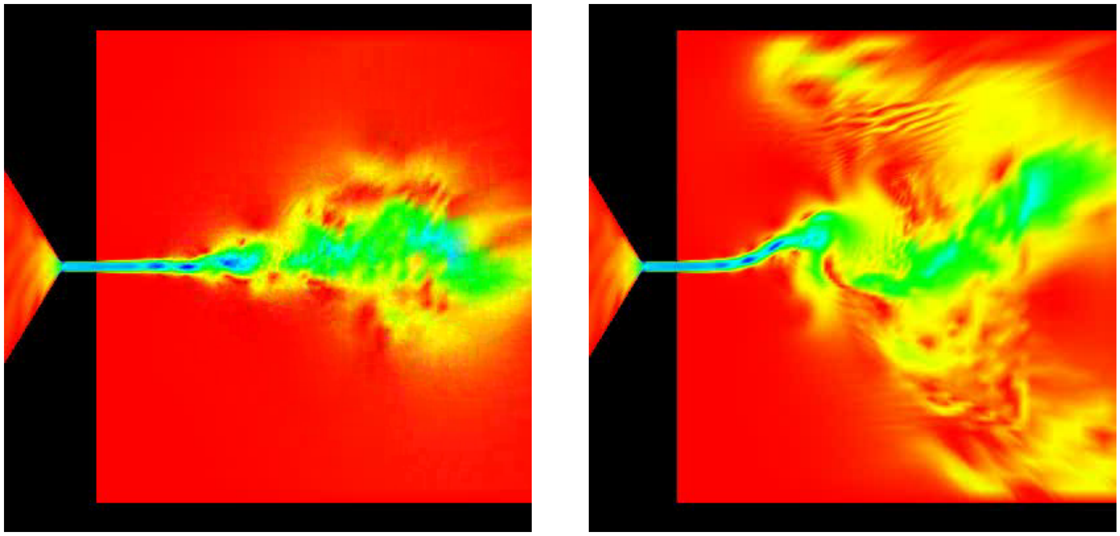
$$u_r^{+1} = u_r^* + \delta u_r, \tag{5}$$
$$u_r^* = u_r + \frac{\Delta t}{2}[F_r(u) + F_r(\hat{u})], \tag{6}$$
$$\delta u_r = \Delta t G_r(p), \tag{7}$$

where u^{+1} denotes velocity at one step ahead in time and $\hat{u} = u + \Delta t F(u)$. By taking divergence of eq. (5) and considering the continuity equation, we find the following Poisson equation is satisfied for p:

$$\Delta p = \frac{\rho}{\Delta t}\partial_s u_s^*. \tag{8}$$

Provided that velocity u_r at time t and pressure p^{-1} at $t - \Delta t$ are known, the algorithm to find u_r^{+1} and p is enumerated as follows:

‡We often find a wrong description of the sounding mechanism of the air-jet driven instrument in literature such as textbooks for musicians. The picture of the instrument as a resonator coupled to an edge-tone generator that amplifies its response is oversimplified.



(a) (b)

Fig. 5: Snapshots of simulated jet movement: (a) No external sound field. (b) With external sound field.

1) Calculate $k_r^{(1)} = F_r(u)$.

2) Calculate $k_r^{(2)} = F_r(\hat{u})$, where $\hat{u} = u + \Delta t k^{(1)}$.

3) Calculate $u_r^* = u_s + \dfrac{\Delta t}{2}(k_r^{(1)} + k_r^{(2)})$.

4) Solve $\Delta p = \dfrac{\rho}{\Delta t}\partial_s u_s^*$ in terms of p.

5) Calculate $\delta u_r = \Delta t G_s(p)$.

6) Calculate $u_r^{+1} = u_r^* + \delta u_r$.

To discretize an equation having an advection term such as the Navier-Stokes equation, the numerical instability sometimes causes a problem. This is most likely for calculations with larger velocity. To prevent this problem, an upwind discretization is needed. The SUPG (Streamline Upwind Petrov-Galerkin) method[16] is adopted here.

Figure 4 depicts a calculation domain where the jet emerges from a slit of 1 mm thickness. The domain is divided into a mesh of the linear 4-node 2D elements. The number of nodes is 7201 and the number of elements is 7056. At the inflow boundary, velocity is specified. At the upper and lower boundaries of the region where the jet travels, sinusoidally oscillating velocity is given. At the outflow boundary, a traction-free condition is imposed.

We use Diffpack[17] for coding the algorithm. This is a tool for developing numerical software, typically on the numerical solution of partial differential equations. Diffpack provides a numerical library consisting of C++ classes. All the classes are highly organized in the object oriented programming style.

SIMULATION RESULTS

Simulations were done with a velocity of 0.4 m/s at the inflow boundary, which makes the mean velocity 16 m/s at the slit. Snapshots of simulations are given in figure 5. Figure 5 (a) shows the jet without the external sound field, and figure 5 (b) depicts that with the external sound field. The amplitude of the sound field is $v_{\text{ext}} = 0.3$ m/s. The sound pressure level corresponding to this amplitude is about 110 dB. The frequency is set to $f = 333$ Hz. The simulations were carried out on a Linux cluster of the Beowulf type with 8 CPU's (Pentium III with 800 MHz clock speed). The time step was set to 2.5μ s. It took about four hours to calculate 8000 steps for jet movement of 20 ms.

In figure 5 (a), where no external sound field is imposed, the jet travels rather straight with irregularity due to the fluid instability. After traveling 10 mm, a boundary layer suddenly develops and the flow becomes turbulent. In figure 5 (b), where the external sound field is imposed, the jet oscillates up and down along with the sound field. It is also observed that vortices are left in the calculation domain.

SUMMARY

Jet deflection was numerically simulated with CFD. When the external sound field is imposed perpendicularly to the jet traveling direction, the jet successfully oscillates up and down along with the sound field.

There is, so far, a technical difficulty in the calculation. A numerical divergence occurs when a large vortex passes through the outflow boundary. This prevents us from continuing a flow simulation beyond the time at which it happens. We are now investigating how to solve this problem by improving the outflow boundary condition.

Simulation with various jet velocities is planned in near future. The amplitude of the jet deflection and the phase difference to the sound field will be numerically obtained. From this measurement, a jet deflection model will be constructed. It is of a great interest to see if the model can replicate the mode transition behavior. Jet deflection is essentially a two-dimensional phenomenon. Accordingly, it was simulated in two-dimensional space. It is, however, worth simulating the jet in three-dimensional space, because the actual flow is three-dimensional.

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