LARGE-BANDWIDTH MEASUREMENT OF ACOUSTIC INPUT IMPEDANCE OF TUBULAR OBJECTS

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ABSTRACT

Experimental determination of the acoustic impedance of tubular objects using a measurement duct with an array of microphones in its wall is considered. A method is proposed in which the influence of the microphone transfer-functions and the discontinuities in the measurement duct are accounted for via a series of calibration measurements with closed tubes. The effects of varying the number of microphones and calibration tubes is studied. Theoretical results show that uniform spacing of both the microphones and calibration tube lengths yields the smallest error within the effective measurement frequency range.

INTRODUCTION

Measurement of the acoustic impedance of tubular objects is often carried out using the capillary method (see for example [1]). However, this method is relatively sensitive to background noise, and usually gives accurate results only at frequencies up to about 5 kHz. For measurement at higher frequencies, an alternative method is required. An essential requirement of such a method is that the effects of non-propagating modes at the microphone positions, which are typically influential at higher frequencies, must be accounted for.

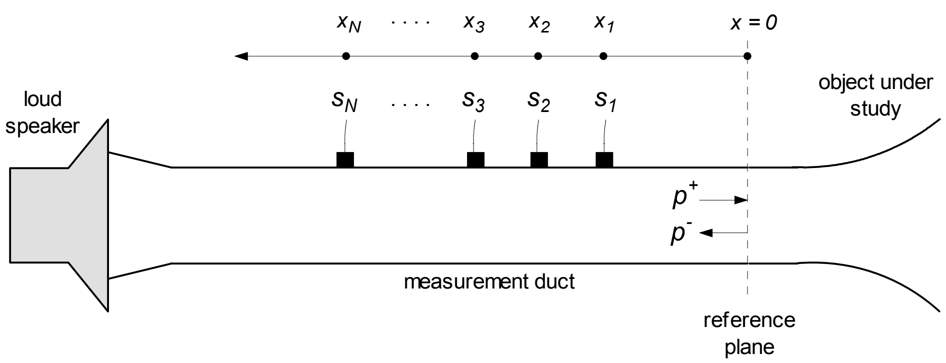

Figure 1. Schematic view of the measurement set-up. The ratio of the reflected and the incident wave at the reference plane is the reflectance of the object under study.

The present study is aimed at determination of the acoustic impedance of tubular objects of circular cross-section at frequencies up to 20 kHz. The approach taken here is to employ a measurement duct with an array of microphones fitted in its wall, as depicted in figure 1. This approach has been used in previous research for the experimental determination of in-duct acoustic properties [2]. The "multiple-microphone method" used in [2] relies on the assumption that no non-propagating modes are excited in the measurement duct at the microphone positions; the effects of the microphone transfer-functions are accounted for by calibrating the microphones. In theory, one can avoid the excitation of non-propagating modes by mounting the microphones perfectly flushed with the wall, but in practice, it is extremely difficult to create such "discontinuity-free" mountings. However, it is in principle possible to include the effects of the non-propagating modes in the calibration procedure. Such an approach has been taken by Gibiat and Laloë [3] for the measurement of woodwind instrument bores, using a measurement duct with two microphones, and performing three calibrations with tubular objects of known impedance in order to fine-tune the experimental procedure. In this paper, we propose a method in which such a calibration procedure is used with the multiple-microphone method.

DEVELOPMENT OF THE MULTIPLE-MICROPHONE MULTIPLE-CALIBRATION METHOD

Theory of Impedance Determination with *N* microphones

Consider the measurement set-up as depicted in Figure 1. At one end of the duct, the air column is excited with a sinusoidal signal, and the other end is passively terminated. The termination load at the reference plane is the impedance of the object under study. Under the assumption of linear propagation of only a single mode in the measurement duct, the microphone signals may be expressed as:

$$s_n = \alpha_n p^+ + \beta_n p^- , \tag{1}$$

where p^+ and p^- are respectively the incident and reflected waves at the reference plane, and α_n and β_n are coefficients that depend on the microphone transfer-functions, and on the effects of the non-propagating modes that are excited at the microphone positions. If we neglect the higher-mode effects, and assume that each microphone is pre-calibrated so that the signal s_n is identical to the total acoustic pressure on the axis at position x_n, the following approximations are found:

$$\alpha_n = \exp(\Gamma x_n) \quad , \quad \beta_n = \exp(-\Gamma x_n) . \tag{2}$$

where Γ is the propagation constant for waves travelling in the measurement duct. The object impedance can be determined from the microphone signals as follows: using equation (1), and by assembling all microphone signals s_n, we obtain the linear matrix equation

$$\mathbf{A}\,\mathbf{p} = \mathbf{b} , \tag{3}$$

where

$$\mathbf{A} = \begin{bmatrix} \alpha_1 & \beta_1 \\ \alpha_2 & \beta_2 \\ . & . \\ . & . \\ \alpha_N & \beta_N \end{bmatrix} \quad , \quad \mathbf{p} = \begin{bmatrix} p^+ \\ p^- \end{bmatrix} \quad , \quad \mathbf{b} = \begin{bmatrix} s_1 \\ s_2 \\ . \\ . \\ s_N \end{bmatrix} . \tag{4}$$

For $N > 2$, this is an over-determined matrix equation. In that case, the "best approximate solution" is:

$$\mathbf{p} = \left(\mathbf{A}^h \mathbf{A}\right)^{-1} \mathbf{A}^h \, \mathbf{b} , \tag{5}$$

which can be computed using Cramer's rule [2]. The reflectance of the object under study is determined from the solution-vector **p** as $R = p^- / p^+$. When no non-propagating modes are excited at the reference plane, the input impedance is then computed as $Z = Z_0 (1+R)/(1-R)$, where Z_0 is the characteristic impedance. Because we only need to know the ratio of the elements of the solution-vector, we may use the first microphone as a reference microphone, and replace the variables in equation (3) with:

$$\tilde{\mathbf{A}} = \begin{bmatrix} 1 & \beta_1/\alpha_1 \\ \alpha_2/\alpha_1 & \beta_2/\alpha_1 \\ . & . \\ . & . \\ \alpha_N/\alpha_1 & \beta_N/\alpha_1 \end{bmatrix} \quad , \quad \tilde{\mathbf{p}} = \left(\frac{\alpha_1}{s_1}\right) \cdot \begin{bmatrix} p^+ \\ p^- \end{bmatrix} \quad , \quad \tilde{\mathbf{b}} = \begin{bmatrix} 1 \\ s_2/s_1 \\ . \\ . \\ s_N/s_1 \end{bmatrix} . \qquad (6)$$

It is advantageous to use this alternative arrangement because (1) it is easier to measure consistent values of the ratio of two measured pressures than an exact pressure, and (2) we now have one unknown coefficient less.

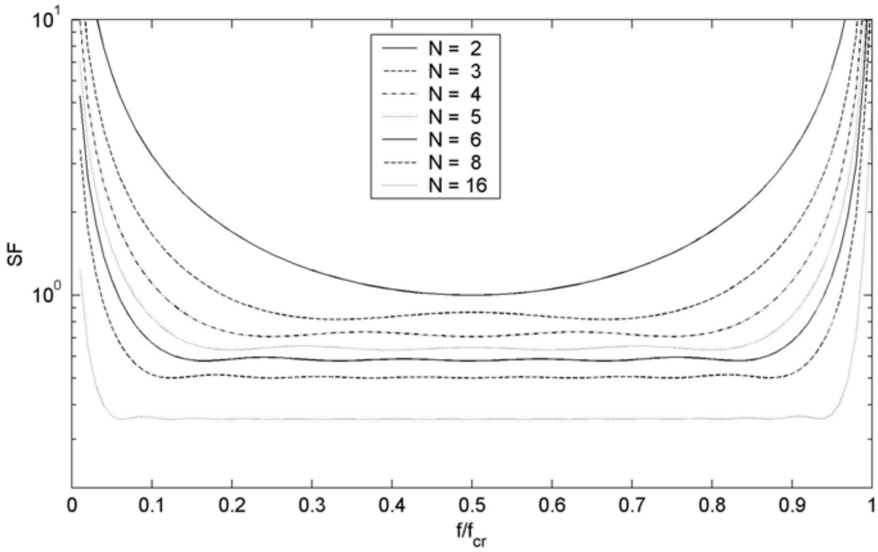

Figure 2: Singularity factors of matrix **A**, for different measurement set-ups with uniform microphone spacing. *N* is the number of microphones.

As explained in [2], the sensitivity of this method to measurement errors may be expressed with the singularity factor *(SF)* of matrix **A**, defined as

$$SF = \sqrt{\sum_j \Lambda_j^{-2}} \; , \qquad (7)$$

where Λ_j are the singular values of the matrix **A**. Using the approximations in equations (2), it can be shown that the widest frequency range with low *SF* values is obtained with uniform spacing of the microphones [2]. Such an equidistant arrangement of the microphones results in measurable frequencies that lie between 0 and f_{cr} , where $f_{cr} = c/(2d)$ is the critical frequency, and *d* is the microphone separation distance. Figure 2 shows the values of *SF* for different numbers of microphones. An increase of the effective frequency range and a decrease of the *SF* can be observed when increasing the number of microphones.

Calibration

In general, the influence of non-propagating modes increases with frequency. Hence for large-bandwidth measurements, the use of the approximations in equations (2) might lead to significant errors. A possible strategy to avoid this is to determine the coefficients via calibration. This can be achieved by measuring the microphone signals for a number of known reflectances.

Using equation (1), it can be found that the microphone signal ratio $y_n^{(m)}$ measured with known reflectance $R^{(m)}$ is

$$y_n^{(m)} = \frac{s_n^{(m)}}{s_1^{(m)}} = \frac{\alpha_n + \beta_n R^{(m)}}{\alpha_1 + \beta_1 R^{(m)}} . \tag{8}$$

By using *M* different known reflectances, and assembling the resulting $M \times (N-1)$ microphone signal ratios, we can obtain the matrix equation

$$\mathbf{B}\,\mathbf{g} = \mathbf{y} , \tag{9}$$

where

$$\mathbf{B} = \begin{bmatrix} -R^{(1)}y_2^{(1)} & 1 & R^{(1)} & 0 & 0 & . & . & 0 & 0 \\ -R^{(2)}y_2^{(2)} & 1 & R^{(2)} & 0 & 0 & . & . & 0 & 0 \\ . & . & . & . & . & . & . & . & . \\ -R^{(M)}y_2^{(M)} & 1 & R^{(M)} & 0 & 0 & . & . & 0 & 0 \\ -R^{(1)}y_3^{(1)} & 0 & 0 & 1 & R^{(1)} & . & . & 0 & 0 \\ -R^{(2)}y_3^{(2)} & 0 & 0 & 1 & R^{(2)} & . & . & 0 & 0 \\ . & . & . & . & . & . & . & . & . \\ -R^{(M)}y_3^{(M)} & 0 & 0 & 1 & R^{(M)} & . & . & 0 & 0 \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ -R^{(1)}y_N^{(1)} & 0 & 0 & 0 & 0 & . & . & 1 & R^{(1)} \\ -R^{(2)}y_N^{(2)} & 0 & 0 & 0 & 0 & . & . & 1 & R^{(2)} \\ . & . & . & . & . & . & . & . & . \\ -R^{(M)}y_N^{(M)} & 0 & 0 & 0 & 0 & . & . & 1 & R^{(M)} \end{bmatrix}, \quad \mathbf{g} = \begin{bmatrix} \tilde{A}_{12} \\ \tilde{A}_{21} \\ \tilde{A}_{22} \\ \tilde{A}_{31} \\ \tilde{A}_{32} \\ . \\ . \\ \tilde{A}_{N1} \\ \tilde{A}_{N2} \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} y_2^{(1)} \\ y_2^{(2)} \\ . \\ y_2^{(M)} \\ y_3^{(1)} \\ y_3^{(2)} \\ . \\ y_3^{(M)} \\ . \\ . \\ y_N^{(1)} \\ y_N^{(2)} \\ . \\ y_N^{(M)} \end{bmatrix}, \tag{10}$$

where $\tilde{A}_{ij}$ are elements of matrix $\tilde{\mathbf{A}}$. Equation (9) can be solved in the same manner as equation (3). Using equations (2), we can calculate approximate values for $y_n^{(m)}$, and compute an approximation of the *SF* of the calibration matrix **B**. This theoretical *SF* value represents a predicted measure of the sensitivity of the calibration procedure to measurement errors. We note the that the exact *SF* value can only be computed after actually measuring all the microphone signals; the theoretical *SF* is intended only as a tool for designing a suitable calibration procedure.

Closed cylindrical tubes are suitable as calibration objects, because their reflectances are known from theory. Computations of the *SF* of **B** have indicated that the "best spacing" is obtained when the length of the tubes are integer multiples of *d.* Therefore, the length of tube *m* in a series of *M* closed tubes was chosen such that its reflectance is

$$R^{(m)} = \exp\left[-2\Gamma(m-1)d\right]. \tag{11}$$

Note the $m = 0$ corresponds to closing of the measurement duct at the reference plane. Figure (3) shows the *SF* for different combinations of *N* and *M*, as computed from the singular values of **B**. It can be observed that (1) the effective frequency range increases with *M*, and (2) the *SF* decreases with *M* but increases with *N*.

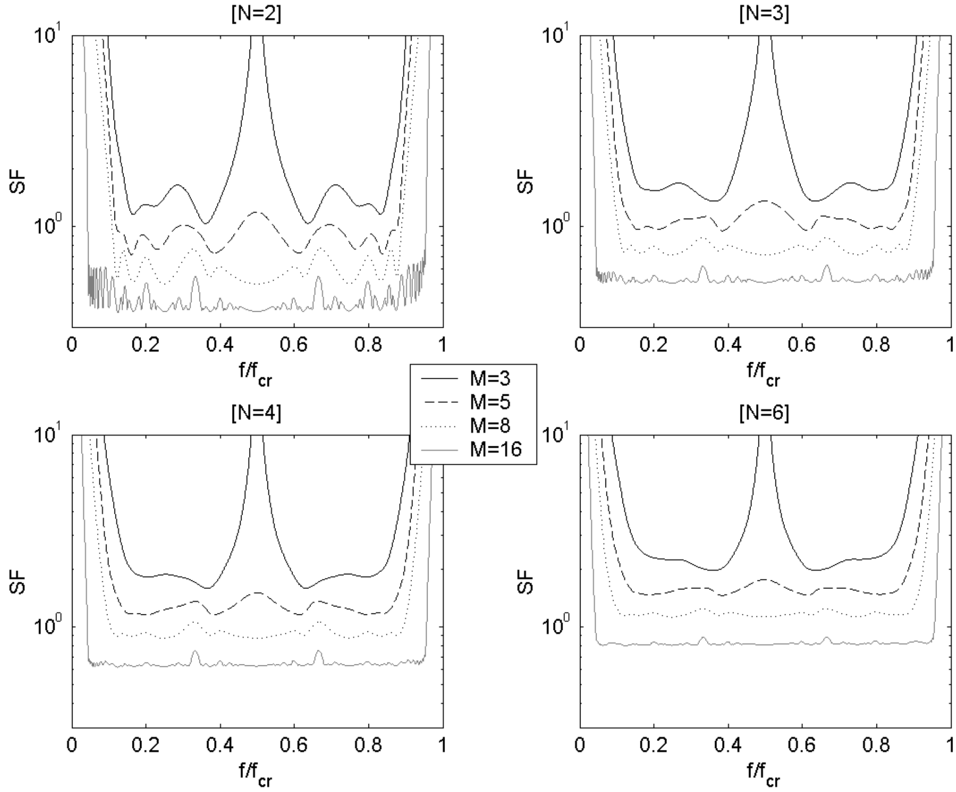

Figure 3: Theoretical singularity factor of the calibration matrix, for different measurement set-ups with uniformly spaced microphones and calibration tube lengths. *N* is the number of microphones, and *M* is the number of calibration tubes.

PRELIMINARY RESULTS WITH TWO MICROPHONES AND THREE CALIBRATIONS

The first experimental results were obtained using the simplest possible case, using $N = 2, M = 3$. The method is then equivalent to the "two-microphone three-calibration method" presented in [3]. The measurement duct that was used is 1 meter in length and has a 9.1 mm diameter. The microphone distance was chosen $d = 37$ mm, thus the critical frequency is 4.68 kHz.

As an alternative to a closed tube, one may use a calibration tube that is effectively anechoic; the input impedance of such a tube is simply its characteristic impedance, although higher modes need be taken into account in cases where there is large discontinuity at the reference plane. In order to determine a suitable calibration procedure, the theoretical *SF* of the calibration matrix was computed for three different combinations: (1) three closed tubes, (2) two closed tubes and an anechoic tube (of the same cross-section as the measurement duct), and (3) one closed tube and two anechoic tubes (of the same and of half cross-section). Figure 4 shows the theoretical *SF* of the calibration matrix for the various combinations. At most frequencies, using two anechoic tubes gives the largest singularity factors. Given that using three closed tubes results in a large *SF* at $f_{cr}/2$, we chose to use one anechoic tube and 2 closed tubes in the experiments. Figure (5) compares the magnitude of the theoretical impedance of a cylindrical tube of the same cross-section as the measurement duct to the impedance determined via measurement. The measured curve that was obtained using calibration largely overlaps with the theoretical curve, and – as could be expected – exhibits discrepancies at frequencies close to $f = f_{cr}$. The result obtained without calibration (i.e. directly using equations (2)) exhibits significantly larger discrepancies, especially at frequencies above 4 kHz, which indicates that the calibration strongly improves the accuracy of the method.

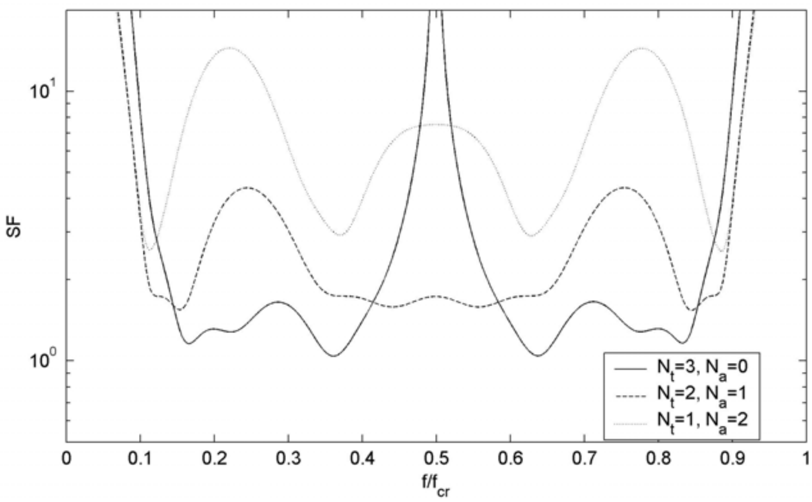

Figure 4: Theoretical singularity factor of the calibration matrix with the two-microphone three-calibration method. N_t and N_a indicate the number of closed and anechoic tubes, respectively.

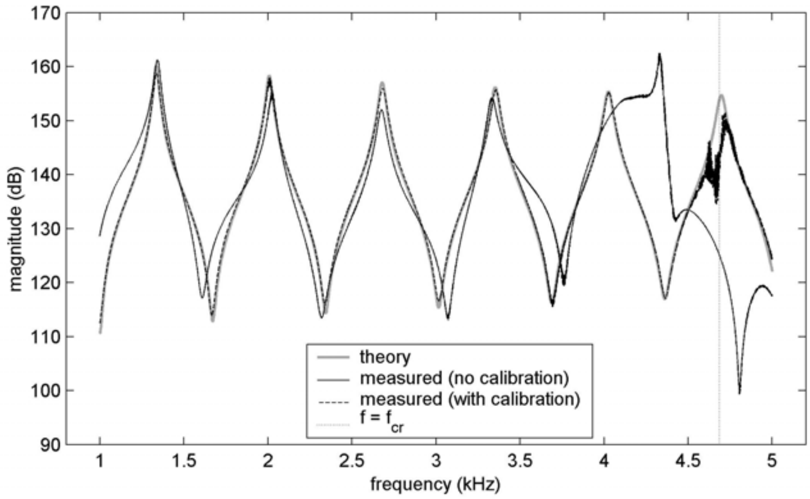

Figure 5: Input impedance of a cylindrical tube of length $L = 257$ mm. The measured curve (with calibration) largely overlaps with the theoretical curve.

CONCLUSIONS AND FUTURE RESEARCH

The errors associated with the calibration of the multiple microphone method have been investigated. The sensitivity of the calibration to measurement errors was expressed in terms of the singularity factor of the calibration matrix. Theoretical results predict that the smallest singularity factors are obtained using a series of calibration tubes which have lengths that are multiples of the microphone separation distance, and that the calibration procedure is improved by increasing the number of calibration tubes. Preliminary measurement results indicate the usefulness of the calibration procedure in the case of using two microphones and three calibrations. Work is underway to assess the feasibility of the method for larger values of *N* and *M*.

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