NUMERICAL MODELLING OF THE TYRE/ROAD CONTACT

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ABSTRACT

In this paper, a numerical method to solve the non-linear contact problem between a tyre and the road in the time domain is presented. The response of the tyre is calculated by a convolution between the contact forces and the tyre's Green's functions. Influence functions for the points in the contact zone are calculated from the tyre model, and used for solving the dynamic contact problem. Thus, coupling between contact points is taken into account. Numerical results show that the coupling is important at high frequencies, and for stiff tyre treads. Additionally, problems regarding the contact resonance are reduced.

INTRODUCTION, PROBLEM DESCRIPTION

Though there has been research conducted in the field of tyre/road noise for more than 30 years there is still a lack of quantitative prediction models for the sound radiation from rolling tyres. This indicates that the tyre/road interaction is a complex process. The tyre is a complicated structure consisting as it does of several layers of various materials, oriented in different ways. Many interesting aspects of contact mechanics exist in the contact zone for example large slips, friction, adhesion and stick-slip motions. In a complete contact model, all of these aspects have to be considered. However, this is yet beyond the state of the art. The size of the contact area is large compared to the wavelength of the vibrations in the tyre structure and the wave speed in the tyre is relatively low. As a consequence, the dynamic effects of the tyre and the roughness variation in the contact zone must be considered. Traditional methods for solving contact problems between elastic bodies can not directly be applied to the tyre/road interaction, since they are restricted to small contact areas and dynamic effects are neglected. The so-called influence functions describing the interaction between adjacent contact points at the tread surface have to be included for a consistent description of the contact. To solve the tyre/road interaction problem requires first a tyre model describing the properties of the tyre surface and secondly a contact model taking into account the non-linear interaction and the coupling between adjacent contact points.

The quality of an acoustic rolling contact model depends on the quality of the modelling of the contact forces. It is relatively easy to measure vibrations and sound radiation from tyres. However, to measure the force distribution in the contact zone for tyres rolling on rough surfaces at high speeds is cumbersome, though indirect measurements of accelerations or deformations in the contact zone might be helpful. Thus, the contact model is hard to validate and the numerical modelling of the contact requires therefore high reliability.

When the tyre is rolling on a rough road surface, the contact forces between the tyre and the road will vary over time. This time variation leads to vibrations of the tyre structure, which radiate sound to the surrounding air. Additionally, the so-called local deformation of the soft rubber tread surface around asperities in the road texture contributes to the sound radiation when air is squeezed out of or sucked into the voids between the tyre and road surface.

The modelling of the sound generation from rolling tyres is divided into three modules. A tyre model provides the dynamic properties of the structure, a contact model describe the forces in the contact zone and a radiation model gives the sound pressure at points outside the tyre.

Existing acoustic rolling models for the tyre/road interaction suffer from some drawbacks. Only the radial contact forces are considered and a bedding, consisting of uncoupled linear springs, is used for the calculation of the contact forces. Recently a different formulation of the contact was presented [1]. This paper presents the updated contact model and shows results obtained using the model.

TYRE MODEL INCLUDING LOCAL DEFORMATION

Measurements of the driving point mobility of a tyre reveal information of the dynamic behaviour of the tyre structure. Figure 1 shows such measurements on a smooth tyre, i.e. a tyre without tread pattern. The measurements were done using four different sizes of the excitation area. As the size of the excitation area decreases, the level of the driving point mobility increases in the same time as it shows a frequency dependency similar to that of a spring. The reason for the resonance like peak at high frequencies is because of the mass loading of the measurement equipment, which is hard to avoid in the rather complicated measurement situation. At low frequencies membrane waves due to the internal inflation pressure determine the response and rather low damped modes appear in the structural response. At higher frequencies modes in the lateral direction (width) of the tyre start to set on, indicating a plate behaviour at high frequencies.

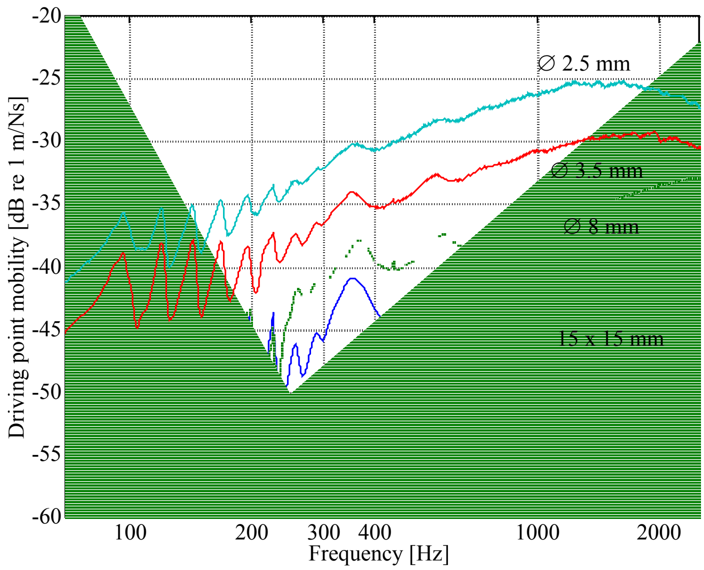


Figure 1. Measured driving point mobility on a smooth tyre for different sizes of the excitation area

The tyre model used for calculating the response of the tyre to external excitations is described in [2]. The tyre is modelled as a double layered flat plate under tension on an elastic foundation. Each layer is described by the elastic field equations, which means that the response can be calculated both radial and tangentially, including the so-called local deformation. This effect regards the response of the structure when the wavelength of the structural vibrations is comparable to the size of the excitation area and the thickness of the plate. Figure 2 shows the calculated driving point mobility for three different excitation areas using the model.

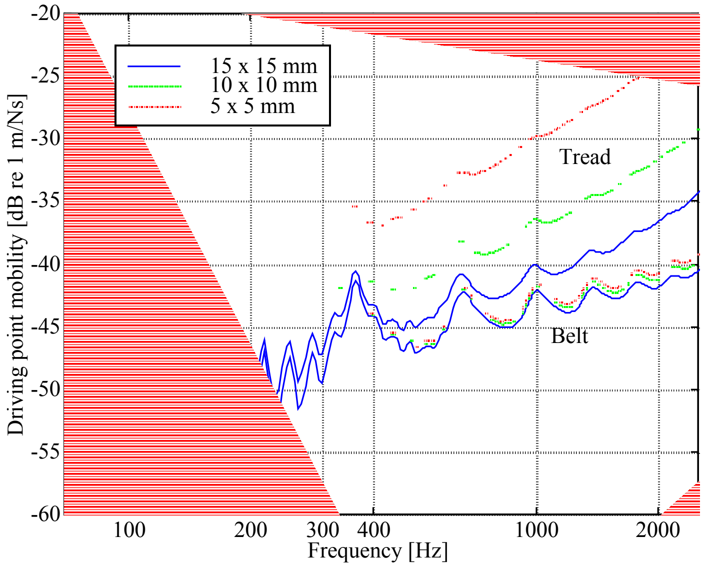


Figure 2. Typical results obtained from the tyre model.

The results show that the model captures the main behaviour of a tyre and that that it is possible to model the tyre tread surface including the local stiffness effects. The smaller the size of the excitation area, the higher the level for the driving point mobility. In addition, the frequency dependency at the tread surface indicates spring behaviour as seen in the measurements. It should be emphasised that the material parameters used in the calculations above are not adapted to the material parameters for the tyre used in the measurements in figure 1. The tyre model is used to calculate the response of the tyre at the outer tread surface in the frequency domain. Since the contact model is formulated in the time domain because of the non-linear character of the contact problem, the results are transformed to the time domain by the use of the inverse Fourier transform. However, this procedure requires some special care in order to obtain a causal impulse response for the contact model.

CONTACT MODEL

In this work, the contact forces are calculated in the time domain since the contact between the tyre and the road is non-linear. The approach is based on the work by McIntyre et al. [3], who studied the non-linear contact between a violin bow and a string. The response of a point on the tyre can be calculated according to equation 1

$$u_e(t) = \sum_m F_m(t) * g_{m,e}(t) = \sum_m \int_{-\infty}^{\infty} F_m(\tau) g_{m,e}(t-\tau)\,\mathrm{d}\tau \tag{1}$$

where u is the displacement at point e on the structure, F is the force at contact point m and g is the so-called Green's function or impulse response function from the contact force to the receiver point.

The integral is rewritten in discrete form for the implementation in a computer code. The problem of solving the integral equation is simplified by using causality and assuming that the system is at rest for times smaller than $t=0$, which is shown in equation 2

$$u_e(t) = \sum_m F_m(N\Delta t) g_{m,e}(0)\Delta t + \sum_m \sum_{n=0}^{N-1} F_m(n\Delta t) g_{m,e}((N-n)\Delta t)\Delta t \tag{2}$$

where the first term represents values at the actual time step and the second term represents known values from previous time steps. A more convenient formulation may be obtained using matrix notations according to equation 3.

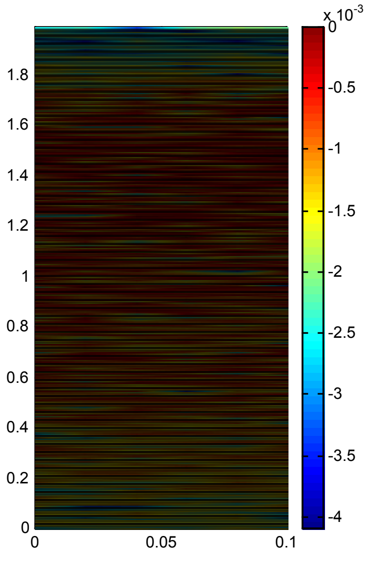
$$\boldsymbol{u} = \boldsymbol{G_0 F} + \boldsymbol{u_{old}} \tag{3}$$

Where $\boldsymbol{G_0}$ is a matrix contacting the influence coefficients, i.e. the coupling terms between the points in contact.

The boundary conditions at the contact points are that the tyre surface has to be on or above the road surface. The deformation needed at each point in contact in order to fulfil the boundary conditions is given by the geometry of the contact. The contact force at the current time step can then be calculated by solving equation 3 for the force. A contact algorithm for the solution of the problem has been developed that uses an active set strategy. An iterative method is used to find the active points, i.e. the points in contact at each time step. When the active set of contact points are found, equation 3 can be inverted directly for those points only. This can be done for all points in the entire contact zone provided that the influence functions of the tyre tread and roughness data of the road surface are given for the complete contact zone. However, the calculation effort for doing this is substantial and simplifications are needed. Therefore, a different approach is used to take the roughness data over the width of the tyre into account, which is described briefly in the following.

ROAD TEXTURE

The idea of simplification of the contact model is to use a quasi three-dimensional model for the roughness variation over the width of the tyre as described in [4]. Figure 3 shows an example of a rough road surface where the colour indicates the height of the road surface. This kind of roughness distribution represents what the tyre sees as it rolls over the road surface. By looking at a single “slice” of the tyre cross-section as it contacts the rough surface, see figure 4, the total force exerted on it can be calculated as a function of the displacement of the cross section. In this way the roughness distribution over the width is replaced by a non-linear stiffness function. In addition the force distribution over the width is replaced by a symmetric excitation, which simplifies the calculation of the response.



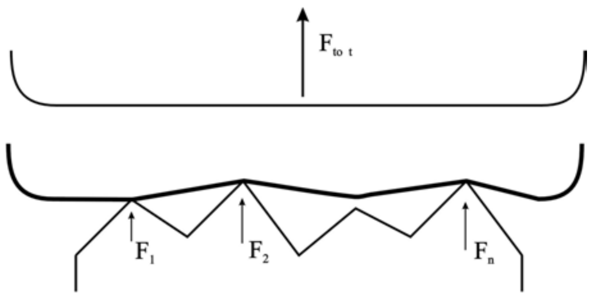


Figure 4. A slice of the cross section of a tyre in contact with a rough road surface used to calculate the contact stiffness.

Figure 3. An example of a road surface profile.

The problem of calculating the total force on each slice is solved by traditional contact mechanics using either a Winkler bedding or an elastic half-space. However, no significant difference has been observed between these two solution methods.

The non-linear stiffness functions relating the total force to the displacement of each tyre slice is normalised in order to avoid any problems of selecting proper material data for calculating the stiffnesses. Incorporating the contact stiffness functions in equation 3 for the calculation of the contact force results in equation 4

$$F(y) = G_0^{-1} \begin{bmatrix} \int_0^{y_1} s_1(\eta)\mathrm{d}\eta \\ \vdots \\ \int_0^{y_m} s_m(\eta)\mathrm{d}\eta \end{bmatrix} \quad (4)$$

where s are the non-linear stiffness functions as functions of the displacements η.

RESULTS

The main results obtained using the contact model is the contact force distribution over the contact length as function of time. Figure 5 shows an example of the contact forces as function of time.

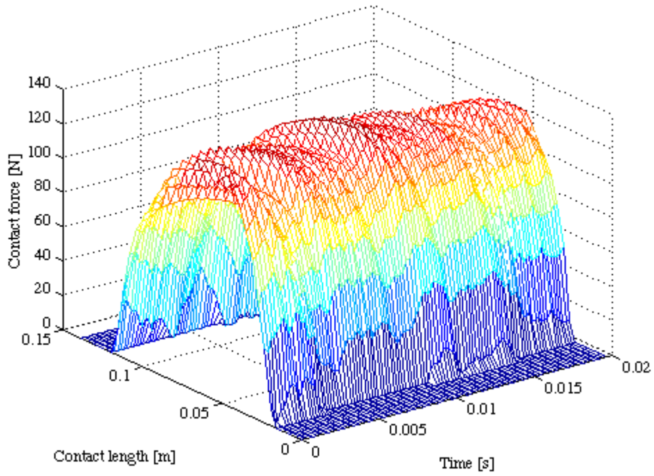


Figure 5. The force distribution in the contact zone as a function of time

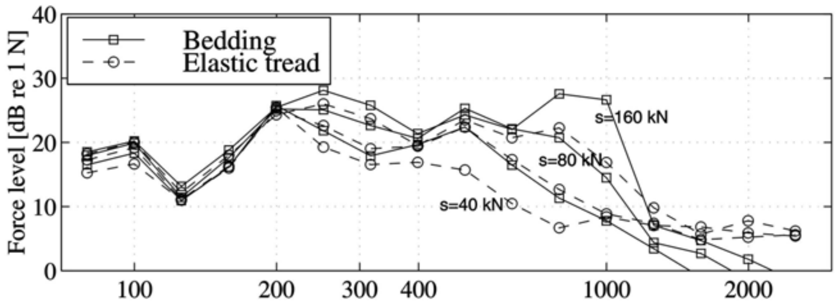


Figure 6 Total contact force as a function of frequency in third octave bands. The

Figure 6 shows the results from the contact model described in this paper as compared to results obtained by the use of a previous model, which is described in for example [5]. In the calculations the influence functions are scaled to represent the same contact stiffness as used in the previous bedding model for comparison. Using the bedding model for the contact, higher levels for the contact force are obtained, especially at about 800 Hz. The results from the contact model using the elastic properties of the tyre tread shows lower level in this frequency range indicating that some problems of contact resonances that was seen in the bedding model is decreased. The reason for this is that the coupling between the contact points is taken into account in the elastic tread model.

When the forces are obtained, the vibrations of the tyre can be calculated both at the tyre belt and at the tread surface. The vibrations are calculated in the frequency domain in order to decrease the computational effort. Since the velocity field can be calculated on the outer surface of the tyre tread including the local deformation, the sound radiation can be calculated including the local deformation. Hence, no additional model for the air pumping because of the local deformation is needed when using this model.

NUMERICAL PERFORMANCE OF THE MODEL

The updated contact model needs more calculation effort as compared to the previous model based on the Winkler bedding, since the solution of the contact problem involves the inversion of the influence matrix and the problem is solved iteratively. However, the number of iterations needed is in most cases very few. If the initial guess of the contact points is close to the solution, only one step is needed (no iteration), while if the initial guess differs substantially from the correct one, one or two iterations are needed for convergence. One should keep in mind that the iteration involves finding the correct contact points, not to find the correct magnitude of the contact forces. Anyhow, in some cases the algorithm is unstable and the search has to be aborted. If this happens, only the positive forces are chosen as the solution.

The most time consuming step in the model is to obtain the influence functions for the tread surface. To get sufficiently high resolution in the results a high number of wavenumbers are required. The number of wavenumbers is determined by the discretisation of the tyre circumference in order to be able to resolve the external force on the tyre structure.

CONCLUSIONS

The work in this paper concerns a model for calculating the non-linear forces in the interaction between the tyre and the road. This contact is complicated by the fact that the contact area is not small in comparison to the wavelength of the vibrations in the tyre and to the roughness variations of the road. Traditional methods used in contact mechanics can not be calculated directly. A tyre model was previously developed that can be used to calculate so-called influence functions of the tyre tread surface describing the interaction between adjacent points. These functions are used in the contact model to calculate the contact forces using the physical properties of the tyre. The advantage with this approach is that the choice of the bedding stiffness in previous models is avoided. In addition the model results in a simplified model for the sound radiation since the vibrations of the outermost tread surface can be calculated directly including the local deformation.

ACKNOWLEDGEMENTS

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