A WAVE MODEL OF A CIRCULAR TYRE

PACS REFERENCE:

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ABSTRACT

The equations of motion of a curved tyre belt are derived for one-dimensional waves propagating around the belt and a standing wave across the belt. The effects of curvature, shear stiffness, rotary inertia, tension, rotational speed and air pressure are included. These are combined to give a sixth order wave equation, the solution of which gives three pairs of wave-number as a function of frequency. The application of the boundary conditions at the contact leads to the input and transfer mobilities for both in-plane and out of plane excitation. Observed are: low frequency rigid-body modes, belt bending modes and in-plane ring modes.

1.INTRODUCTION

When a rotating tyre interacts with the road surface, the time varying deformations are transmitted causing noise interior and exterior to the vehicle. To calculate this interaction with the road and also to determine the resulting vibration of the tyre surface it is first necessary to make a dynamic model of the tyre. The main exterior noise occurs between 500Hz and 3000Hz, a region where there is little modal behaviour of the belt flexural waves. An infinite flat belt wave model that included tension, transverse shear, rotary inertia and bending was made for this region [1]. However a full tyre model should embrace the whole low frequency range to include the vehicle interior noise between 50Hz and 500Hz, and the quasi-static slip of road-tyre interaction.

The objective here is therefore to extend the previous wave model to include: curvature, asymmetric belt and tyre rotation to the other parameters, and thus make a complete circular belt model. As the main effect of curvature is to couple transverse and longitudinal motion the response to both transverse and in-plane forces are obtained. The waves in the air cavity are neglected here, as they are only noticed at the first cavity resonance.

Using the equations for radial and circumferential equilibrium a sixth order wave equation is constructed. The solution is three pairs of roots or waves at each frequency. These waves can then be added to satisfy the chosen boundary conditions at the excitation zone. These boundary conditions can be displacements, rotations, forces or moments. Presented here are only the frequency response functions for radial and circumferential forces.

2: EQUATIONS OF MOTION

The tyre could be described as a curved, tensioned, Mindlin plate on distributed two-directional stiffness. Figure 1 shows a segment of length δs and width δz, in a tyre belt of radius a and width b, displaying the sign convention for positive directions, rotations, forces and moments. The belt is subjected to a net static pressure P which causes a static tension/length N_S, N_Z in the circumferencial and transverse directions s, z.. Q_S, Q_Z are the shear forces/length. N is the total static and dynamic circumferencial force/width. M_S, M_Z are the bending moments/length. The accompanying displacements are u,w, in the circumferential and radial directions. θ describes the geometric position and is related to the circumferential co-ordinate s as: $s = a\theta$. The tyre rotates in the positive θ direction with angular velocity Ω. The belt is restrained either side by a side-wall of stiffnesses/ belt length of K_s , K_r , in the circumferential and radial directions.

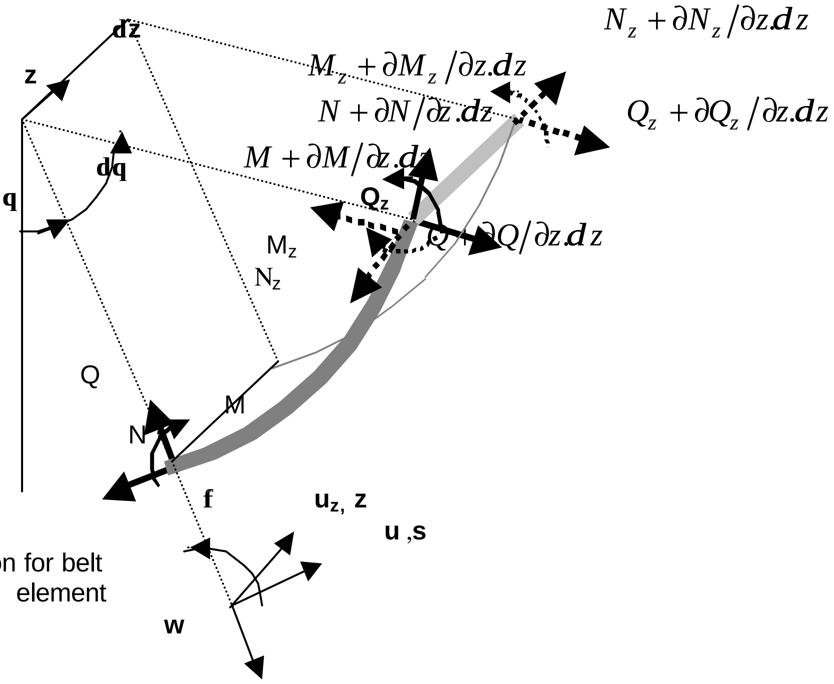

Figure 1: Sign Convention for belt element

In this section three groups of equations are presented: kinematic relationships, equilibrium of forces and moments, and the linking Hooke's Law relationships. It is also assumed that the averaged material properties of the cross-section are known, as reference will only be made to these single material values for the cross-section. The segment motion can be described by three variables; displacements u,w and the kinematic rotation Φ. For a Mindlin plate that can deform in both bending and shear, the slope at any position s in the s and z directions are respectively:

$$\frac{\partial w}{\partial s} = \beta_s + \gamma_s , \quad \frac{\partial w}{\partial z} = \beta_z + \gamma_z \qquad (1a,b)$$

where β_s ,β_z and γ_s ,γ_z are the slopes due to bending and shear respectively. The kinematic rotations Φ_S, ,Φ_Z, in the s and z direction, of the element seen in Figure 1, including that due to the circumferential displacement u, are therefore:

$$\Phi_s = \frac{u}{a} - \frac{\partial w}{\partial s}, \quad \Phi_z = -\frac{\partial w}{\partial z} \tag{2a,b}$$

The total angle j in the s direction is the sum of the geometric rotation q and the kinematic rotation Φ_s**.** The small change in slope over the length ds is therefore:

$$dj = \left(\frac{1}{a} + \frac{\partial \Phi_s}{\partial s} \right) ds \tag{3}$$

The circumferencial strain e_s and transverse strain e_z are:

$$e_s = \frac{\partial u}{\partial s} + \frac{w}{a}, \quad e_z = \frac{\partial u_z}{\partial z} \tag{4a,b}$$

There are four equations of equilibrium: forces in the radial direction, forces in the circumferencial direction, and for moments in the s and z directions. The equilibrium of radial forces taken in line with the circumferencial shear force/width Q_s. Substitution of the segment rotation in equation (2a) yields:

$$P + m\Omega^2 a + p(s,z) + \frac{\partial Q_s}{\partial s} + \frac{\partial Q_z}{\partial z} - N\left(\frac{1}{a} + \frac{\partial \Phi_s}{\partial s} \right) + N_z \frac{\partial^2 w}{\partial z^2} = m\ddot{w} - \left(\frac{P}{a} + m\Omega^2 \right) w \tag{5}$$

where P is the net static pressure. $p(s,z)$ is the dynamic pressure due to the side-wall and external radial force. N_z is the static transverse tensile force/length, N is the total static and dynamic circumferencial tensile force/width, Q_z is the transverse shear force/width. m is the belt mass/area. The last term is the extra centrifugal force due to displacement w.

By resolving forces in the s direction in line with the circumferencial force/width N in Figure 2a, and substitution of the segment rotation in equation (2a), gives:

$$t\left(s,z\right) + \frac{\partial N}{\partial s} + Q_s \left(\frac{1}{a} + \frac{\partial \Phi_s}{\partial s} \right) = m\ddot{u} + 2m\dot{w}\frac{c}{a} + \frac{2K_s}{b} u \tag{6}$$

where K_s is the single side-wall circumferencial stiffness/belt length. The tangential external stress is t. . The second term from the left is the Coriolis force, which gives gyroscopic coupling between the axial and radial motion.

The net moment, taken about the right hand end z axis of the segment is responsible only for the angular acceleration due to bending $\ddot{b}_s$ as seen in equation 7a. A similar relation holds for the z direction:

$$Q_s - \frac{\partial M_s}{\partial s} = I_s \ddot{b}_s, \quad Q_z - \frac{\partial M_z}{\partial z} = I_z \ddot{b}_z \tag{7a,b}$$

The three types of forces in equations 5-7 are assumed to be linearly related to the three deformation types in equations 1-4 by the various elastic moduli of the belt section. There are the following three groups of these Hookes Law relationships.

The circumferential force/width N has a static component N_s and a dynamic component arising from the circumferential strain ε_s :

$$N = N_s + A_s\varepsilon_s, \quad N_s = Pa\left(1-\frac{l_s}{b}\frac{\sin\phi_1}{\phi_c}\right) + \mu\Omega^2 a^2, \quad N_z = \frac{1}{2}\frac{Pl_s}{\phi_c} \tag{8a,b,c}$$

where A_s is the belt axial stiffness/width. The static tensions N_s, N_z calculated in [2], are determined from the pressure, side-wall geometry and from resisting the centrifugal force $\mu\Omega^2 a$. $2\phi_c$ is the angle subtended by the side-wall, ϕ_1 is the angle the angle between side-wall and the ground. In the circumferencial and transverse directions the shear force/width Q_s, Q_z is related to the shear strain by the belt shear stiffness/width S_s, S_z.:

$$Q_s = S_s\gamma_s, \quad Q_z = S_z\gamma_z \tag{9a,b}$$

The bending moment in the circumferencial and transverse directions can be written from [2] as equations 10. This concludes the component equations for the tyre belt.

$$M_s = -B_s\left(\frac{\partial\beta_s}{\partial s} + \frac{w}{a^2}\right), \quad M_z = -B_z\left(\frac{\partial\beta_z}{\partial z}\right) \tag{10a,b}$$

Equations 1-10 are reduced in [2] to a single sixth order wave equation. The solution selected here is a harmonic solution for a wave travelling in the positive s direction with m transverse half wavelengths, of the form $w_m exp(-ik_m s)$, $u_m exp(-ik_m s)$. The sixth order wave equation is obtained in the normalized wave-number $z_m = k_m a$:

$$\begin{aligned}
0 = {} & z_m^{6}\left(\bar{S}_s + \bar{N}_s\right) \\
& - z_m^{4}\left[z_L^2 + \bar{P}a + Z_{ce} - \bar{K}_m - \bar{N}_s\bar{R}_s + \left(\bar{S}_s + \bar{N}_s\right)\left(1 + z_c^2 + z_L^2 - \bar{K}_s\right)\right] \\
& + z_m^{2}\left[\begin{array}{r}\left(\bar{K}_s - z_L^2\right)\left(1 + \bar{K}_m - \bar{P}a - \bar{S}_s - z_L^2 + Z_{ce} + \bar{N}_s\bar{R}_s - z_c^2\left(\bar{S}_s + \bar{N}_s\right)\right) \\ + \left(z_c^2 - \bar{R}_s\right)\left(\bar{N}_s - \bar{K}_m + \bar{P}a + Z_{ce} + z_L^2\right)\end{array}\right] \\
& + \left(1 + \bar{K}_m - \bar{P}a - Z_{ce} - z_L^2\right)\left(z_c^2 - \bar{R}_s\right)\left(z_L^2 - \bar{K}_s\right) \\
& \pm i\left(1 + \bar{N}_s\right)\left(z_m^3 - \left(z_c^2 - \bar{R}_s\right)z_m\right)Z_{co}
\end{aligned} \tag{11}$$

The normalized cross section properties of shear stiffness, tension, pressure, rotational stiffness, sidewall and section modal stiffness, and centrifugal force are:

$$\bar{S}_s = \frac{S_s}{A_s}, \quad \bar{N}_s = \frac{N_s}{A_s}, \quad \bar{P}a = \frac{Pa}{A_s}, \quad \bar{R}_s = a^2\frac{S_s}{B_s}, \quad \bar{K}_m = 2a^2\frac{K_{rm} + K_{zm}}{A_s}, \quad Z_{ce} = a^2\frac{\mu\Omega^2}{A_s}$$

The two normalised non-dimensional wave-numbers z_L, z_c are for the longitudinal wave and the 'rotational wave' [1], which could also be called the first asymmetric Lamb wave [3]. The wave-numbers are defined by:

$$z_L^2 = (a\omega)^2\frac{\mu}{A_s}, \quad z_c^2 = (a\omega)^2\frac{I_s}{B_s}$$

The final term with the odd wavenumber orders is the Coriolis coupling, the $\pm$ is for the anti-clockwise and clockwise waves respectively. This term may be significant in the contact patch. Figure 2a and 2b shows the modulus and phase of the wave-number for the m=0 waves.

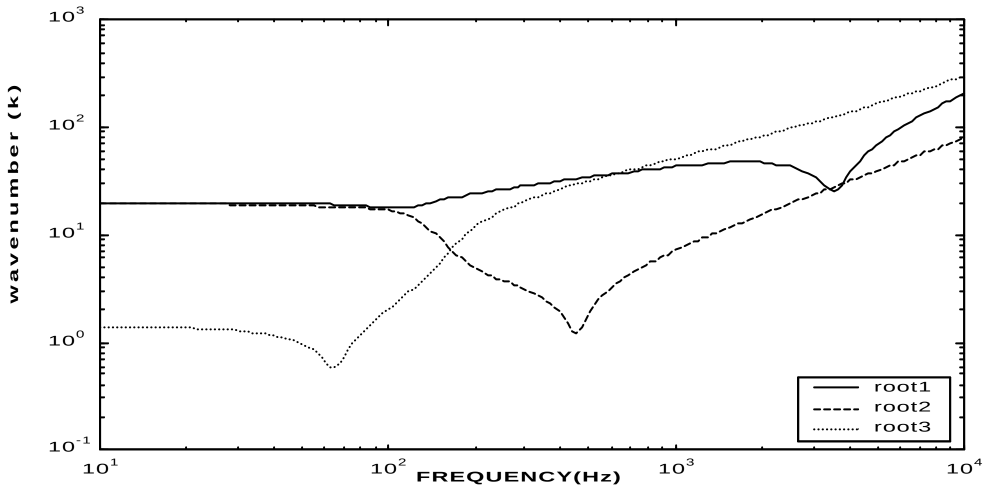

Figure 2a: Wavenumber Modulus

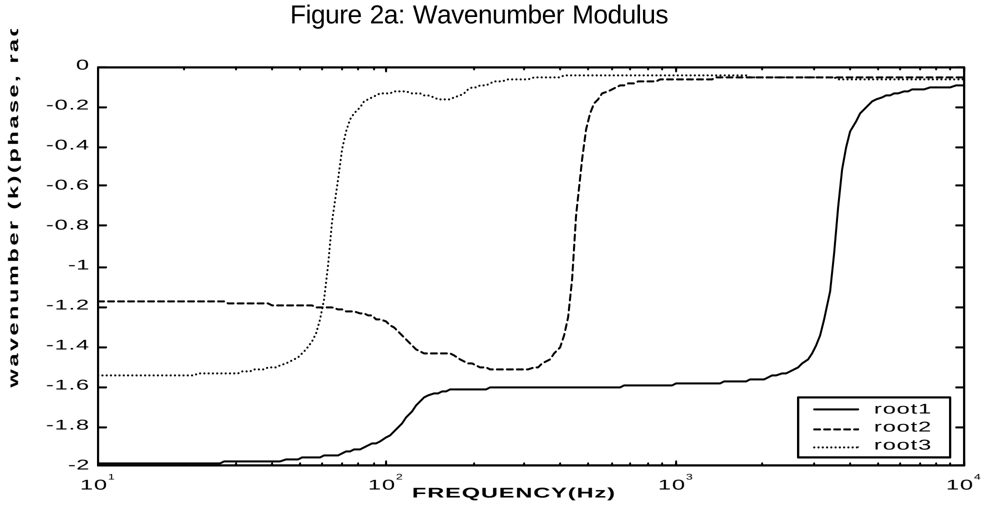

Figure 2b: Wavenumber Phase

The equation in z_m^2 is solved at each frequency to give three pairs of roots *p=1,2,3* for each transverse mode group *m.* The normalised wave-numbers are of the form z_{pm} i.e. $\pm z_{1m}, \pm z_{2m}, \pm z_{3m}$, for the anti-clockwise and clockwise wave. Here the three selected wave-numbers k_{pm} will take the sign and form of the wave that exists in the anticlockwise direction. The roots are complex in general and the true roots or waves are those that *decay in the anti-clockwise direction* and have the possibilities $(\pm k_r - ik_i)$, where k_r and k_i are the real and imaginary wave-numbers. This wave can have three forms seen in Figures 2a,2b:

1. $k_r >> k_i$, a propagating or travelling wave with a small negative phase from the damping. At 100Hz the root 3 wave cuts on and becomes a travelling bending-tension wave.
2. $k_i >> k_r$, the evanescent bending wave has a phase of $-\pi/2$ between 100Hz and 3kHz.
3. $k_r \approx k_i$ termed the 'complex wave', which always occurs in a pair $\pm k_r - ik_i$, e.g.root 2 and 3 below 60Hz, form a rapidly decaying standing wave at the contact zone.

3: TRANSFER FUNCTIONS

The transfer functions may now be found as each response is a sum of six waves, the amplitude of which is found from the six boundary conditions. These are continuity of displacement, continuity of the slope due to bending, and the input force and moment. For a stationary wheel, and the same material data as Figure 2, the input and transfer radial line mobility is given in Figure 3 and the tangential line mobility in Figure 4.

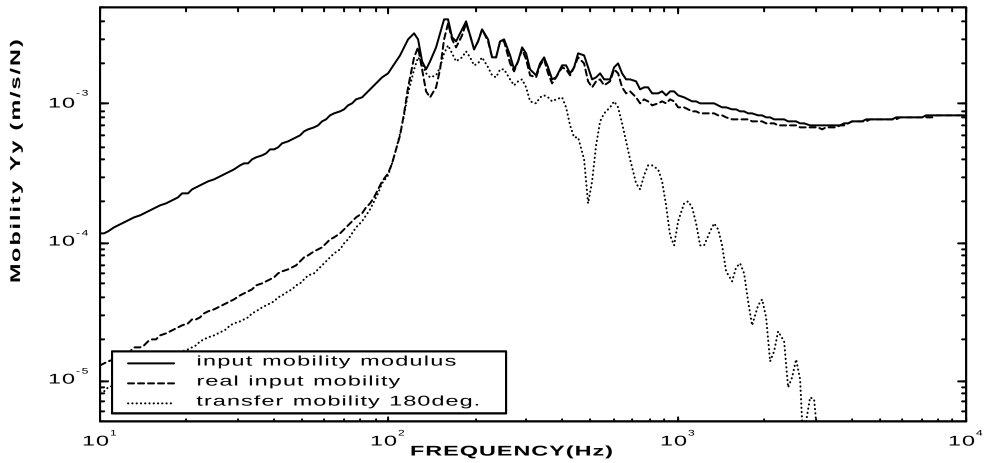

Figure 3: Input and Transfer Mobility for radial line excitation

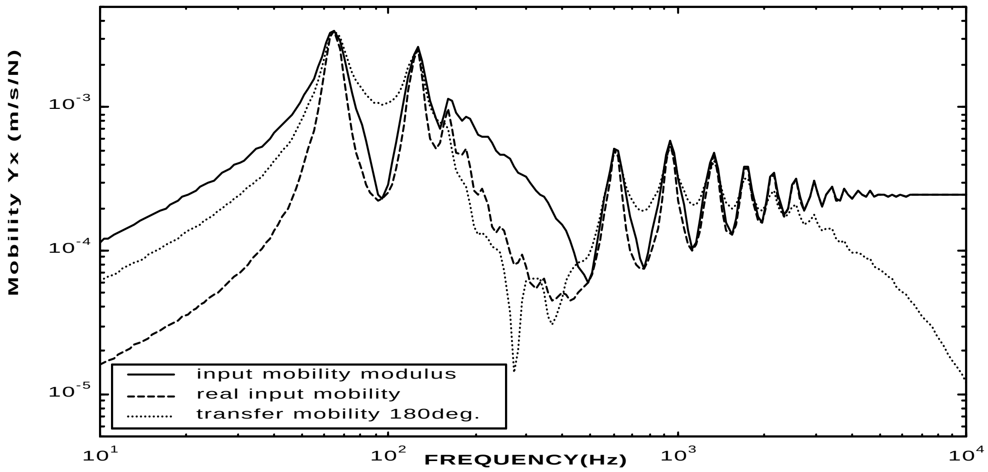

Figure 4: Input and Transfer Mobility for tangential line excitation

4: CONCLUSIONS

A tyre model for the whole working range from 0-3kHz has been made using a wave approach. This can provide transfer functions in the normal and tangential directions and can accommodate rotational effects such as centrifugal and Coriolis forces.

5:REFERENCES

1: R.J.Pinnington and A.R.Briscoe, A wave model for a pneumatic tyre belt. Submitted to Journal of Sound and Vibration.

2: R.J.Pinnington, A wave model for a circular pneumatic tyre, Submitted to Journal of Sound and Vibration.

6: ACKNOWLEGEMENT

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