NUMERICAL OPTIMIZATION OF GEOMETRICAL DIMENSIONS AND DETERMINATION OF MATERIAL PARAMETERS FOR VIOLIN TOPS

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ABSTRACT
Violin tops are made out of the wooden material spruce and the material parameters, as density, Young's modulus, Poisson's ratio, and others, have a natural variation. These variations of course influence the vibration properties of the top. In this paper optimization is used to retain a vibration property when the material parameters are changed and as variables in these optimization analyses the arch height and the thickness distribution over the top is used. The possibility to estimate important material parameters for individual tops by use of optimization and knowledge of eigenfrequencies, geometric dimensions, and density is also investigated. A code based on the stochastic optimization method simulated annealing (SA) [1] is linked together with a modified version of the finite element code FEMP [2] to achieve an optimization program, which incorporates structural design changes in an automatic fashion.

INTRODUCTION
The vibration properties, as mode shapes and eigenfrequencies, and the characteristics of the sound emanating from a vibrating structure are modified if structural design variables such as geometric dimensions [3], shell thickness [4], material parameters [5], discrete masses [6], and, for fiber reinforced material, fiber orientation [7] are changed. Of course, changes to one or more of these variables will results in changes to other structural characteristics. To find the best design, i.e. the one that satisfies all the demands put upon is a question of optimization. This often requires a multidisciplinary approach i.e. analytical tools from different disciplines must be used in concert. The material parameters, which have a natural distribution, have great influence on the vibration properties of the violin plates. Earlier studies [8] on blanks for violin tops and backs gives some idea of the sensitivity of the vibration properties with respect to the material parameters. In the present work we use optimization to give some suggestions on how to compensate, through changes in thickness distribution and rise of the arch for variations in the material parameters, that is: is it possible to change the material parameters and retrieve the initial vibration properties. Also the possibility to estimate important material parameters in a particular top by use of optimization methods is investigated. The search for the optimum solution can be performed in many different ways. A common approach is to use mathematical programming techniques. This technique has been used in earlier analysis [7] where, a gradient-based method called MMA [9] was used to achieve optimization. MMA has been used with great success for a variety of problems. Another method, which is conceptually quite different from the mathematical programming techniques, is to optimize using some form of

natural selection process. One such technique is the simulated annealing, SA, a stochastic method based on the simulation of metal (or solids) annealing [10]. Annealing is the physical process of heating up a solid and then cooling it down slowly. The slow and controlled cooling of the solid ensures proper solidification with a highly ordered crystalline state. At high temperature the atoms in the heated material have high energies and more freedom to arrange themselves. Annealing results in a material with an atom arrangement that corresponds to the lowest internal energy. There are many other optimization methods, such as genetic algorithms, neural network and so on, that is based on natural selection of solutions to achieve an optimum. In this paper simulated annealing is used as the optimization algorithm. A code based on this algorithm [1] is linked together with a modified version of the finite element code FEMP [2] to achieve an optimization program, which incorporates structural design changes in an automatic fashion. In this paper, the thickness variation and the rise of the arch of the top are used as variables in order to retrieve the three first eigenfrquencies on a violin top plate when the material parameters are changed. The possibility to estimate important material parameters in an individual top based on knowledge of the geometric dimensions, the density, and the three first eigenfrequencies is also investigated by use of the above mentioned optimization system.

PROBLEM DEFINITION

THICKNESS AND ARCH HEIGHT AS VARIABLES

For the purpose of optimization, two violin tops with different material parameters were analyzed. First a top with a chosen material parameter set and an initial thickness and arch height distribution emanating from a Stradivarius Cremona 1720 [11] was analyzed with respect to the first three eigenmodes. In the text the top with these initial settings is referred to as the initial top and the results from the analysis on this top are referred to as the initial results. On a second top the material parameters were changed and the objective with the optimization analysis was to, with this new material set, retrieve the first three eigenmodes obtained for the initial top. The variables in this optimization were the thickness and arch height distribution over the top with initial values according to the initial top. The analysis involves modal analysis of the top by use of FE-calculations and it was discretizised with triangular orthotropic shell elements according to Figure 1. In the FE-analyses the nodes along the inner edge (the bold line in Figure 1) was simply supported, i.e. prevented to move perpendicular to the plane but free in all other directions. In the analyses the eigenmodes according to Figure 2 were studied and the objective function was formulated as:

$$\textit{Minimize:} \qquad z = \alpha_k \cdot \left(f_k^{\,initial} - f_k^{\,actual} \right)^2, \qquad k = 1, 3 \tag{1}$$

where $f_k^{\,initial}$ are the eigenfrequencies for the initial top and $f_k^{\,actual}$ are the eigenfrequencies for the top with the new material parameters and with actual variable set. α_k are so-called penalty parameters. The only constraints used in this investigation are on the variables. The material in a violin top is spruce and has a cellular structure. If knots, annual rings and other deviations are neglected, the hexagonal honeycomb model according to Gibson and Ashby [12] gives a good description of the mechanical properties of wood and the model has been used to estimate the material parameters for the tops in this investigation. The parameters for the second top are estimated with respect to a 5% decrease of the density compared to the initial top.

The formulation according to eq. (1) with constraints only on the variables is well suited for simulated annealing which is a nongradient (zeroth-order) stochastic optimization technique based on random evaluation of the objective function in such a way that transitions away from local minimum are possible. Although the method usually requires a large number of function evaluations to find the optimum design, it will find the global optimum with a high probability even for problems with numerous local minima [10].

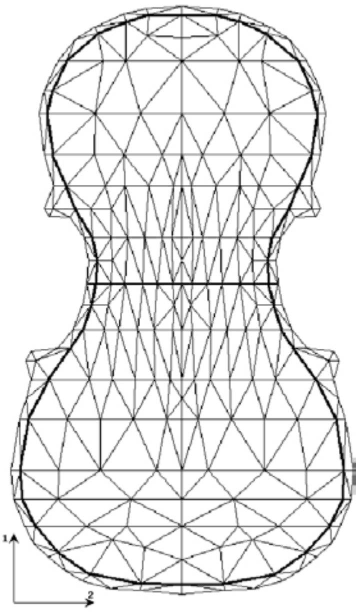

Fig. 1. Discretization of violin top.

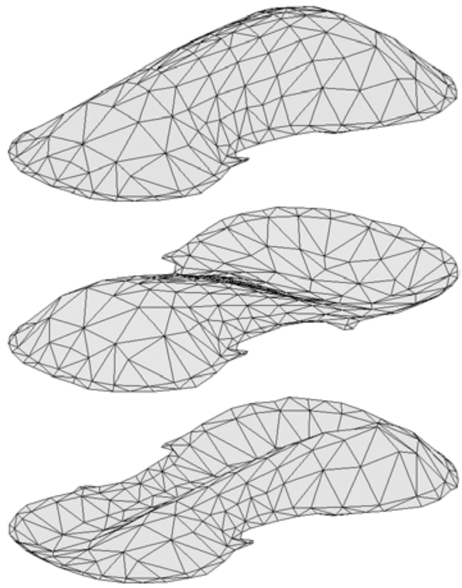

Fig. 2. Studied eigenmodes. Top: mode 1, middle: mode 2, bottom: mode 3.

MATERIAL PARAMETERS AS VARIABLES

The material parameters for wood have a natural variation, which makes it impossible for the makers of violins to create a template of a successfully created violin top and use it over and over again. The violinmaker has to adjust the thickness and arch height depending on the material parameters on the actual blank being worked on. Some guidelines, with respect to the thickness and arch height, could perhaps be created if the material parameters where known for every individual blank, or blanks could be selected based on knowledge of individual material parameters. In the followings a method, based on the above-mentioned optimization system, for estimation of important material parameters for individual violin tops is proposed. The hypothesis is that if the geometric dimensions, the density, and the value of the three first eigenfrequencies are given for the top, then it is possible to estimate, thru optimization, what the material parameters must be. In this optimization analysis the geometry is held constant and the variables are the material parameters. The optimization problem is formulated as:

$$\textit{Minimize:} \qquad z = \alpha_k \cdot \left(f_k^{\,measured} - f_k^{\,actual} \right)^2, \; k = 1, 3 \tag{2}$$

where $f_k^{\,measured}$ are the measured eigenfrequencies for the existing top and $f_k^{\,actual}$ are the eigenfrequencies for the top with the actual material parameters. α_k are the so-called penalty parameters as before. In order to test the idea, a violin top with a given geometry and given material parameters where analyzed with respect to the first three eigenfrequencies. The resulting eigenfrequencies are called $f_k^{\,measured}$ and simulates measured frequencies. After this the optimization analysis was performed on a top with the same geometry and with the material parameters as variable and the objective with the optimization was to retrieve the first three eigenfrequencies as obtained in $f_k^{\,measured}$. The initial values of the material parameters were of course changed from those that gave $f_k^{\,measured}$.

RESULTS

THICKNESS AND ARCH HEIGHT AS VARIABLES

With shell thickness and arch height as variables a maximum variation of $\pm$10% from the initial variable value was allowed. The initial material (spruce) parameters are taken as: $E_1 = 9.567 \cdot 10^9$ Pa, $E_2 = 5.789 \cdot 10^8$ Pa, $G_{12} = 7.08 \cdot 10^8$ Pa, $\nu_{12} = 0.03$, and $\rho = 410$ kg/m^3. With initial thickness, initial rise and these material parameters the studied eigenfrequencies were determined to: $f_1 = f_1^{initial} = 283.26$ Hz, $f_2 = f_2^{initial} = 510.55$ Hz, and $f_3 = f_3^{initial} = 542.45$ Hz.

The new material parameters were chosen with respect to a 5% decrease of the density, from 410 to 389.5 kg/m^3. This affected the other parameters and resulted in new material parameters according to: $E_1 = 9.092 \cdot 10^9$ Pa, $E_2 = 4.963 \cdot 10^8$ Pa, $G_{12} = 6.726 \cdot 10^8$ Pa, $\nu_{12} = 0.03$, and as mentioned above $\rho = 389.5$ kg/m^3. The new material set together with the initial thickness and initial arch height resulted in the following eigenfrequencies: $f_1 = 279.84$ Hz (-1.2%), $f_2 = 504.78$ Hz (-1.1%), and $f_3 = 532.51$ Hz (-1.8%). These are the first f_k^{actual} , (k = 1, 3), in the optimization process and with the penalty parameters α_k taken as 1.0 it follows that the objective function is z = 143.79 in the beginning of the optimization process.

Thickness distribution

In this case the shell thickness at nodes along the bolded line in Figure 1 are collected to one variable and the thickness at nodes outside the line collected to another variable. The shell thicknesses at all other nodes are separately variables. Utilization of symmetry with respect to the vertical mid-axis gives the total number of thickness variables to 68. The optimization process converged to the following eigenfrequencies: $f_1 = f_1^{\text{optimal}} = 283.25$ Hz (-0.004%), $f_2 = f_2^{\text{optimal}} = 510.54$ Hz (-0.002%), and $f_3 = f_3^{\text{optimal}} = 542.44$ Hz (-0.002%). The change in thickness distribution is illustrated in Figure 3 where the values are in meter and shall be added to the initial thickness distribution to get the distribution at optimum.

Arch height distribution

Here the arch height for the top was taken as variable. The arching for the nodes along the bolded line in Figure 1 and the nodes outside this line are kept constant during the optimization process. The arching at all other nodes is separately variables. Also here symmetry with respect to the vertical mid-axis is utilized which gives the total number of variables to 66. The optimization process converged to the following eigenfrequencies: : $f_1 = f_1^{\text{optimal}} = 283.33$ Hz (+0.025%), $f_2 = f_2^{\text{optimal}} = 510.80$ Hz (+0.049%), and $f_3 = f_3^{\text{optimal}} = 542.54$ Hz (+0.017%). The change in arch height distribution is illustrated in Figure 4 where the values are in meter and shall be added to the initial arch height to get the distribution at optimum.

MATERIAL PARAMETERS AS VARIABLES

The material parameters for an individual blank or violin top is difficult to measure. For that reason and for the purpose of testing the idea of using knowledge of the geometric dimensions, the density, and the value of the three first eigenfrequencies together with optimization methods to determine the relevant material parameters a numerical analyses was performed to simulate the measurements or knowledge of these parameters. In this analyses the material parameters was given the following values: $E_1 = 9.092 \cdot 10^9$ Pa, $E_2 = 4.963 \cdot 10^8$ Pa, $G_{12} = 6.726 \cdot 10^8$ Pa, $\nu_{12} = 0.03$, and $\rho = 389.5$ kg/m^3 and the resulting eigenfrequencies, here called $f_k^{measured}$, was determined, as above, to: $f_1 = f_1^{measured} = 279.84$ Hz, $f_2 = f_2^{measured} = 504.78$ Hz, and $f_3 = f_3^{measured} = 532.51$ Hz. These values on material parameters and frequencies are, consequently, the values we want to achieve with the optimization process. The initial value for the material parameters was set to: $E_1 = 9.567 \cdot 10^9$ Pa, $E_2 = 5.789 \cdot 10^8$ Pa, $G_{12} = 7.08 \cdot 10^8$ Pa, $\nu_{12} = 0.03$ and the eigenfrequencies at these values together with $\rho = 389.5$ kg/m^3 was determined to: $f_1 = 290.62$ Hz, $f_2 = 523.81$ Hz, and $f_3 = 556.54$ Hz. The optimization converged to the following values: $E_1 = 9.061 \cdot 10^9$ Pa (-0.34%), $E_2 = 4.938 \cdot 10^8$ Pa (-0.50%), $G_{12} = 6.823 \cdot 10^8$ Pa (+1.5%), $\nu_{12} = 0.0296$ (-1.3%), $f_1 = f_1^{\text{optimal}} = 279.86$ Hz (+0.005%), $f_2 = f_2^{\text{optimal}} = 504.76$ Hz (+0.004%), and $f_3 = f_3^{\text{optimal}} = 532.50$ Hz (-0.001%).

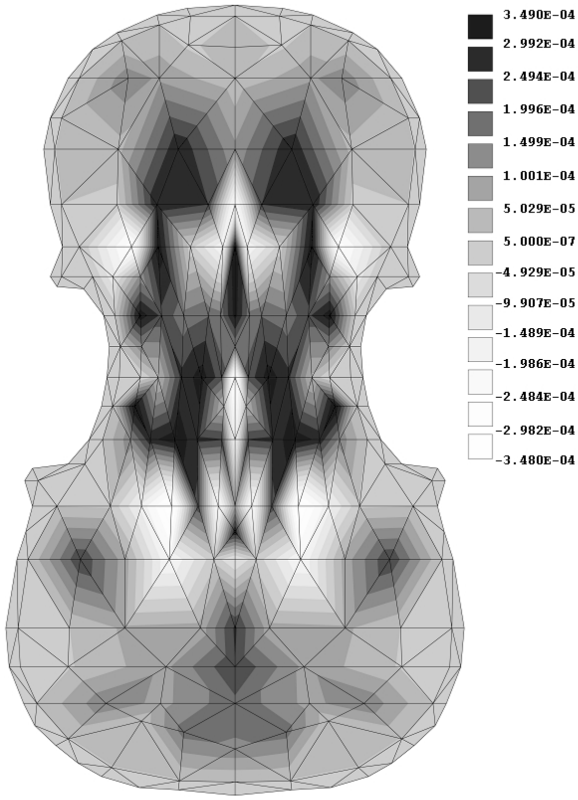

Fig. 3. Change in thickness distribution [m].

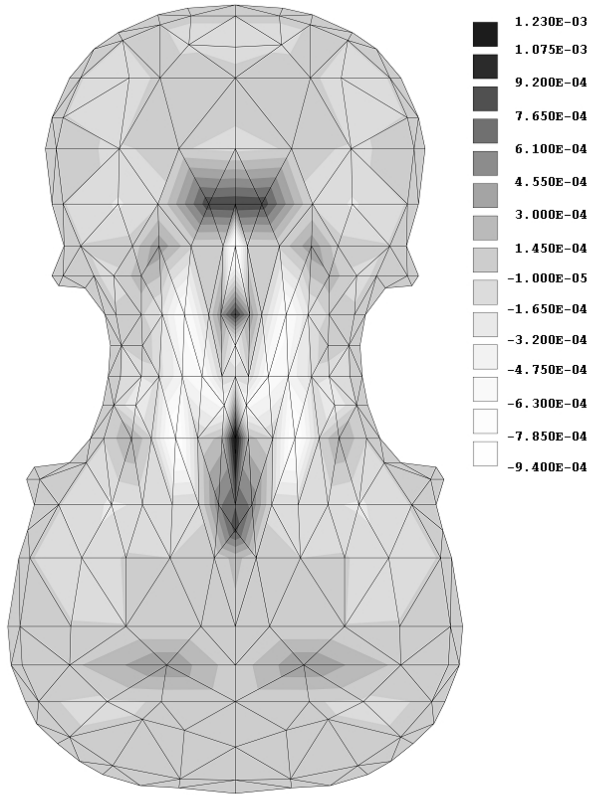

Fig. 4. Change in arching [m].

DISCUSSION AND CONCLUSION

The optimization problem was to minimize the difference in eigenfrequencies for violin top plates with different material parameters and to investigate the possibility to retrieve important material parameters for an individual top if the geometric dimensions, the density, and the values of the eigenfrequencies are known. The three lowest eigenmodes were studied, and the problem was formulated as: Minimize the quadratic sum of the difference for each pair of eigenfrequencies multiplied by a penalty parameter. In this study the penalty parameters was set to unity, α_k=1.0, which implies that no weighting between different eigenfrequencies was utilized. The results were quite inspiring. For the case with variable thickness, the differences in eigenfrequencies are within 0.004% and for the case with arch height as variable, the differences are within 0.049% (with the same computational time used). The material parameters were estimated by a hexagonal honeycomb model according to [12] and the difference in material parameters, when comparing two tops, was determined from a decrease of the density from 410 to 389.5 kg/m^3 (-5%). The 5% decrease in density is considered to be a moderate change in density for spruce relevant for violin makers. The variation of the Young's modulus is not only correlated to the variation of density but depends also on the microfibril angle which gives a larger variation in Young's modulus than the variation in density [13]. The variation of material parameters within a top will be considered in future work. The results indicate that it is possible to retain specific eigenfrequencies for the top plate when the material parameters are changed, both thru thickness and arch height compensation. Performing optimization with both thickness and arch height distribution as variables simultaneously is of course possible and would probably produce smoother results. Another possibility to compensate for changes in the material parameters is to include variables concerning the shape of the bass bar, together with variables on thickness and arch height in the optimization. This will be considered in future work. The results also indicate that it is possible to determine important material parameters for an individual top if knowledge of the density, the geometric dimension and the three first eigenfrequencies is at hand. However, an ongoing work shows that it is not enough to know the values of the eigenfrequencies but also the shapes of the eigenmodes must be considered in the optimization process. The boundary conditions used in the analyses in this study was not chosen to simulate real boundary conditions for a violin top plate accurately. Our primary purpose was to investigate the possibilities of using optimization to compensate for differences in the material properties and the possibilities of retrieving important material parameters. The FE discretization according to Figure 1 was chosen to give reasonable good results with relatively short computational time. A denser discretization and/or the use of higher order FE-element would produce smoother results. Also the use of additional constraints on the variation of thickness and arch height distribution would produce smoother and more realistic results. This will be considered in future work. In order to produce more useful results which could serve as guidelines for violin makers, we believe that it is necessary to include constraints on other significant behavior characteristics as modal mass or the sound intensity in different points outside the vibrating top.

FUTURE CONSIDERATIONS

Although the results are inspiring much work remains to be done in order to understand the behavior of a good violin and to be able to guide the makers of violins. Future work will comprise optimizations of violin top plates with different boundary conditions and different materials. Hopefully, optimization of a complete violin using additional variables besides thickness and arch height distribution of the top and back plates will be possible. Variables that could be interesting to include are for example the position of the sound post, the shape and position of the bass bar, and the shape of the *f*-holes. Another interesting item to investigate would be the interaction, through the sound post, between the top and back plate. The material model must also be refined in order to consider the influence of different proportions of early- and latewood in the annual rings, the reinforcement due to wood rays in the radial direction etc.

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