DYNAMICAL PROPERTIES OF FINITE CYLINDRICAL STEEL SHELLS WITH HEMISPHERICAL END CAPS

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ABSTRACT:

Straight, finite, cylindrical steel shells with hemispherical end caps were modeled with simple 4 node shell elements. A numerical modal analysis was performed on the models, systematically varying a number of parameters, like radius, length, shell thickness. The eigenfrequencies of the circumferential modes were plotted versus their corresponding axial wave numbers (k-ω-diagrams). The resulting curves, with the circumferential wave numbers as parameter, indicate the existence of a constant limiting velocity. The value of this velocity increases with decreasing radius of the shell. The comparison with analytical calculated eigenfrequencies for simply supported finite cylindrical shells shows very good agreement.

INTRODUCTION

In technical acoustics, cylindrical structures, like pipes or fluid containers play a major role. Pipe structures usually can be approximated by infinite cylinders, where analytical solutions for many applications are available. Finite cylindrical structures are in general more difficult to treat, especially if certain boundary conditions are required. If the structure includes end caps with particular geometry, finite element models are in general more suitable to the problem. Additional effort is needed in case of fluid loading of the cylinders. If the fluid is inside of the structure, the volume of the cylinder has to be discretised with finite elements, too. Submerged shells can be treated either by a finite volume around the structure with absorbing elements at the boundary, or with semi infinite elements at the boundary, or with the coupling of Finite Element Methods and Boundary Element Methods (FEM/BEM). The absorbing element approach is only an approximate solution of the problem, since exact absorption usually occurs only for certain incident angles [1]. Different numerical approaches for the semi infinite element solution are available e.g. [2], [3]. They present the solution for the most general types of problems for submerged structures, since the whole finite element model is present throughout the complete calculation and even inhomogeneous fluid properties can be treated. However, the necessary numerical effort for the discretisation of the fluid volume depends on the nature of the problem and the number of degrees of freedom increases rapidly with the upper frequency limit. In this paper, a coupled FEM/BEM algorithm, based on the added mass approach for the fluid loading is used, since it requires only marginal additional effort for the numerical model and the calculation of coupled eigenfrequencies [4].

NUMERICAL MODEL

The FE-package ANSYS, which was used to model and calculate the dynamical properties of the cylindrical steel shells with hemispherical end caps offers the opportunity to build and calculate models based on the ANSYS parametric design language (APDL) [5]. Thus, different models with varying parameters, like radius, length, and shell thickness could be handled very easily. However, the evaluation of the results had to be done by hand. Therefore, the variation of parameters was confined to a certain number of values in order to reduce the overall evaluation time. Every model comprises roundabout 13,000 nodes and shell elements, which corresponds to a number of almost 80,000 degrees of freedom. A number of 200 modes was calculated for each of the models with free-free boundary conditions. The ratio of length to radius varied from 0.067 to 0.096. Table 1 shows a matrix of the different geometrical parameters. Standard material properties for steel were used in the models:

Density:	7800 kg/m³
Poisson ratio:	0.3
Young Modulus:	2.1e11 Pa;

length L [m]	radius a [m]	shell thickness h [mm]	velocity [m/s]
35	2.4	16	326.1
30	2.4	16	322.4
30	2.2	16	340.8
30	2.0	16	362.2
30	2.0	18	379.6
30	2.0	20	399.8
25	2.4	16	323.5

Table 1.- Geometrical parameters of the calculations with corresponding limiting velocity

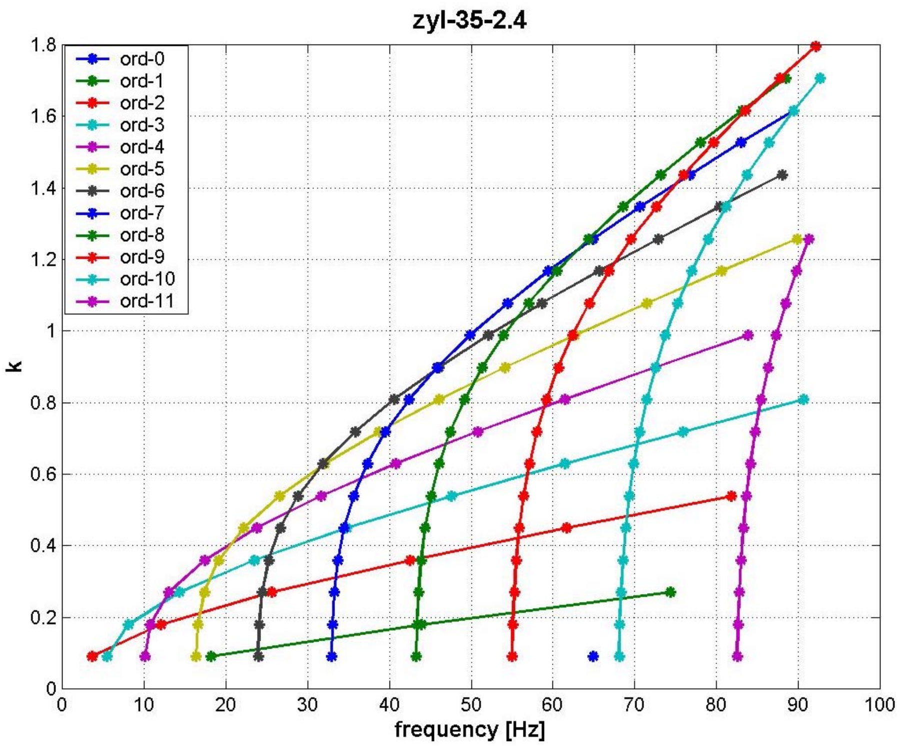

Fig. 1.- Eigenfrequencies of a finite cylindrical steel shell with hemispherical end caps in vacuum vs. wave number along the cylinder axis, FE-solution

RESULTS

The calculated eigenfrequencies of the shells are plotted versus the axial wave number of the vibrating cylinder. Modes of common circumferential order are marked by colored lines. The convention for the numbering of circumferential orders follows Cremer, Heckl, [6], (breathing mode is order 0, bending mode is order 1, etc.). Figure 1 shows the diagram of the cylinder with a length L of 35 m, a radius a of 2.4 m and a shell thickness h of 16 mm. The eigenfrequencies in the diagram are grouped below a straight line. The slope of this line in the k-ω-diagram corresponds to a constant velocity of 326.1 m/s. Since no eigenvalues are found above this line, we can assume a limiting velocity for the cylindrical shells. For the different geometrical parameters of the calculated cylinders, the limiting velocity varies with the geometrical parameters of the cylinders. The last column of Table 1 shows the limiting velocities of the corresponding combination of parameters. It is obvious, that the velocity doesn't vary much with the length of the cylinder. However, both, the radius and the shell thickness cause significant variations of the velocity. The velocity increases proportional with the shell thickness and indirect proportional with the radius of the cylinders.

Junger & Feit, [7], give an exact derivation for the analytical calculation of the natural frequencies of simply supported finite cylindrical shells. However, for our purposes, an approximate equation for the calculation of eigenvalues of predominantly radial modes is sufficient:

$$\Omega_{mn} \cong \left[\left(1-\nu^2\right) \frac{k_m^4 a^4}{k_s^4 a^4} + \beta^2 k_s^4 a^4 \right]^{1/2} \left(1+n^{-2}\right)^{-1/2}, \qquad \text{for } n>0,$$

(Equ. 1)

where

$$\beta = \frac{1}{\sqrt{12}} \frac{h}{a},$$

(Equ. 2)

k_m is the axial wave number

$$k_m = (2m+1)\frac{\pi}{2L},$$

(Equ. 3)

and k_s is the helical wave number

$$k_s = \left(k_m^2 + \frac{n^2}{a^2} \right)^{1/2}.$$

(Equ. 4)

Equation 1 was applied to calculate the radial eigenfrequencies of a corresponding simply supported shell. The length L of this shell is similar to the strictly cylindrical part of the FE-model. Figure 2 shows the results of the analytical calculation. Again, the circumferential orders of the eigenmodes are grouped by colored lines. Markers for the discrete values have been omitted, to distinguish the plot of the analytical values from the FE-results. The qualitative impression of both figures is very similar. The limiting velocity of the analytic solution of the simply supported shell is 327.6 m/s. This is only 0.4 % more than the one of the FE-calculation. It seems, that a simply supported cylindrical shell is an excellent approximation for cylinders with hemispherical end caps and free-free boundary conditions.

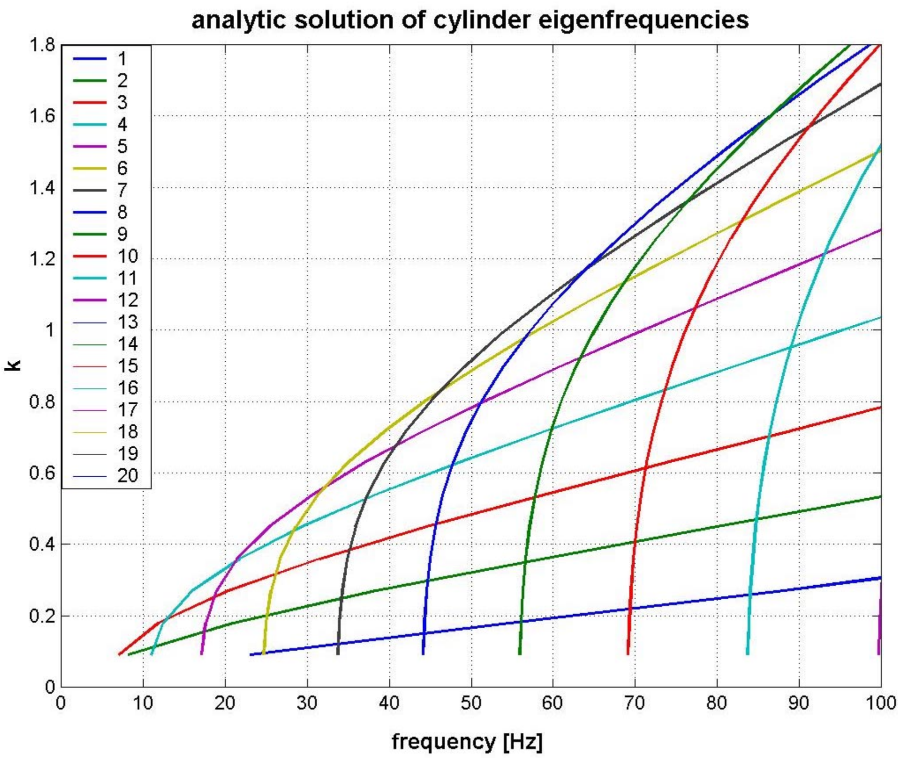

Fig. 2.- Eigenfrequencies of a simply supported finite cylindrical steel shell in vacuum vs. wave number along the cylinder axis, approximate analytic solution (continuous curves for convenience, only)

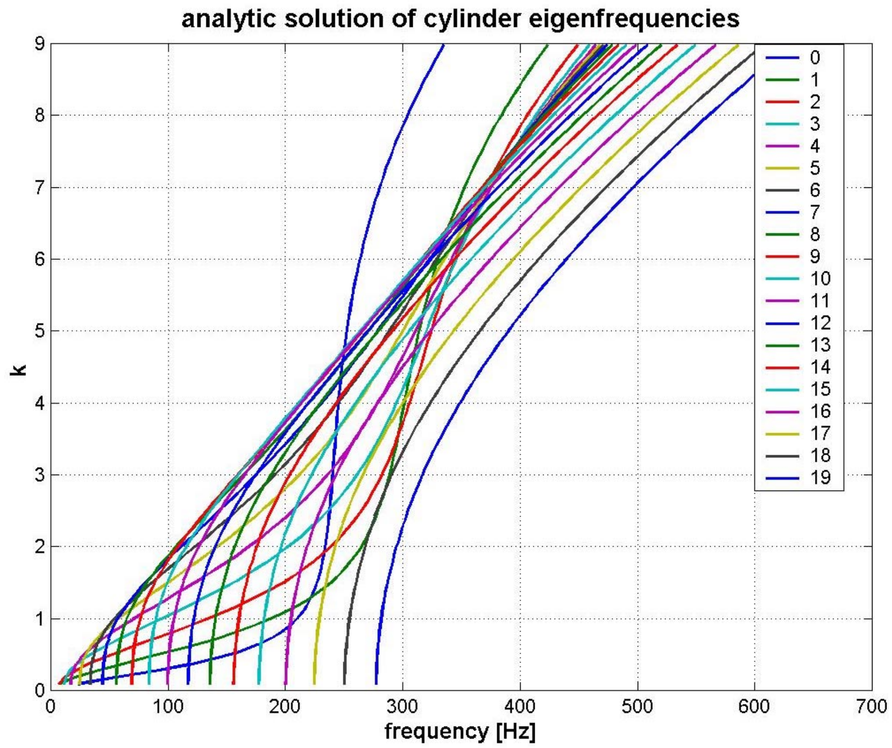

Fig. 3.- Eigenfrequencies of a simply supported finite cylindrical steel shell in vacuum vs. wave number along the cylinder axis, approximate analytic solution up to 700 Hz (continuous curves for convenience, only)

The analytical solution can be easily extended to higher frequencies without much additional effort. Figure 3 shows the k-ω-diagram of the same shell up to frequencies of 700 Hz. Three things are remarkable in this figure:

- the concept of the limiting velocity is only valid for low frequencies, in this case about 200 Hz,
- the dispersion of the circumferential order modes changes with increasing axial order
- higher order circumferential modes show the same dispersion behavior as bending modes on plates, which means that the curvature of the cylinder surface becomes negligible.

The boundary element package SYNOISE offers a coupled FEM/BEM-solution, using an added mass approach to calculate the dynamical behavior of submerged structures, [4]. In principle, the influence of the fluid loading of structures is strictly frequency dependent. For a given frequency within the range of interest, the added mass will be calculated on the basis of a coupled system of equations, including FE-matrices, BEM influence matrices, and coupling matrices. As long as the considered frequency range is small, this is a reasonable approximating approach. In order to reduce the number of degrees of freedom of the unsymmetrical coupled system of equations, the FE-system was replaced by a modal basis. This is again an approximation, which requires a high number of modes, up to at least twice the frequency of interest. The already available eigenmodes of the shells in vacuum served as a modal basis for the coupled calculation. As an example, the k-ω-diagram of the 35-2.4-shell is given in Fig. 4. Due to the fluid loading, the coupled eigenfrequencies are lower than the values in vacuum. The exact percentage of the frequency shift depends on the type and order of the mode, respectively. The strongest effect occurs at the low order modes. Therefore, more of the submerged eigenvalues are grouped at lower frequencies. The density of the higher order modes seems to be less than in vacuum. However, this is mainly caused by the fact, that a fixed number of uncoupled modes was used for the calculation of the submerged modes. Hence, unlike the shell in vacuum, the limiting curve in the diagram is no longer a straight line and thus a constant limiting velocity cannot be defined, even at frequencies below 100 Hz.

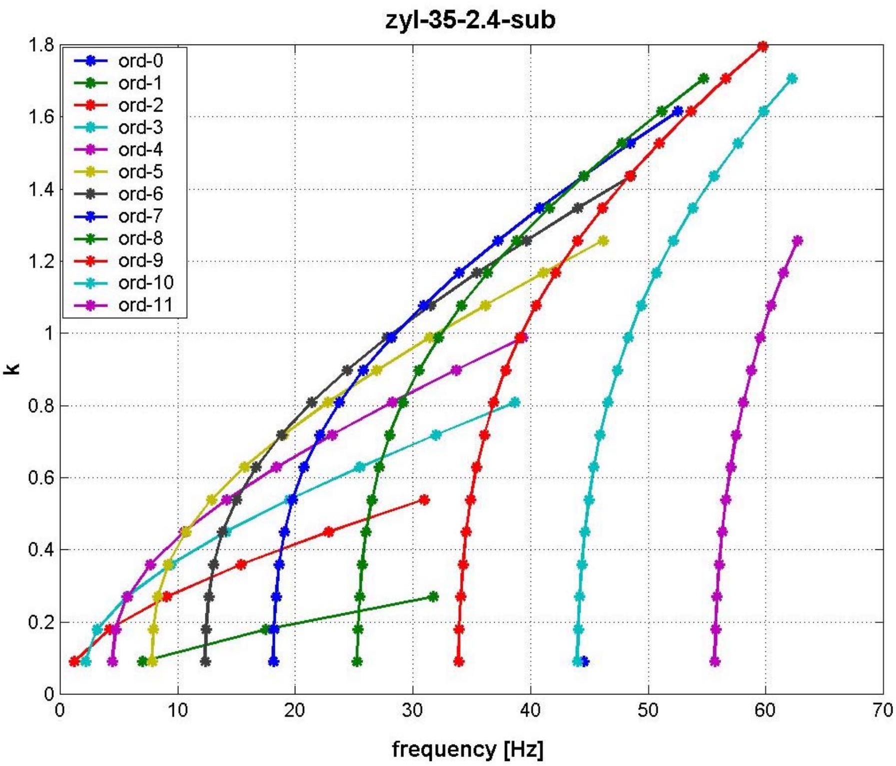

Fig. 4.- Eigenfrequencies of a submerged finite cylindrical steel shell with hemispherical end caps vs. wave number along the cylinder axis, coupled Fem/BEM-solution

CONCLUSION

A number of straight, finite, cylindrical steel shells with hemispherical end caps were modeled with simple 4 node shell elements. Parameters, like radius, length, and shell thickness were systematically varied. After the numerical evaluation of the modal analysis, the eigenvalues were plotted versus their corresponding axial wave numbers (k-ω-diagrams). The resulting curves, with the circumferential wave numbers as parameter, indicate a constant limiting velocity. The value of this velocity increases proportional with the shell thickness and indirect proportional with the radius of the shell. The comparison with analytical calculated eigenvalues for simply supported finite cylindrical shells shows very good agreement. However, the concept of a limiting velocity makes only sense for low frequencies. For higher order modes, the curvature of the cylindrical shell becomes negligible and the modes show similar behavior as bending waves on plates.
No constant limiting frequency can be defined for submerged shells. Even at very low frequencies, the limiting curve in the k-ω-diagrams is not a straight line.

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