BOUNDARY ELEMENT METHODS IN FLUID-STRUCTURE PROBLEMS. APPLICATION TO ACOUSTIC WAVE SCATTERING.

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ABSTRACT

In this paper Boundary Element Meted (BEM) is used to solve a coupled fluid-structure problem. BEM is applied usually to calculate acoustic fields in fluid medium, specially in open domains, while Finite Element Meted (FEM) is preferred to solve elastodynamic equations in closed domains. There are only a few works where BEM has been used in both cases, coupling the resulting equations.

This work presents an application of BEM to an acoustic wave scattering problem. BEM is used in both media: fluid and elastic target. The complex is highly coupled because the target can be hollow, with a fluid inside. Matrix formulation for each equation is presented, and a technique based on acoustic impedance is used, in order to decrease the computational cost.

INTRODUCTION

This paper deals with solution of the acoustic wave scattering of an incident wave from an elastic target inmersed in a fluid medium by the Boundary Element Method (BEM). There are many works in the scientific literature [1, 2, 3] about this subject, and even some commercial software application, like SYSNOISE. They usually studied the problem using BEM for the acoustic equations and Finite Elements Methods (FEM) for the elastodynamic ones. Even they usually use FEM for the inner fluid in the hollow case. It seems that the tender is to use FEM for closed (bounded) domains and BEM for open domains.

However, others authors [4] have applied successfully the BEM in both media, the fluid and the target. The aim of this paper is to apply the BEM for all the media and to evaluate the results for different kinds of target: rigid, solid and hollow with outer and inner fluid. In order to achieve this objective a numerical application to an infinite circular cylinder is given.

In other works [5] author has integrated BEM in both media linking different matrix equations. However, in this paper an impedance couple technique is used. This procedure has the advantage of decrease the computation time because the final matrix to invert has a less dimension than in direct procedure.

BOUNDARY INTEGRAL EQUATIONS

The problem assumes an incident harmonic and plane wave ($e^{-j\omega t}$) from back to the target. This perturbation interact with the target, which is modelled as elastic (longitudinal and transversal waves), and a scattered acoustic field is produced. Target can be solid or hollow, with a fluid inside.

Outer Fluid

The integral equation for acoustic field in outer fluid is derived from Helmholtz equation [6]:

$$p(\vec{x}) = p_i(\vec{x}) - \int_S j\,\omega\,\rho_1\,p^*(\vec{x},\vec{Y})\,v_n(\vec{Y})\,d\partial\Omega + \int_S p(\vec{Y})\,q^*(\vec{x},\vec{Y})\,d\partial\Omega \quad (1)$$

where normal vector is directed toward outer fluid. To solve eq. (1) is necessary to know pressure and normal velocity over the exterior boundary of the target. This is possible by a limit process over eq. (1) which leads to:

$$c(\vec{X})\,p(\vec{X}) = p_i(\vec{X}) - \int_{\partial\Omega} j\,\omega\rho_1\,p^*(\vec{X},\vec{Y})\,v_n(\vec{Y})\,d\partial\Omega + \int_{\partial\Omega} p(\vec{Y})\,q^*(\vec{X},\vec{Y})\,d\partial\Omega \quad (2)$$

After discretization eq. (2) produces the next system matrix, assuming N nodes at the exterior boundary of the target:

$$\begin{bmatrix} \underset{N\times N}{A^p_{fe}} & \underset{N\times N}{A^v_{fe}} \end{bmatrix} \underset{2N\times 1}{\begin{Bmatrix} p^e \\ v^e_n \end{Bmatrix}} = \underset{N\times 1}{\{p_i\}} \quad (3)$$

where "e" indicates the exterior boundary of the target. To solve eq.(3) are necessary extra conditions. In this paper impedance conditions (relation between normal velocity and pressure) will be used.

Elastic Target

Elastic behaviour of the target is described by Navier equation of elastodynamic, which in integral form is expressed as [6, 7]:

$$c(\vec{X})\,\delta_{ij}\,v_j(\vec{X}) + \int_{\partial\Omega} v_j(\vec{Y})\,t^*_{ij}(\vec{X},\vec{Y})\,d\partial\Omega + \int_{\partial\Omega} j\,\omega\,t_j(\vec{Y})\,u^*_{ij}(\vec{X},\vec{Y})\,d\partial\Omega = 0 \quad (4)$$

where normal vector is directed toward fluid, either outer and inner one. Assuming both outer and inner boundaries have N nodes, eq. (4) can be expressed in matrix form as:

$$\underset{2N\times 2N}{\left[A^v_{ij}\right]}\;\underset{2N\times 1}{\{v_j\}} + \underset{2N\times 2N}{\left[A^t_{ij}\right]}\;\underset{2N\times 1}{\{t_j\}} = \underset{2N\times 1}{\{0\}} \quad (5)$$

Previous equation can be transformed through the relation between fluid and target at boundaries:

$$\begin{aligned} v_n &= v_i\,n_i \\ t_i &= -\,p\,n_i \end{aligned} \quad (6)$$

So, eq. (5) can be expressed as:

$$\underset{2N\times 2N}{\left[A^v_{ij}\right]}\;\underset{2N\times 1}{\{v_j\}} = \underset{2N\times 2N}{\left[A^t_{ij}\right]}\;\underset{2N\times 1}{\{n_j\,p\}} = \underset{2N\times 2N}{\left[A^t_{ij}\right]}\;\underset{2N\times 2N}{\left[I_j\right]}\;\underset{2N\times 1}{\{p\}} \quad (7)$$

where:

$$[I_k] = \begin{bmatrix} n_k^1 & & 0 \\ & \dots & \\ 0 & & n_k^{2N} \end{bmatrix} \tag{8}$$

In a 3D configuration the components of velocity vector are:

$$\underset{6N\times 1}{\begin{Bmatrix} v_x \\ v_y \\ v_z \end{Bmatrix}} = \underset{6N\times 6N}{[A^v]^{-1}} \ \underset{6N\times 2N}{[A^t]} \ \underset{2N\times 1}{\{p\}} \tag{9}$$

In order to obtain normal velocity (acoustic variable) operator given by eq. (8) can be applied to eq. (9):

$$\underset{2N\times 1}{\{v_n\}} = \underset{2N\times 2N}{[I_k]} \ \underset{2N\times 1}{\{v_k\}} = \underset{2N\times 2N}{[Y]} \ \underset{2N\times 1}{\{p\}} \tag{10a}$$

where Y can be interpreted as admitance of target with vacuum inside of its hole. Eq. (10a) can be expanded as:

$$\begin{bmatrix} \underset{N\times N}{Y_1} & \underset{N\times N}{Y_2} \\ \underset{N\times N}{Y_3} & \underset{N\times N}{Y_4} \end{bmatrix} \begin{Bmatrix} \underset{N\times 1}{p^e} \\ \underset{N\times 1}{p^i} \end{Bmatrix} = \begin{Bmatrix} \underset{N\times 1}{v_n^e} \\ \underset{N\times 1}{v_n^i} \end{Bmatrix} \tag{10b}$$

where "i" represents the nodes in the inner surface. If target were solid a similar treatment could have been madden, giving:

$$\underset{N\times N}{[Y_1]} \underset{N\times 1}{\{p^e\}} = \underset{N\times 1}{\{v_n^e\}} \tag{10c}$$

<u>Inner Fluid</u>

Problem in inner fluid is a radiation type one. Boundary integral formulation is [6]:

$$c(\vec{X})\, p(\vec{X}) = \int_{\partial\Omega} p(\vec{Y})\, q^*(\vec{X},\vec{Y})\, d\partial\Omega - \int_{\partial\Omega} j\,\omega\,\rho_2\, p^*(\vec{X},\vec{Y})\, v_n(\vec{Y})\, d\partial\Omega \tag{11}$$

where normal vector is directed toward fluid. After discretization, eq. (11) can be expressed as:

$$\begin{bmatrix} \underset{N\times N}{A_{fi}^p} & \underset{N\times N}{A_{fi}^v} \end{bmatrix} \underset{2N\times 1}{\begin{Bmatrix} p^i \\ v_n^i \end{Bmatrix}} = \underset{N\times 1}{\{0\}} \tag{12}$$

IMPEDANCE COUPLIG METHOD

One alternative to solve the problem is linking a matrix using eqs. (3), (10b or c) and (12), producing:

$$\begin{bmatrix} A_{fe}^p & 0 & A_{fe}^v & 0 \\ Y_1 & Y_2 & -1 & -1 \\ Y_3 & Y_4 & -1 & -1 \\ 0 & A_{fi}^p & 0 & A_{fi}^v \end{bmatrix} \begin{Bmatrix} p^e \\ p^i \\ v_n^e \\ v_n^i \end{Bmatrix} = \begin{Bmatrix} p_i \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{13a}$$

for hollow target or

$$\begin{bmatrix} A_{fe}^{p} & A_{fe}^{v} \\ Y_1 & -1 \end{bmatrix} \begin{Bmatrix} p^e \\ v_n^e \end{Bmatrix} = \begin{Bmatrix} p_i \\ 0 \end{Bmatrix} \tag{13b}$$

for solid one. Solution of Eqs. (13) gives pressure and normal velocities values at exterior boundary of target, so it's possible to solve the numerical version of eq. (1), which determines the acoustic pressure in any point of the outer fluid. This alternative has been used in [6]. This method has a high computational cost, due to the inversion of a large matrix. In this paper another strategy is formulated, based on acoustic impedance. Code used is described by Dominguez [7], but it only allows Neuman or Dirichlet boundary condition. Impedance type are Robin condition, so it has been necessary to modify the code to include impedance conditions. This has been madden through relations between pressure, velocity potential (variable used in the code) and normal velocity:

$$\phi = \frac{p}{\rho_o\ \omega} = \frac{Z}{\rho_o\ \omega} v_n = Z' v_n \tag{14}$$

Assuming a boundary with N1 nodes with Dirichlet conditions ($\phi = \hat{\phi}_1$), N2 con Neuman ones ($v_n = \hat{v}_2$) and N3 with impedance ones ($Z' = \hat{Z}'_3$) and using the typical notation [7], the matrix formulation of the problem gives:

$$\left[\underset{N\times N1}{\left(H_1\right)} \ \underset{N\times N2}{\left(H_2\right)} \ \underset{N\times N3}{\left(H_3\right)} \right] \begin{Bmatrix} \underset{N1\times 1}{\hat{\phi}_1} \\ \underset{N2\times 1}{\phi_2} \\ \underset{N3\times 1}{\left(\hat{Z}' v\right)_3} \end{Bmatrix} = \left[\underset{N\times N1}{\left(G_1\right)} \ \underset{N\times N2}{\left(G_2\right)} \ \underset{N\times N3}{\left(G_3\right)} \right] \begin{Bmatrix} \underset{N1\times 1}{v_1} \\ \underset{N2\times 1}{\hat{v}_2} \\ \underset{N3\times 1}{v_3} \end{Bmatrix} \tag{15}$$

Rearranging the previous system:

$$\left[\underset{N\times N1}{\left(-G_1\right)} \ \underset{N\times N2}{\left(H_2\right)} \right] \begin{Bmatrix} \underset{N1\times 1}{v_1} \\ \underset{N2\times 1}{\phi_2} \end{Bmatrix} + \left[\underset{N\times N3}{H_3} \right] \left\{ \underset{N3\times 1}{\left(\hat{Z}' v\right)_3} \right\} - \left[\underset{N\times N3}{G_3} \right] \left\{ \underset{N3\times 1}{v_3} \right\} = \left[\underset{N\times N1}{\left(-H_1\right)} \ \underset{N\times N2}{\left(G_2\right)} \right] \begin{Bmatrix} \underset{N1\times 1}{\hat{\phi}_1} \\ \underset{N2\times 1}{\hat{v}_2} \end{Bmatrix} \tag{16}$$

Solving eq. (16) normal velocity in type 3 nodes is known. Velocity potential or acoustic pressure can be evaluated by eq. (14).

Procedure described by eqs. (14) to (16) is applied in first place to inner fluid. So, an unitarian pressure or normal velocity is assumed in eq. (12), so the another unknown is solved. With this two variables eq. (14) is applied in each node, so acoustic impedance at boundary nodes is evaluated. After changing the sign to this impedance, it's imposed as impedance condition at the interior surface of target. Then, an unitarian pressure or normal velocity is assumed again, this time in exterior nodes of target in eq. (10b), with the impedance previously found in the interior ones. After solving eq. (10b) eq. 14 is applied again, so nodal impedance is obtained at exterior surface of the target. Sign of this new impedance is changed again. At last, it's possible to solve eq. (3), and so solve the acoustic field in the outer fluid by eq. (1).

Application of impedance condition to eq. (10b) and (3) implies to modify them as in eqs. (15) and (16). Of course, if solid target is considered the previous process begins with the target, and eq. (10a) is used instead of eq. (10b).

The previous sequential process reduce the computational cost due to small dimension in each step. Three matrix has been inverted, but it's speeder than inverting only one of the global dimension.

NUMERICAL RESULTS

The proposed method has been successfully applied to an infinite circular cylinder (2D problem) in both cases, solid and hollow. It has also been studied the rigid case. Cuadratic isoparametric elements has been used, in a different number depending on the type of problem. So, 14 and 20 elements has been used in the rigid and solid case, and 20, 30 and 50 in the hollow one.

The cylinder has an exterior radius (a) of 11 mm and an interior of 6 mm. It's madden of cooper, with 8.9 g/cm^3 of density, 5000 m/s of speed of longitudinal waves and a Poisson's ratio of 0.35. Both fluids are water of 1 g/cm^3 of density and 1500 m/s of sound speed. The relative frequency studied is ka = 4, so the ratios of wavelength to distance between nodes are 7 for 14 elements, 10 for 20, 15 for 30 and 25 for 50.

In Fig.1 the results for the hollow case are shown. It's possible to see how the convergence of the solution is improved when the number of the elements increases.

Some results are shown in Table 1 to evaluate the accuracy of the method. The analytical (exact) and numerical solution for scattering pressure at 1 m from the cylinder has been evaluated for certain angles (the incident wave arrives to the cylinder at 180º azimuth angle). It's seen that when the type of the target is more complicated more elements are necessary to achieve the same accuracy. So, the hollow target is shown as the most expensive target to compute.

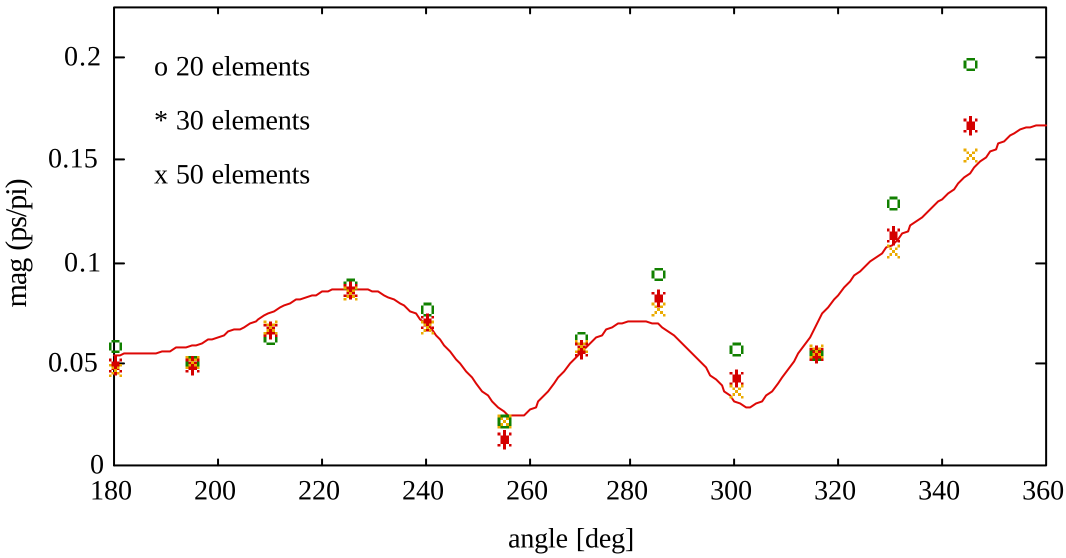

FIG.1. Scattering pressure versus azimuth angle for a hollow infinite cylinder.

CONCLUSIONS

The aim of this paper has been showing that it's possible to solve a coupled elastodynamic-acoustic problem by means only of BEM. Results about the convergence of the technique with different types of targets are given. This method gives an alternative to the "traditional" one, which uses FEM for the target and BEM for the fluid.

There are two basic advantages in the proposed method. One is that the size of the problem reduces in one dimension (a 3D problem is solved by surface integrals and a 2D problem by line ones), which is typical of the BEM. The other important advantage is that only the necessary data are calculated, this is, only data in the boundary. However, if FEM is used to

Case \ Angle (deg)	0	45	90	135	180
Rigid					
Numerical (N = 14)	0.1398	0.0571	0.0666	0.0752	0.0678
Numerical (N = 20)	0.1395	0.0563	0.0666	0.0751	0.0687
Exact	0.1379	0.0546	0.0666	0.0748	0.0702
Elastic Solid (4)					
Numerical (N = 14)	0.1798	0.0582	0.0719	0.0752	0.0560
Numerical (N = 20)	0.1806	0.0581	0.0734	0.0749	0.0576
Exact	0.1797	0.0571	0.0762	0.0743	0.0595
Elastic Hollow (4)					
Numerical (N = 20)	0.2265	0.0555	0.0636	0.0895	0.0599
Numerical (N = 30)	0.1922	0.0555	0.0584	0.0863	0.0495
Numerical (N = 50)	0.1770	0.0562	0.0595	0.0851	0.0475
Exact	0.1649	0.0672	0.0555	0.0872	0.0546

Table 1. Comparison between numerical solution and analytical one in different configurations.

solve the target, the pressure and velocity field have to be calculated in all the domain, not only in the boundary.

The disadvantage of BEM is that it produces full matrices. When different governing equations have to be linked it's possible some zeros appears, as it's shown at eq. (13a). In spite of this, the direct treatment of this matrix system spends very resources. Impedance method proposed in this paper allows decrease the computational resources and solving time, without losing accuracy.

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