NONLINEAR INTERACTION OF ULTRASONIC WAVES IN AN INHOMOGENEOUSLY PREDEFORMED ELASTIC MATERIAL

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ABSTRACT Simultaneous propagation of two finite amplitude ultrasonic waves in an inhomogeneously predeformed nonlinear elastic material is investigated theoretically. The analytical solution to describe the propagation, reflection and interaction of waves with arbitrary smooth initial profile excited simultaneously on two opposite parallel surfaces of the material is derived. Nonlinear effects that accompany two wave simultaneous propagation, reflection and interaction are studied in detail. It is clarified that the ultrasound nonlinear response has different sensitivity to the variation of the material and predeformed state parameters. This enables to propose algorithms for nondestructive material characterization. The corresponding numerical experiment is posed.

INTRODUCTION Utilization of wave propagation data in nondestructive testing (NDT) became possible after discovery of the piezoelectric effect by the brothers Pierre and Jacques Curie in 1880 [1]. The first attempt to use this possibility was made in 1913 [2]. Later on, NDT has progressed at a remarkable rate as a result of rapid advancements in theoretical treatment of the problem, in computer technology and in electronics. The effective methods for evaluating mechanical properties of materials, for determination of its structure and state are developed [3].

This paper deals with the results on development of mathematical basis of NDT of nonlinear elastic materials undergoing inhomogeneous predeformation. The mathematical basis and the experimental set-up of the ultrasonic NDT of materials with homogeneous predeformation is developed in detail [4, 5]. Inhomogeneity of predeformation introduces additional problems in NDT. The amount of information to be determined increases essentially. The necessity to extract more information from wave propagation measurements data arises. This problem is under intensive investigation [6, 7]. Most proposed solutions in this direction pose the problem of acoustodiagnostics of inhomogeneously predeformed materials on the basis of solitary nonlinear wave propagation data [8, 9]. Another possibility is to increase the number of simultaneously propagating waves in the material [10, 11].

In this paper the both possibilities are considered. Nonlinear propagation of longitudinal wave or simultaneous propagation of two waves in inhomogeneously predeformed material is described analytically. Evolution of nonlinear effects of wave propagation is studied. The dependences wave characteristics versus parameters of the predeformed state of the material are clarified. These dependences form the basis for an analysis of the sensitivity of the wave propagation on the predeformed state parameters and may therefore be used in nondestructive characterization of the predeformed state of the material.

ACOUSTOELASTIC EFFECT AND INHOMOGENEITY The acoustoelastic effect - the dependence of wave velocity on the value of initial stress in solids was described by Bergman and Shanbender in 1958 [12] and Benson and Raelson in 1959 [13]. After that, this effect was intensively studied theoretically and experimentally and numerous methods for nondestructive evaluation of homogeneous predeformation in various materials were developed in detail [4, 5]. It was proved that in nonlinear elastic material the influence of the geometrical nonlinearity and the physical nonlinearity on the acoustoelastic effect has in many cases the same order [14]. In addition, the geometrical nonlinearity enables to bind the stress fields caused by the prestress and the wave propagation. This is the reason why by description of the wave motion in the predeformed material it is necessary to determine the initially isotropic and homogeneous elastic material on the basis of the physically nonlinear theory of elasticity involving both kinds of nonlinearity [15].

NDT of inhomogeneously predeformed material on the basis of the acoustoelastic effect only may lead to serious mistakes in interpreting of experimental data. For example, in the case of the material undergoing pure bending the average wave velocity is equal to the wave velocity in the predeformation free material. To distinguish these two cases the additional information is needed. One possibility is to consider besides the acoustoelastic effect also the nonlinear effects of wave propagation [16]. This possibility is studied below theoretically.

A nonlinear elastic material is considered. Deformations of the material are described on the basis of the nonlinear theory of elasticity [15]. An attention is confined to small but finite deformations. The physical and geometrical nonlinearity is taken into account. The physical nonlinearity corresponds to the nonlinear theory of elasticity with five elastic constants. The geometrical nonlinearity is involved by the Lagrangian deformation tensor.

The quasi one-dimensional problem of one-dimensional wave propagation in two-dimensional predeformed material is described in Lagrangian coordinates by the equation

$$f_1 U,_{XX} + f_2 U,_X + f_3 U,_X U,_{XX} - f_4 U,_{tt} = 0 \tag{1}$$

Here *U* denotes displacement and indices after the comma indicate differentiation with respect to the coordinate *X* or the time *t*. The coefficients f_j, $j=1, 2, \ldots, 4$ are functions of the predeformation and the physical properties of the material [16].

Equation (1) is solved by assumption that the predeformed state of the material is known. The perturbation method is used and the solution is sought in the form of series

$$U = \sum_{n=1}^{\square} \text{å}^{n} U^{(n)} \tag{2}$$

where *å* is a positive small perturbation parameter.

The initial conditions are taken equal to zero and the condition on the boundary is expressed in terms of particles velocity by the expression

$$U_{,t}\ (0, t) = \text{å} a_0\ \text{ö}(t)\ H(t). \tag{3}$$

Here *H(t)* denotes Heaviside unit step function, and a_0 is constant. The smooth arbitrary initial wave profile is determined by the function *ö(t)* with *max |ö(t)| = 1*.

A solution to the equation (1) is derived in [16]. The solution describes nonlinear propagation of the longitudinal wave with arbitrary smooth initial profile into the depth of inhomogeneously predeformed nonlinear elastic material. Consequently, the solution determines the initial stage of wave propagation and it is supposed that in this initial stage the distortion of the initially smooth wave profiles is weak and the shock wave is not generated.

Utilization of nonlinear effects of wave propagation for NDT of inhomogeneously predeformed material is studied on the basis of sine wave propagation by introducing *ö(t)=sin ùt*, where *ù* denotes the frequency into the condition (3). It is assumed that the inhomogeneous predeformed state of the material corresponds to pure bending with compression or tension characterized by the component $T_{22} = a + b\ X$ of the Kirchhoff pseudostress tensor. The constant a characterizes the constant part of stress and the constant b the linearly variable part of it.

The analytical solution to describe nonlinear propagation of sine wave in the material undergoing two parametric predeformed state is derived. The solution determines the dependencies of the nonlinear wave characteristics (valus of nonperiodic term, amplitudes and phase shifts of harmonics) on the properties of the elastic material (density, second and third order elastic constants) and the parameters of the predeformed state (a and b). Unfortunately, these nonlinear expressions turned out to be too cumbersome for the analytical analyses.

The following scheme for NDT is proposed. The numerical experiment is posed and the plots wave characteristics versus material or its state parameters are plotted. The analysis of the results of computation indicates that the variation of the amplitude of the first harmonic is strongly sensitive to the inhomogeneity of predeformation, i.e. to the parameter b and less sensitive to the homogeneous part of predeformation (the parameter a). The variation of the amplitude of the second harmonic, the phase shifts of the first and the second harmonics and the phase velocities of both harmonics (acoustoelastic effect), all are strongly sensitive to both parameters of predeformation. Significant is that these dependencies are rather linear at the range of parameters variation and with respect to various parameters they are qualitatively different. These different relationships permit to propose an algorithm to ultrasonic diagnostics of the considered predeformed state of the material.

For example, the preliminary information about the physical properties of the material is available and the intension is to evaluate the parameters of the described above two parametric predeformed state. Taking into consideration the results of computation presented above it is decided to solve the problem of nondestructive evaluation of the predeformed state on the basis of the amplitude and velocity measurement data of the first harmonic. The plots amplitude and velocity of the first harmonic versus parameters of the predeformed state are composed for the real material. Then the parameter b with some precision is determined on the basis of the first amplitude measurement data making use of the corresponding plot. After that the measured value of the velocity of the first harmonic (acoustoelastic effect) and the known value of the parameter b make it possible to determine the parameter a resorting to the second plot.

It is important for NDT that the recorded nonlinear effects of wave propagation contain maximum information about the inhomogeneous predeformation. The amount of this information is dependent on the ratio of strain intensities in the material caused by the propagating wave and the predeformation, correspondingly. The evolution of nonlinear effects and the interaction of the effects of nonlinearity and inhomogeneity are analyzed for various values of intensity of the excited wave.

If the displacements of the material points of the material caused by wave motion are much smaller than the corresponding displacements caused by predeformation then nonlinear effects of wave propagation are small and it is difficult if not impossible to use them in nondestructive evaluation of the predeformed state of the material. In contrary, if the displacements caused by wave motion are much larger than the corresponding displacements caused by predeformation then the large displacements caused by wave propagation are superposed on small displacements caused by of predeformation. The result is that large nonlinear effects of wave propagation are very weakly affected by predeformation and in NDT it is difficult to extract information about the predeformed state from the large nonlinear effects of wave propagation.

The conclusion is that in ultrasonic NDT of inhomogeneous materials it is important to pay attention to the intensity of the applied excitation. The best correction for nonlinear effects of wave propagation may be achieved by choosing the wave intensity such that deformations of the material caused by the wave propagation are of the same order as deformations of the predeformed state.

NONLINEAR WAVE INTERACTION In comparison with the through transmission technique, application of multi-wave propagation in NDT enhances essentially the effectiveness of nondestructive characterization of materials and their states. The simplest possible realization of the nonlinear interaction technique for inhomogeneously predeformed nonlinear elastic material follows.

The specimen (structural element) has two parallel traction free surfaces. The wave process is excited simultaneously on these surfaces. Mathematically, following the ideas presented above the solution to equation (1) is derived under the initial conditions taken equal to zero and the conditions on the boundaries

$$U_{,t}\,(0,\, t) = \text{å}a_0\ \text{ö}(t)\ H(t),$$

$$U_{,t}\,(h,\, t) = \text{å}a_h\ \text{ø}(t)\ \ H(t). \qquad (4)$$

Here a_0 and a_h are constants. The arbitrary smooth initial wave profiles are determined by functions ö(t) and ø(t) with max |ö(t)| = 1 and max | ø(t)| = 1.

The perturbation technique is used. The analytical solution to describe simultaneous nonlinear propagation, reflection and interaction of two longitudinal waves with arbitrary smooth initial profiles in the material is obtained. The solution is valid in some time interval and similarly to the through transmission problem, it describes the initial stage of the waves propagation, reflection and interaction.

The possibility to use this solution for nondestructive characterization of inhomogeneous predefomed state of the material is studied on the basis of sine wave propagation. The corresponding cumbersome analytical solution is derived.

In NDT the sine wave propagation is excited in the material in terms of particle velocity (see the boundary conditions (4)). The evolution of wave profiles is recorded on the same surfaces in terms of stress, induced by the wave motion. The time interval between the instant of the wave front excitation and the second reflection of it from the opposite surface is considered. The recorded evolution of the wave profiles have in this interval two clearly distinguishable intervals – interval of propagation and interval of wave interaction.

The two wave nonlinear interaction technique is illustrated by the numerical experiment with the duralumin sample. It is assumed that the preliminary information about the physical properties of the material and the kind of predeformed state is available. The two parametric predeformed state coincides with the predeformed state described above. It is characterized by the component $T_{22} = a + b\,X$ of the Kirchhoff pseudostress tensor, where *a* and *b* are unknown constants.

Interesting is that the recorded in NDT amplitude of the wave induced stress depends essentially on the initial value of the wave frequency in the interaction interval [17]. If the number of wave periods in the interval of propagation is equal to integer, the maximal amplification occurs and the wave amplitude in the interval of interaction is three times greater than in the interval of propagation. This amplification phenomenon is caused mainly by the superposition of wave amplitudes described by the linear part of the solution.

The most important information from the point of view of NDT may be obtained from the analyses of the nonlinear effects of wave propagation, reflection and interaction. These effects characterize the influence of predeformation and nonlinearity on the wave motion. The evolution of nonlinear effects is governed by the oscillation with the double frequency. There is also a strong amplification of the wave amplitude in the interval of interaction but this amplification is not very sensitive to the value of the initial wave frequency.

Analysis of nonlinear effects of two sine waves simultaneous propagation, reflection and interaction data on both surfaces and in two intervals on both surfaces of the material enables to solve several qualitative and quantitative NDT problems. It is easy to distinguish qualitatively the following special cases of the predeformed state of the material:
(i) a homogeneous predeformation-free nonlinear elastic material,
(ii) a homogeneously predeformed nonlinear elastic material,
(iii) a nonlinear elastic material undergoing pure bending,
(iv) a nonlinear elastic material undergoing pure bending with tension or compression.

In the case (i) of a predeformation-free nonlinear elastic material oscillations on both surfaces have different constant amplitudes in both time intervals. The oscillation is not modulated. The homogeneous predeformation (case (ii)) modulates these amplitudes. The profile of modulation is dependent on the sign of the parameter *a* and the depth of modulation is very sensitive to the value of predeformation. The inhomogeneity of predeformation characterized by the parameter *b* (cases (iii) and (iv)) causes disparity in amplitude modulations on different surfaces. The peculiarity of the case (iii) is that this disparity is characterized by the phase shift of similar modulations caused by the different sign of stress on different surfaces.

The quantitative nondestructive evaluation of the considered predeformed state (evaluation of the parameters *a* and *b*) may be posed making use of the dependence of the depth of modulation of the oscillation of nonlinear effects on the boundary on the values of the predeformed state parameters. The quantitative NDT problem may be solved as follows. The plots predeformed state parameters versus modulation depth value are composed. The modulation depth value is determined on the basis of measurement data on the surfaces of the material. Resorting to the composed plots the parameters of the predeformed state may be evaluated.

CONCLUSIONS The through transmission technique of NDT of predeformed materials is used in experiments where there is an access to two parallel surfaces of the specimen (structural element). The conventional approach of predeformed state evaluation is based on utilization of the acoustoelastic effect. As it is shown above the acoustoelastic effect is strongly sensitive to the homogeneous predeformation as well as to the inhomogeneity of it. Inhomogeneity of predeformation may lead to incorrect interpretation of the data of NDT based only on the acoustoelastic effect. The additional information is needed to determine the inhomogeneity of the predeformed state. Utilization of nonlinear effects of wave propagation for his purpose is demonstrated in this paper.

The analysis of information obtained from multi-wave simultaneous propagation data in inhomogeneously predeformed media enables to enhance the efficiency of NDT. In comparison with single wave experiments the amount of information extracted from the recorded multi-wave simultaneous propagation, reflection and nonlinear interaction data increases essentially. It becomes possible to use the wave profile distortion data in qualitative and quantitative NDT of the properties and the state of the material. These possibilities are illustrated by the numerical experiment. It is shown that the qualitative and quantitative analysis of the recorded nonlinear oscillation profile data on the boundaries of the material enable to distinguish special cases of the two-parametric predeformed state of the material and to evaluate the parameters of the predeformed state.

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