TYRE NOISE HORN EFFECT ON AN ABSORBING ROAD SURFACE – SEMI-ANALYTICAL MODELLING USING THE MULTIPOLE SYNTHESIS

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Philippe Klein
INRETS
25, avenue François Mitterrand – Case 24
BRON
FRANCE
Tel: +33 4 72 14 24 06
Fax: +33 4 72 37 68 37
E-mail: philippe.klein@inrets.fr

ABSTRACT

This paper deals with the modelling of the horn effect created between the tire and the road surface. For perfectly reflecting conditions, existing models use the multipole synthesis associated to an image multipole source. This principle is here extended to address the condition of an acoustically absorbing road surface. In this paper, the 2D (cylindrical) and 3D (spherical) modelling are presented. Results are expressed in terms of power amplifications. For both cases, a comparison with the perfectly rigid surface is given.

INTRODUCTION

Two main phenomena for tyre-road noise generation were identified a long time ago: tyre tread excitation, vibration and radiation due to the interaction with the road surface, compression and expansion of air near the contact area (air-pumping). The first phenomenon predominates in low and medium frequency ranges (below 1kHz), the second one in high frequency ranges (above 1kHz). While air-pumping sources are concentrated near the horn apex, tire tread vibrations decay progressively when moving away from the contact area [1]. Whatever the origin of the source, it has been shown that tyre-road noise sources are amplified by the geometry made by the tyre and the road surface. This amplification not only applies to punctual pressure (directivity effect) but also to the total radiated power [2]. Different approaches have been used to model the horn effect: analytical (or semi-analytical) and numerical modelling (FEM and BEM). Within the analytical approaches one finds the equivalent sources method. Multipole synthesis has been used by W. Kropp [3] for the 2D cylindrical case by introducing an image multipole to fit the perfectly reflecting condition on the road surface. This method also permits the introduction of a reflexion coefficient affecting the image source to take into account an absorption of the road surface [3], [4]. Otherwise the absorbing acoustical properties of the road surface are taken into account in BEM codes by modelling the coupling between the air and a porous layer (extended reaction) [5].

In this paper the multipole synthesis approach is extended to take into account the absorbing properties of the road pavement by introducing a boundary condition of localized reaction impedance on the road surface. In the first part of this paper the principle of multipole synthesis in the presence of a locally reacting surface is brought in mind for the 2D case of an infinite cylinder [6]. In the second part, the method of resolution is described for the 3D case of a sphere. A special attention is made to how to introduce the acoustical impedance of the road surface in the multipole synthesis method. In the third part, power amplifications due to the horn effect for reflecting or absorbing surfaces are calculated and compared to each other in the 2D and 3D cases.

In this paper, we will consider harmonic sources. The time dependency is chosen to be e^{jwt}. The wave number in air is noted as $k = w/c$ (c is the celerity of sound in air).

2D MULTIPOLE SYNTHESIS FOR A CYLINDER PLACED NEAR A PLANE ABSORBING SURFACE:

Principle:

An infinite cylinder of radius a placed near a plane surface is considered . The distance between the centre of the cylinder and the plane surface is $b/2$ ($b/2 > a$) (see figure 1). The harmonic radial velocity of the cylinder $v(q_1)$ is the input of the problem which consists in determining the acoustic pressure outside the cylinder taking into account the reflecting condition on the plane surface.

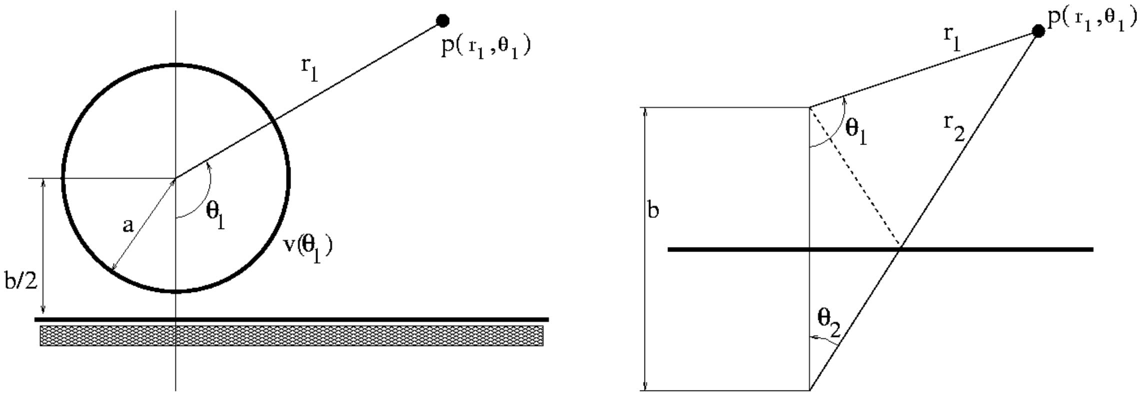

Figure 1 – Notations – Definition of the coordinates systems

The principle of resolution of this radiation problem is described in [3] for a perfectly reflecting condition on the plane surface. The 2D multipole synthesis consists in expanding the pressure field in terms of outgoing wave functions $H_m^{(2)}(kr)e^{jmq}$, where $H_m^{(2)}$ denotes the Hankel function of second kind of order m. A primary cylindrical multipole is then placed at the centre of the cylinder. Moreover to fulfill the reflecting condition on the plane surface, an image cylindrical multipole is introduced (see figure 1 for the definition of the coordinates systems) .

The pressure field is then expressed as the sum of the pressure contributions of the primary source p_1 and its image source p_2:

$$p(r_1,q_1) = p_1(r_1,q_1) + p_2(r_2,q_2) = \sum_m a_m H_m^{(2)}(kr_1)e^{jmq_1} + \sum_m b_m H_m^{(2)}(kr_2)e^{jmq_2}$$

where $r_2 = \left(r_1^{\,2} + b^2 - 2r_1 b\cos q_1\right)^{1/2}$ and $q_2 = \sin^{-1}\left(r_1 \sin q_1 / r_2\right)$.

Determination of the Modal Coefficients:

The modal coefficients a_m and b_m are determined by fulfilling the boundary conditions on the cylinder and on the plane surface. The method is described in [6]. The main equations governing the problem are given here after.

Boundary Condition on the Cylinder:

The velocity condition on the cylinder is given by $-\frac{1}{jwr}\frac{\partial p}{\partial r_1}\Big|_{r_1=a} = v(q_1)$. Using addition theorem for Hankel functions, this equation leads, for all m, to

$$a_m H_m^{(2)'}(ka) + \left(\sum_n b_n H_{m+n}^{(2)}(kb) \right) J_m{}'(ka) = -j\boldsymbol{r}cv_m \, , \quad (1)$$

where v_m is the Fourier coefficient of $v(\boldsymbol{q}_1)$ of order m.

<u>Boundary Condition on the Plane Surface:</u>

The locally reacting condition on the plane surface is written as $p - Zv_\perp = 0$, where $v_\perp$ is the normal ascending velocity on the surface (where $r_1 = r_2$, $\boldsymbol{q}_1 = \boldsymbol{q}_2$). $v_\perp$ is given by

$$v_\perp = -\frac{1}{j\boldsymbol{wr}} \sum_m (a_m - b_m) \left[-k\cos\boldsymbol{q}_1 H_m^{(2)'}(kr_1) + jm \frac{\sin\boldsymbol{q}_1}{r_1} H_m^{(2)}(kr_1) \right] e^{jmq_1} \, . \quad (2)$$

The pressure on the surface is $p = \sum_m (a_m + b_m) H_m^{(2)}(kr_1) e^{jmq_1}$.

For a perfectly reflecting condition on the plane surface ($v_\perp = 0$), one finds from equation (2) that $a_m = b_m$ for all m.

For an absorbing surface characterised by the normalised impedance $z = Z / \boldsymbol{r}c$, one gets

$$a_m + b_m + \frac{z}{2}\left[(a_{m-1} - b_{m-1}) - (a_{m+1} - b_{m+1}) \right] = 0 \, . \quad (3)$$

The set of linear equations (2) and (3) permit to determine the modal coefficients a_m and b_m. It is to be noted that the velocity distribution on the cylinder is not perfectly fulfilled (see [6]).

MULTIPOLE SYNTHESIS FOR A SPHERE PLACED NEAR A PLANE SURFACE

A similar appoach from that used for the cylinder has been applied to the case of a spherical radiator vibrating with the velocity distribution $v(\boldsymbol{q}_1, \boldsymbol{j}_1)$. The sound pressure is expressed as the sum of contributions from the sphere and its image: $p(r_1, \boldsymbol{q}_1, \boldsymbol{j}_1) = p_1(r_1, \boldsymbol{q}_1, \boldsymbol{j}_1) + p_2(r_2, \boldsymbol{q}_2, \boldsymbol{j}_2)$. Each contribution is expanded in spherical outgoing wave functions :

$$p_1 = \sum_{m \ge 0} \sum_{n=-m}^{m} a_{mn} h_m^{(2)}(kr_1) P_{m|n|}(\cos\boldsymbol{q}_1) e^{jnj} \quad , \quad p_2 = \sum_{m \ge 0} \sum_{n=-m}^{m} b_{mn} h_m^{(2)}(kr_2) P_{m|n|}(\cos\boldsymbol{q}_2) e^{jnj}$$

where $h_m^{(2)}$ represents the m-th order spherical Hankel function of second kind and P_{mn} the Legendre function of order (m,n). The coordinates systems are chosen in order to have $\boldsymbol{j}_1 = \boldsymbol{j}_2 = \boldsymbol{j}$ (see figure 2). The relationships between the coordinates $(r_2, \boldsymbol{q}_2)$ and $(r_1, \boldsymbol{q}_1)$ are then the same as those for the cylinder.

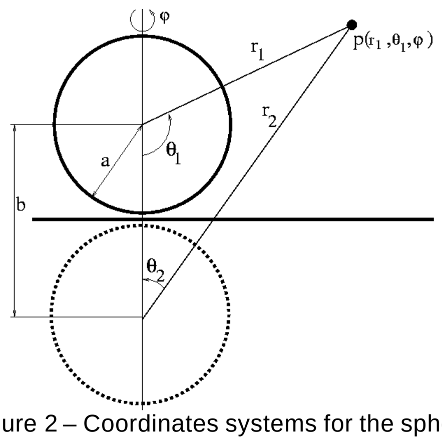

Figure 2 – Coordinates systems for the sphere

The determination of the modal coefficients a_{mn} and b_{mn} is done by expressing the boundary conditions on the sphere and on the plane surface. The choice of the coordinates systems implies that there is no interaction between the primary sphere and its image for the longitudinal modes (indices n). It is then more convenient to express the pressure expansion as:

$$p(r_1,q_1)=\sum_n e^{jnj}\cdot\left(\sum_{m\geq|n|} a_{mn}h_m^{(2)}(kr_1)P_{m|n|}(\cos q_1)+b_{mn}h_m^{(2)}(kr_2)P_{m|n|}(\cos q_2)\right).$$

Boundary Condition on the Sphere:

The boundary condition on the sphere is expressed as $-\frac{1}{jwr}\frac{\partial p}{\partial r_1}\Big|_{r_1=a}=v(q_1,j)$. Using the addition theorem for spherical waves functions is possible as for the cylinder and its development is under progress. We give here another method which consist in calculating the expression of the normal velocity on the sphere $v_r(q_1,j)$ given by

$$v_r(q_1,j)=-\frac{1}{jwr}\sum_n e^{jnj}\sum_{m\geq|n|}\left\{\begin{array}{l} a_{mn}kh_m^{(2)'}(ka)P_{m|n|}(\cos q_1)+ \\ b_{mn}\left[k\frac{a-b\cos q_1}{r_2}h_m^{(2)'}(kr_2)P_{m|n|}(\cos q_2)-\frac{b\sin q_1\sin q_2}{r_2^2}h_m^{(2)}(kr_2)P_{m|n|}^{'}(\cos q_2)\right]\end{array}\right\}.$$

The components of the velocity distribution of the sphere $v(q_1,j)$ are noted as v_{mn}:

$$v_{mn}=\frac{2m+1}{4p}\frac{(m-|n|)!}{(m+|n|)!}\int_0^{2p}e^{-jnj}\,dj\int_0^{p}v(q,j)P_{m|n|}(\cos q)\sin q\,dq\,.$$

By expanding the term of $v_r(q_1,j)$ affecting the coefficients b_{mn} on the $e^{jnj}P_{m|n|}(\cos q_1)$ basis ($g_{mn}(a,b)$ coefficients), one gets for each longitudinal mode n_0 a set of linear combinations involving the coefficients a_{mn_0} and b_{mn_0} such as

$$a_{mn_0}kh_m^{(2)'}(ka)+\sum_p b_{pn_0}g_{pn_0}(a,b)=-jwrv_{mn_0}\,. \qquad (4)$$

Boundary Condition On The Plane Surface:

The expression of the normal ascending velocity $v_\perp$ is here given by:

$$v_\perp=-\frac{1}{jwr}\sum_n e^{jnj}\cdot\left[\sum_{m\geq|n|}(a_{mn}-b_{mn})\left(\begin{array}{l}-k\cos q_1h_m^{(2)'}(kr_1)P_{m|n|}(\cos q_1)\\ \quad+jm\frac{\sin^2 q_1}{r_1}h_m^{(2)}(kr_1)P_{m|n|}^{'}(\cos q_1)\end{array}\right)\right].$$

Perfectly Reflecting Surface:

Assuming a perfectly reflecting condition on the plane surface ($v_\perp=0$), equation yields the equality $a_{mn}=b_{mn}$ for all (m,n).

Impedance Plane Surface:

Assuming a locally reacting boundary condition characterised by a normalised impedance z and considering the following relationships involving the Hankel and Legendre functions,

$$h_m^{(2)'}(x)=\frac{m}{2m+1}h_{m-1}^{(2)}(x)-\frac{m+1}{2m+1}h_{m+1}^{(2)}(x)\,,$$

$$\frac{h_m^{(2)}(x)}{x}=\frac{1}{2m+1}h_{m-1}^{(2)}(x)+\frac{1}{2m+1}h_{m+1}^{(2)}(x)\,,$$

$$xP_{mn}(x) = \frac{m+n}{2m+1} P_{m-1,n}(x) + \frac{m-n+1}{2m+1} P_{m+1,n}(x) \ ,$$

$$(x^2-1)P'_{mn}(x) = (m-n+1) P_{m+1,n}(x) - (m+1)xP_{m,n}(x) \ ,$$

the expression of $v_\perp$ becomes

$$v_\perp = -\frac{1}{jrc}\sum_n e^{jnj} \cdot \left[\sum_{m \geq |n|} \left(a_{mn} - b_{mn}\right) \left(\begin{array}{l} \frac{m+|n|}{2m+1} h^{(2)}_{m-1}(kr_1) P_{m-1,|n|}(\cos q_1) \\ \quad - \frac{m-|n|+1}{2m+1} h^{(2)}_{m+1}(kr_1) P'_{m+1,|n|}(\cos q_1) \end{array} \right) \right] ,$$

which can be rewritten, after shifting the indices m of +/-1, as,

$$v_\perp = -\frac{1}{2jrc}\sum_{n,m} \left[\frac{m+|n|+1}{m+3/2} \left(b_{m+1,n} - a_{m+1,n}\right) - \frac{m-|n|}{m-1/2} \left(b_{m-1,n} - a_{m-1,n}\right) \right] h^{(2)}_m(kr_1) P_{m|n|}(\cos q_1) e^{jnj} \ .$$

The pressure on the plane surface is

$$p = \sum_{n,m} (a_{mn} + b_{mn}) h^{(2)}_m(kr_1) P_{m|n|}(\cos q_1) e^{jnj} \ .$$

As for the case of the cylinder, the impedance condition on the plane surface $p - Zv_\perp = 0$ leads to a set of linear equations involving the modal coefficients a_{mn} and b_{mn} :

$$a_{mn} + b_{mn} + \frac{z}{2}\left[\frac{m-|n|}{m-1/2} \left(a_{m-1,n} - b_{m-1,n}\right) - \frac{m+|n|+1}{m+3/2} \left(a_{m+1,n} - b_{m+1,n}\right) \right] = 0 \ . \quad (5)$$

Calculation of the Modal Coefficients:

Solving the problem requires the truncation of the infinite summation over the indice m. All modal coefficients are considered to be zero for $m > M$. As mentioned above, for each longitudinal mode n_0, one gets from equations (4) and (5) a linear system involving the $2(M - |n_0| + 1)$ coefficients a_{mn_0} and b_{mn_0}. As for the case of the cylinder, the impedance condition on the plane surface leads to a system with more equations than unknowns (see [6]). Perfectly fulfilling the boundary condition on the plane surface renders an approximately fulfilled velocity condition on the sphere. However for a piston-like distribution defined on the sphere, the quadratic error between the prescribed and the actual velocity is sufficiently small.

COMPARISON BETWEEN A FINITE IMPEDANCE SURFACE AND THE PERFECTLY REFLECTING SURFACE FOR THE CYLINDRICAL AND SPHERICAL RADIATORS

Calculations have been done to evaluate the influence of a surface of finite impedance on the horn effect. A velocity distribution on the cylinder and the sphere is chosen to simulate a piston located at an angle q_1 of 20 degrees and with an angular size of 10 degrees. The amplification due to the horn effect is defined here by the ratio between the power radiated by the cylinder or the sphere in the upper half-space in the presence of the plane surface and the power radiated by the cylinder or the sphere in free field. The calculations of the amplification have been done for a perfectly reflecting surface and for an absorbing surface. Figure 3-Left shows the normalized impedance and the absorption coefficient of the absorbing surface as a function of frequency. The effects of this particular case of a finite impedance condition are shown in figure 3-Right where the power amplifications of both cases (finite and infinite impedance) are compared. For this case, the power amplification due to the horn effect is strongly reduced. However there is no simple relationship between the difference of amplifications observed with the cylinder and the absorption coefficient of the surface for plane waves in normal incidence, while it seems to be a better accordance for the case of the sphere.

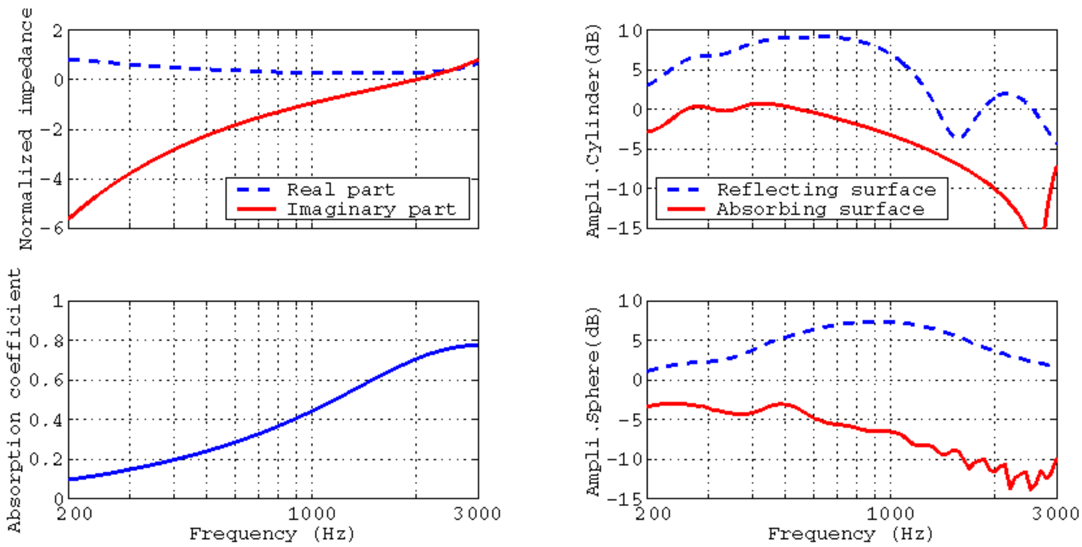

Figure 3 – Left: Absorbing properties of the finite impedance surface –
Right: Power amplifications for the cylinder and the sphere

CONCLUSION

The multipole synthesis method has been used for the modelling of the radiation of a cylinder or a sphere placed near a plane surface. It has been shown how to take into account a finite impedance condition in the formulation of the multipole synthesis method. The method has been applied to the tyre-road noise horn effect. The example given here shows a great influence of the finite impedance on the horn effect with respect to perfectly reflecting conditions. The influence of the use of an impedance model of localized reaction instead of an extended one is yet to be clarified.

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