SOUND REFLECTION FROM A FLEXIBLE STRUCTURE WITH IMPEDANCE DISCONTINUITIES USING LOCAL/GLOBAL HOMOGENIZATION

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ABSTRACT

A novel homogenization method for complex structures utilizes a local/global separation of the low and high wavenumber spectrum. The low-wavenumber global problem has infinite-order operators. The local problem provides transfer functions for the global problem. The global problem is self-contained; local solutions are reconstructed afterwards. Using Local/Global Homogenization, the global problem is solved at a resolution lower than the flexural wavelengths on the structure. As an example, oblique sound reflection from a flexible barrier with impedance discontinuities is analyzed. Radiating acoustic modes are contained in the smooth global problem, and evanescent acoustic modes are contained within the global structural operator.

INTRODUCTION

Many important structures have discontinuities such as ribs, stringers, braces, and/or attachments placed at regular intervals. Ribbed hulls, aircraft fuselages, and truss structures are examples. These structures may be periodic, or quasi-periodic, depending on whether the discontinuities are identical and equally spaced. When forced at a single frequency, the response occurs in a broad spectrum of spatial wavenumbers due to the discontinuities. Furthermore, structures such as fuselages and hulls are fluid loaded, which alters their response. The structural motion and the acoustic radiation, scattering, and/or interior sound field may be of interest.

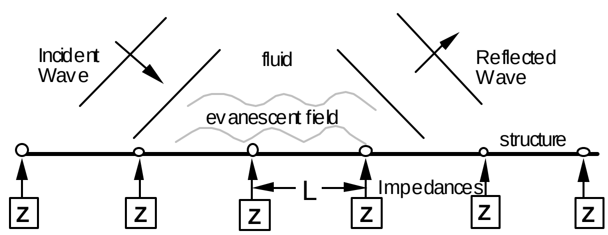
Calculating the motion of such structures is a complex and computationally expensive task. The disparity of scales requires high numerical resolution. The forcing may be a series of locally applied forces or continuously distributed forces having a spectrum of wavenumbers. If the structure is spatially periodic, the response will exhibit stop-and-pass bands and the wavenumber spectrum will be discrete. If it is not strictly periodic and certain forms of coupling are present, there will be a distributed wavenumber spectrum, and the response may exhibit localization.

The low wavenumber (long wavelength) portion of the response is often of primary interest, since it models the gross vibratory response of the structure. Also, for fluid loaded structures, the low wavenumber part of the response corresponds to supersonic phase speeds, which are most efficiently coupled to the acoustic field. This paper presents results of ongoing research[1,2,3] to formulate *self-contained governing equations* that describe *directly* the low wavenumber response of spatially periodic structures. It is important to emphasize that this method leads to *direct* formulations of the low wavenumber problem. In particular, it is not a matter of solving the full problem and subsequently isolating the low wavenumber part of the solution. This formulation for the low wavenumber part of the solution is self-contained, and indeed, the high wavenumber content can be reconstructed after the fact using information from the low wavenumber solution. The present work shows that it is possible to include fluid loading effects in a convenient manner.

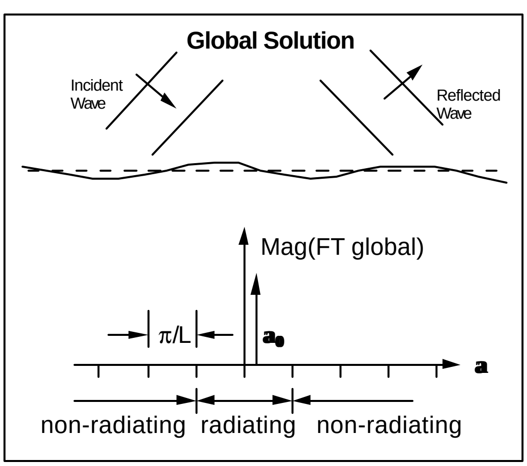
Because the low wavenumber problem is smooth and contains transfer function information from the high wavenumber part of the problem, the approach is a type of homogenization method. However, it differs from classical homogenization, and it is valid for the full frequency range. The low wavenumber problem is smooth and *global*; namely, it spans the structure. The high wavenumber problem involves waves typically shorter than the discontinuity spacing, and it can be thought of as a series of contiguous *local* solutions between the discontinuities. Therefore the method is called *Local-Global Homogenization*. Since the global problem has a known degree of smoothness, there are potential advantages in accuracy and efficiency if the approach can be extended to numerical methods. The approach is an analytical reformulation method for complex problems, prior to solution, to allow the calculation of the important aspects directly and efficiently.

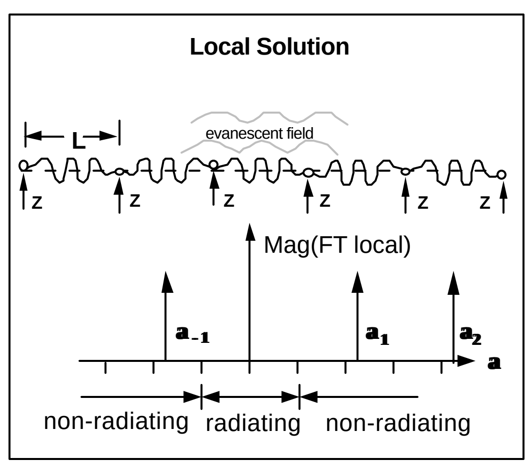
TECHNICAL APPROACH

In this section, the approach will be developed in general. Then it will be applied to the problem of acoustic scattering from a fluid loaded membrane, as illustrated below.



The goal is to divide the original problem into a global solution and a local solution distinguished by their low and high wavenumber content, respectively. As shown below, the global problem will be smooth and have only low wavenumber content. In the scattering problem to be solved, the global problem will contain a single wavenumber falling in the interval $-\pi/2 < \alpha_0 < \pi/2$, which corresponds to the wavenumber range that produces acoustic radiation for sufficiently low frequencies. Note, however, that the approach can be readily generalized to include more wavenumber intervals in the global solution, and thereby be extended to higher frequencies. The local problem, also shown below, is seen to have rapid spatial variation and to contain the remainder of the wavenumbers, which excite only evanescent (non-radiating) fluid modes.





First, consider a continuous structure, such as a membrane or plate with no fluid, having displacement $\eta(x)$ governed by a linear differential operator $D(x)$ and subjected to applied forcing $F(x)$. Assuming harmonic motion removes the time dependence. Identical discontinuities having impedance $Z(\omega)$ are attached at intervals $x_n = nL$. The governing equation is given by:

$$D(x)\,\eta(x) = i\omega Z \sum_n \eta(x)\,\delta(x - x_n) + F(x)$$

This equation will be used to illustrate the general approach to local/global decomposition. The Fourier transform is used to rewrite the equation in wavenumber space:

$$D(\alpha)\,\bar{\eta}(\alpha) = \frac{i\omega Z}{L} \sum_n \bar{\eta}\left(\alpha - \frac{2\pi n}{L}\right) + \bar{F}(\alpha)$$

The global and local parts of displacement are defined in terms of their wavenumber domains:

$$\bar{\eta} = \bar{\eta}_{global} + \bar{\eta}_{local}; \quad \bar{\eta}_{global}(\alpha) = 0, when\, \alpha \notin \left[-\frac{\pi}{L}; \frac{\pi}{L}\right]; \quad \bar{\eta}_{local}(\alpha) = 0, when\, \alpha \in \left[-\frac{\pi}{L}; \frac{\pi}{L}\right]$$

Considering the equation separately in the global and local intervals gives the expressions:

$$D(\alpha^*)\,\bar{\eta}_{global}(\alpha^*) = \frac{i\omega Z}{L} \sum_{n \neq 0} \bar{\eta}_{local}\left(\alpha^* - \frac{2\pi n}{L}\right) + \frac{i\omega Z}{L} \bar{\eta}_{global}(\alpha^*) + \bar{F}(\alpha^*);\ \alpha^* \in \left[-\frac{\pi}{L}; \frac{\pi}{L}\right]$$

$$D(\alpha)\,\bar{\eta}_{local}(\alpha) = \frac{i\omega Z}{L} \sum_{n \neq n^*} \bar{\eta}_{local}\left(\alpha - \frac{2\pi n}{L}\right) + \frac{i\omega Z}{L} \bar{\eta}_{global}\left(\alpha - \frac{2\pi n^*}{L}\right) + \bar{F}(\alpha);\ \alpha \notin \left[-\frac{\pi}{L}; \frac{\pi}{L}\right]$$

where $\alpha^* = \alpha - 2\pi n^*/L$, with n* such that $-\pi/L < \alpha^* < \pi/L$, the wavenumber interval of η_{global}. Subtracting these equations and using the definitions of local and global displacements yields:

$$\bar{\eta}_{local}(\alpha) = \frac{1}{D(\alpha)}\left[D(\alpha^*)\,\bar{\eta}_{global}(\alpha^*) + \bar{F}(\alpha) - \bar{F}(\alpha^*)\right]$$

This important result is the Local-Global Relationship. It allows the local displacement η_{local} to be eliminated to obtain a new governing equation involving only the global displacement η_{global}. The local short-wave solution can be reconstructed once the global long-wave solution is known. Therefore, the governing equation for the global displacement in transform space is:

$$D(\alpha)\left[1 - \frac{i\omega Z}{L} \sum_n \frac{1}{D\left(\alpha - \frac{2\pi n}{L}\right)}\right] \bar{\eta}_{global}(\alpha) = \bar{F}(\alpha) + \frac{i\omega Z}{L} \sum_{n \neq 0} \frac{\bar{F}\left(\alpha - \frac{2\pi n}{L}\right) - \bar{F}(\alpha)}{D\left(\alpha - \frac{2\pi n}{L}\right)}$$

Since the global solution is defined only in the narrow wavenumber interval $-\pi/L < \alpha < \pi/L$, the right hand side may be considered as a modified forcing *(MF)* in that interval only. Nevertheless, this modified forcing depends on the original excitation through the whole range of wavenumbers. The left hand side contains the transform of a new differential operator, called the global operator.

The procedure beyond this point is best illustrated by the specific example of a 1-D membrane with attached impedances, namely

$$D(\alpha) = k_s^2 - \alpha^2;\ \sum_n \frac{1}{D\left(\alpha - \frac{2\pi n}{L}\right)} = \sum_n \frac{1}{k_s^2 - \left(\alpha - \frac{2\pi n}{L}\right)^2} = \frac{L}{2k_s} \frac{\sin k_s L}{\cos \alpha L - \cos k_s L}$$

Thus, in this case, the transform of the global equation can be written as:

$$\left(\cos \alpha L - \cos k_s L - \frac{i\omega Z}{2k_s} \sin k_s L\right) \bar{\eta}_{global}(\alpha) = \frac{\cos \alpha L - \cos k_s L}{k_s^2 - \alpha^2} MF$$

Without forcing, the wavenumber operator on the left side yields the dispersion relations for Bloch waves on the structure. The forcing term on the right is a filtered version of the original forcing.

After a series expansion of the cosines, the left side can be written as a power series in wavenumber α with frequency dependent coefficients. Using the inverse Fourier transform to return to physical space leads to a governing equation with an infinite-order differential operator acting on the unknown $\eta_{global}(x)$. The functional form of the forcing on the right side can generally be viewed as a convolution of the original forcing and the inverse transform of the filter function. However, in some important cases, e.g. harmonic or periodic, it has a much simpler form. The same global equation was derived earlier by Bliss and Franzoni[1,2,3] using a *smooth force* method. The present *wavenumber filtering* approach has advantages for fluid loading cases.

ACOUSTIC REFLECTION FROM A MEMBRANE WITH DISCONTINUITIES

A fluid-loaded membrane with attached impedances is described by the preceding membrane equation with the addition of pressure forcing. This equation is coupled to the acoustic wave equation through the boundary condition that matches normal velocities at the interface:

$$\begin{cases} \dfrac{\partial^2 \eta}{\partial x^2} + k_s^2 \eta = i\omega Z \sum\limits_n \eta\delta(x - x_n) + F(x) + \dfrac{1}{T} p\Big|_{z=0} \\ \dfrac{\partial^2 p}{\partial x^2} + \dfrac{\partial^2 p}{\partial z^2} + k^2 p = 0; \quad w\big|_{z=0} = \dfrac{i}{\rho\omega} \dfrac{\partial p}{\partial z}\Big|_{z=0} = i\omega\eta \end{cases}$$

where c and c_s are wave speeds, and $k = \omega/c$ and $k_s = \omega/c_s$ are wavenumbers, in the fluid and on the membrane, respectively; T is the membrane tension. In the wavenumber-frequency space this system can be written as:

$$\begin{cases} \left(k_s^2 - \alpha^2\right)\bar{\eta} = \dfrac{i\omega Z}{L} \sum\limits_n \bar{\eta}\left(\alpha - \dfrac{2\pi n}{L}\right) + \bar{F} + \dfrac{1}{T}\bar{P}\Big|_{z=0} \\ \dfrac{\partial^2 \bar{P}}{\partial z^2} + \left(k^2 - \alpha^2\right)\bar{P} = 0; \quad \dfrac{\partial \bar{P}}{\partial z}\Big|_{z=0} = \rho\omega^2\bar{\eta}. \end{cases}$$

Introducing the global displacement and pressure, and applying the same procedure as before to derive the appropriate Local-Global Relationship, leads to the following global system:

$$\begin{cases} D(\alpha)\left[1 - \dfrac{i\omega Z}{L} \sum\limits_{n \neq 0} \dfrac{1}{D_f\left(\alpha - \dfrac{2\pi n}{L}\right)} - \dfrac{i\omega Z}{L}\dfrac{1}{D(\alpha)}\right]\bar{\eta}_{global}(\alpha) = MF + MP \\ \dfrac{\partial^2 \bar{P}_{global}}{\partial z^2} + \left(k^2 - \alpha^2\right)\bar{P}_{global} = 0, \quad \dfrac{\partial \bar{P}_{global}}{\partial z}\Big|_{z=0} = \rho\omega^2\bar{\eta}_{global} \\ MF = Modified\ \ forcing,\ as\ before \\ MP = Modified\ \ pressure = \dfrac{\bar{P}_{global}}{T}\left[1 - \dfrac{i\omega Z}{L}\sum\limits_{n \neq 0} \dfrac{1}{D_f\left(\alpha - \dfrac{2\pi n}{L}\right)}\right] \\ Operators: \quad D(\alpha) = k_s^2 - \alpha^2; \quad D_f(\alpha) = k_s^2 - \alpha^2 - \dfrac{\rho\omega^2}{T\mu(\alpha)}; \quad \mu(\alpha) = -\sqrt{\alpha^2 - k^2}. \end{cases}$$

This system governs the exact long-wave content of membrane displacements and pressure in fluid. The Local-Global relationship can be used afterwards to reconstruct short-wave content.

Writing the global system in physical space variables requires an approximation to evaluate the sums involving the operator D_f above in closed form. The difficulty arises from the fluid loading term involving $\mu(\alpha)$. If only long waves from the global interval propagate in the fluid, then all other waves are evanescent. The terms in the sum corresponding to the effect of the evanescent modes on the global solution can be approximated as follows:

$$\mu(\alpha) = -\sqrt{\alpha^2 - k^2} \;\rightarrow\; \mu = -\sqrt{k_s^2 - k^2};$$

$$D_f\left(\alpha - \frac{2\pi n}{L}\right) \rightarrow k_s^2 - \left(\alpha - \frac{2\pi n}{L}\right)^2 - \frac{\rho\omega^2}{T\mu} = \tilde{k}^2 - \left(\alpha - \frac{2\pi n}{L}\right)^2;$$

$$\sum_{n\neq 0} \frac{1}{D_f\left(\alpha - \frac{2\pi n}{L}\right)} \rightarrow \frac{L}{2\tilde{k}} \frac{\sin \tilde{k}L}{\cos \alpha L - \cos \tilde{k}L} - \frac{1}{\tilde{k}^2 - \alpha^2}$$

This approximation can be justified as follows. For an infinite continuous membrane with fluid loading, when the membrane waves are subsonic, the effect of fluid loading is like a frequency dependent added mass, since the fluid waves are evanescent. The wave speed of the fluid loaded membrane is approximately $c_{sf} = \omega/k_{sf} = [T/(\rho_s + \rho_f/k_{sf})]^{1/2}$ when the wave Mach number is small. It is possible to approximate the unknown k_{sf} inside the radical with k_s with good accuracy for fairly high fluid loading. Then, the effective density of the membrane is $(\rho_s + \rho_f/k_s)$.

In the problem under consideration, there are discontinuities that produce a broad spectrum of wavenumbers on the membrane. However, between discontinuities (neglecting fluid loading) the waves on the membrane will have a dominant wavelength associated with k_{sf}. If the membrane waves were very subsonic, there would typically be many such wiggles between discontinuities. Neglecting end effects near the discontinuities, the effect of fluid loading on this part of the membrane could be approximated by using the effective density described above. All of the wavenumbers on the membrane with discontinuities, none of which are typically k_s, are simply adding up to make the wiggly membrane motion that looks approximately like k_s between discontinuities. The approximate simplification of the sum of operators above embodies this effect. Although, strictly speaking it applies to lightly loaded waves that are very subsonic, it will be seen to work remarkably well for heavy fluid loading and higher subsonic wave speeds.

It is now possible to state the global problem for the coupled system in wavenumber space:

$$\begin{cases} \left(k_s^2 - \alpha^2\right)\left[\left(\cos\alpha L - \cos\tilde{k}L\right)\left(1 + \frac{i\omega Z}{L}\frac{k_s^2 - \tilde{k}^2}{\left(k_s^2 - \alpha^2\right)\left(\tilde{k}^2 - \alpha^2\right)}\right) - \frac{i\omega Z}{2\tilde{k}}\sin\tilde{k}L\right]\bar{\eta}_{global}(\alpha) = \left(\cos\alpha L - \cos\tilde{k}L\right)(MF + MP) \\ \frac{\partial^2 \bar{P}_{global}}{\partial z^2} + \left(k^2 - \alpha^2\right)\bar{P}_{global} = 0, \qquad \left.\frac{\partial \bar{P}_{global}}{\partial z}\right|_{z=0} = \rho\omega^2\bar{\eta}_{global} \\ MP = \frac{\bar{P}_{global}}{T}\left[1 - \frac{i\omega Z}{2\tilde{k}}\frac{\sin\tilde{k}L}{\cos\alpha L - \cos\tilde{k}L} + \frac{i\omega Z}{L}\frac{1}{\tilde{k}^2 - \alpha^2}\right] \end{cases}$$

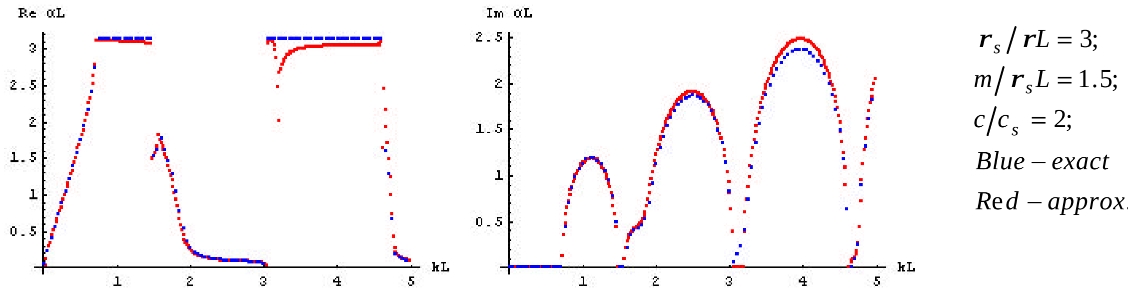
Note that this important result has a form similar to the original problem formulation for the coupled fluid and structure, only now the operators are different and the variables are smooth. A similar form also occurs upon returning to physical-space variables. After some manipulation, a series expansion in powers of α, and a transform inversion, the structural equation can be re-expressed in physical space variables in a very simple form involving infinite order operators:

$$\sum_{n=0}^{\infty} A_n \frac{\partial^{2n}\eta_{global}}{\partial x^{2n}} = \sum_{n=0}^{\infty} B_n \frac{\partial^{2n} F}{\partial x^{2n}} + \sum_{n=0}^{\infty} C_n \frac{\partial^{2n} P_{global}}{\partial x^{2n}}$$

where the coefficients A_n, B_n, and C_n depend on frequency. Along with the easily inverted acoustic equation and boundary condition, this equation provides a closed system for global displacement and pressure. When radiation occurs only in the global interval, this method accounts for propagating pressure waves in the fluid with the evanescent mode effect contained in the operator. The summations can be accurately truncated after a few terms (typically n=4).

RESULTS

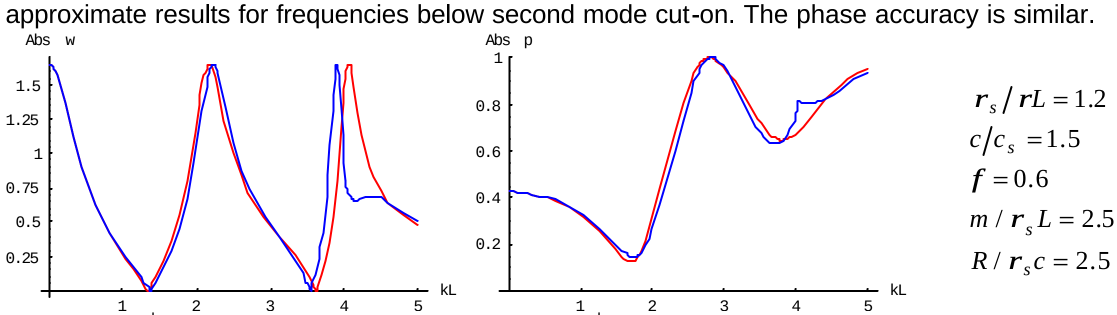
First, results are presented for free vibration (no incident waves or applied forcing) of the fluid-loaded membrane with impedance discontinuities. The dispersion curves of global wavenumber versus frequency were solved numerically for the exact formulation, and for the approximate formulation for evanescent fluid loads, as described above. The real and imaginary parts of global wavenumber, shown below, exhibit the expected stop-band and pass-band behavior, however, when the global wave speed is supersonic, there is a non-zero imaginary part in the pass bands due to damping from fluid radiation. The curves were found by a numerical root-finding scheme.



Next, acoustic scattering from the membrane with discontinuities is considered. The forcing comes from the global pressure field having prescribed incident pressure wave of amplitude P^+:

$$P_{global}(x,z,t) = P^+ e^{i\omega t} e^{-ik_x x} e^{ik_z z} + P^- e^{i\omega t} e^{-ik_x x} e^{-ik_z z}$$

where $k_x = k \sin\phi$ and $k_z = k \cos\phi$, with incidence angle ϕ measured from the normal. Waves outside the global interval start to propagate at frequency $kL = 2\pi/(1+\sin\phi)$. For the incidence angle $\phi = 0.6$ used in the example below, only results for $kL < 4.0$ should be in good agreement, since only one propagating mode was included in the global analysis with the fluid loading approximation. More propagating modes can be readily included, but this was not done for the example shown. The results shown below are magnitudes of dmensionless displacements and reflected pressure versus frequency. There is excellent agreement between exact and approximate results for frequencies below second mode cut-on. The phase accuracy is similar.



$w = \rho c \omega \eta / P^+$, mass impedance case; $p = P^- / P^+$, damper impedance case

Finally, the effect of truncating the infinite-order operators in the global solution is discussed. For this scattering problem, numerical simulations show a good convergence for truncation beyond the 8th derivative (five terms in the operators since only even derivatives are present). The truncated results are then indistinguishable from the approximate results shown above, within the appropriate range of validity ($kL < 4.0$). The use of the fluid loading approximation and operator truncation leads to a very simple representation of the scattering problem in physical space.

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