HIGHER-ORDER MODER EFFECTS OF THE NONUNIFORMLY PERFORATED PIPE

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ABSTRACT Nonuniformly perforated pipe is frequently used to reduce the acoustic power of the flow noise and its existence is dispensable in the muffler system. The higher-order mode resonance and flow noise of such pipe when located inside the circular cavity are investigated in this report. There are many patterns of non-uniformly perforated pipe in the muffler systems. In this report, we deal with a pipe which has a fixed hole diameter and fixed pitch, with the orifices arranged in a belt configuration.

1. INTRODUCTION

The acoustic power of flow noise increases in proportion from the 4th to 8th power of its velocity. To attenuate the acoustic power of flow noise, the perforated pipe is frequently used to reduce the flow velocity and its existence is dispensable in the muffler system.

Contemporary automotive muffler chambers consist of more than one resonance room and expansion room in which one or several perforated pipe is installed. The sound propagated into the resonance room has a plane wave component and the higher-order mode components. Resonance of the latter is contingent on the acoustic impedance of the perforated pipe and the shape of resonance room. Therefore, the muffling effect will be decreased by generating high-level resonance of higher-order modes in the chamber when perforated pipe is not correctly used.

We have been examining these resonance frequencies theoretically and experimentally [1]. The calculation results agree closely with measurement in the well, and the effectiveness of our method of analyzing the resonance mechanism has been confirmed. In this analysis, relying on the wave equation, sound pressure at the outlet was computed with one perforated pipe installed inside the chamber. A pipe having orifices uniformly distributed around the surface was the object of our analysis. However, in real muffler system, perforated pipes with orifices arranged in a belt configuration are widely used instead of the described pipe . Such a pipe will be called a nonuniformly perforated pipe. Theoretical analysis concerning nonuniformly perforated pipe is extremely difficult and a suitable analytical technique has not yet been found. Because the distribution of orifice is different from the case of an uniformed perforated pipe, it is possible to generate different resonances in the chamber. However, it is thought that the resonances of the two configurations do differ greatly when the length of the perforated pipe is short. In practical muffler system, the length of the perforated pipe is a maximum of 20cm.

In order to discover the cause of the resonance generated when such perforated pipe is used, the resonances of nonuniformly perforated pipe is first examined experimentally. Next, these resonances are assumed in the proposed resonance mechanism used in our analysis for uniformed perforated pipe. There are many patterns of nonuniformly perforated pipe in the muffler systems. In this paper, we deal with a pipe which has a fixed hole diameter and fixed pitch, with the orifices arranged in a belt configuration. After the uniformed perforated pipe theory is outlined, resonances of such pipe will be described in detail.

2. MODEL FOR ANALYSIS

Consider a circular cylindrical chamber of radius and length L that has an input, output and a thin perforated pipe as shown in Fig. 1. The radius of input, output and perforated pipe are r_0, r_L and r_p, respectively.

The wave equation with uniform flow velocity V in terms of the velocity potential is

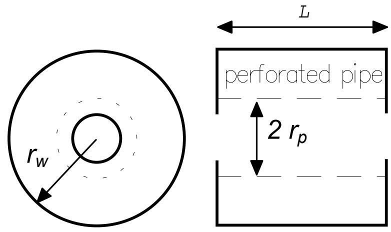

Fig. 1 Model of analysis

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\Phi=\frac{1}{c^2}\left(\frac{\partial^2}{\partial t^2}+2\,V\cdot\nabla\frac{\partial}{\partial t}+(V\cdot\nabla)^2\right)\Phi \tag{1}$$

where $\Phi=\sqrt{2}\phi\exp(j\omega t)$, $j^2=-1$, is the angular frequency. Let the velocity potential inside the perforated pipe be ϕ_{in} and that outside the pipe be ϕ_{out} and let $k=\omega c$, $M=V/c$. Then the solution of ϕ_{in} and ϕ_{out} become

$$\phi_{in} = \left\{ A^{in} \exp(\beta+\gamma)z + B^{in} \exp(\beta-\gamma)z \right\} \sum_{m=1}^{\infty} C_m^{in} J_m(\lambda_m r / r_w) \cos m\theta$$

(2)

$$\phi_{in} = \left\{ A^{out} \exp(\beta+\gamma)z + B^{out} \exp(\beta-\gamma)z \right\} \left\{ \sum_{m=1}^{\infty} C_m^{out} J_m(\lambda_m r / r_w) \cos m\theta \right.$$

$$\left. - \sum_{m=1}^{\infty} D_m^{out} N_m(\lambda_m r / r_w) \sin m\theta \right\} \qquad (3)$$

where

$$\gamma = \sqrt{\mu^2 \left(1-M^2\right) - k^2 M^2} \;/\; \left(1-M^2\right) \qquad (4)$$

$$\beta = jkM \;/\; \left(1-M^2\right) \qquad (5)$$

$J_m(x)$ and $N_m(x)$ are Bessel functions. Other symbols are constants.

The boundary conditions are

[1] $V_z = -\partial\phi_{in} / \partial z = V_0 F_0(r)$ at $z = 0$

[2] $V_z = -\partial\phi_{in} / \partial z = V_L F_L(r)$ at $z = L$

[3] $V_r = -\partial\phi_{out} / \partial r = 0$ at $r = r_w$

[4] $V_z = -\partial\phi_{out} / \partial z = 0$ at $z = L$

[5] $\phi_{in} - \phi_{out} = Z_{pn} \partial\phi_{out} / \partial r$ at $r = r_p$

[6] $\partial\phi_{in} / \partial r = \partial\phi_{out} / \partial r$ at $r = r_p$

where

$$Z_{pn} = Z_{pipe} \;/\; (jk\rho c)$$

(6)

Z_{pipe} is the impedance of the perforated pipe [1], ρ is mass density and *c* is the sound velocity, respectively.

The sound pressure at the output corresponding to ϕ_{in} becomes

$$P_{in} = -jZ_w \frac{1}{\sin k_m L} \left[\left\{ U_0 \exp(jMk_m L) + U_L (jMk_m L - \cos k_m L \right\} \right.$$

$$- \sum{}' \left\{ \frac{U_0 \exp(jMk_m L)\, Q_{0,L}}{\left(\dfrac{\gamma^2{}_{m,i} - \beta^2}{\gamma_{m,i}}\right) \sinh \gamma_{m,i} L + \tilde{Z}_{m,i} \left(\dfrac{\bar{\gamma}^2{}_{m,i} - \beta^2}{\bar{\gamma}_{m,i}}\right) \sinh \bar{\gamma}_{m,i} L} \right.$$

$$+\frac{U_L Q_{L,L}}{\dfrac{(\gamma^2{}_{m,i}-\beta^2)\sinh\gamma_{m,i}L}{\gamma_{m,i}\cosh\gamma_{m,i}L-\beta\sinh\gamma_{m,i}L}+\overline{Z}_{m,i}\dfrac{(\overline{\gamma}^2{}_{m,i}-\beta^2)\sinh(\overline{\gamma}_{m,i}L)}{\overline{\gamma}_{m,i}\cosh\overline{\gamma}_{m,i}L-\beta\sinh\overline{\gamma}_{m,i}L}}\Bigg\}\Bigg]\Bigg] \tag{7}$$

where

$$\gamma_{m,i}=\sqrt{\mu_{m,i}{}^2(1-M^2)-k^2M^2}\,/\,(1-M^2) \tag{8}$$

$$k_M=k/(1-M^2) \tag{9}$$

Z_w is the characteristic acoustic impedance of the chamber, $\tilde{Z}_{m,i}$ is the constant depends on the shaped of the perforated pipe [1], U_0 and U_L are the volume velocity at the input and output piston, respectively. Σ' means $\Sigma_{m=0}^{\infty}\Sigma_{i=0}^{\infty}$ without *m=I=0*. Other symbols are constants. When *M=0*, the mean flow is absent, Eq. 7 reduce to

$$P_{in}=-jZ_w\frac{1}{\sin kL}\Bigg[\Big\{U_0-U_L\cos k\ L\Big\}+\sum{}'\Bigg\{\frac{U_0\,Q_{0,L}}{\mu_{m,i}\sinh r_{m,i}L+\tilde{Z}_{m,i}\overline{\mu}_{m,i}\sinh\overline{\mu}_{m,i}L}$$

$$+\frac{U_L\,Q_{L,L}}{\mu_{m,i}\tanh r_{m,i}L+\tilde{Z}_{m,i}\overline{\mu}_{m,i}\tanh\overline{\mu}_{m,i}L}\Bigg\}\Bigg] \tag{10}$$

The first term represents the sound pressure of mode (0,0) and the second ones denotes the sound pressure of higher-order modes (m,i). The resonances of P_{in} occur at the frequency where these denominators are zero, namely

$$\mu_{m,i}\sinh r_{m,i}L+\tilde{Z}_{m,i}\overline{\mu}_{m,i}\sinh\overline{\mu}_{m,i}L=0 \tag{11}$$

$$\mu_{m,i}\tanh r_{m,i}L+\tilde{Z}_{m,i}\overline{\mu}_{m,i}\tanh\overline{\mu}_{m,i}L=0 \tag{12}$$

Note that $\tilde{Z}_{m,i}$ is a function of the perforated pipe impedance. With the example of Eq. 11, when $\tilde{Z}_{m,i}$ << 0, the resonances occur at

$$\mu_{m,i}\sinh r_{m,i}L=0 \qquad \therefore\ f=\frac{c}{2\pi w_w}\sqrt{\lambda_{m,i}^2+(n\pi\ r_w/L)^2} \tag{13}$$

when $\tilde{Z}_{m,i}$ >> 0, the resonances occur at

$$\overline{\mu}_{m,i}\sinh\overline{\mu}_{m,i}L=0 \qquad \therefore\ f=\frac{c}{2\pi w_w}\sqrt{\overline{\lambda}_{m,i}^2+(n\pi\ r_w/L)^2} \tag{14}$$

where n=0,1,2,…, other symbols are constants [1].

3. EXPERIMENTS AND CONSIDERATION OF NONUNIFORMLY PERFORATED PIPE

Seven perforated pipes of the same size were created in order to examine the resonance corresponding to the orifices distribution. Each pipe is 20cm in length, 2.5cm in radius, respectively. Orifices 6mm in diameter were arranged in a belt configuration as shown in Fig. 2. Let x be the distance from a location on the pipe to the center of the belt; x has dimension of 4, 6, 8, 10, 12, 14, and 16cm in our experiment. Each pipe has the pitch of 2.5 cm and the porosity, ratio of the total hole areas to the pipe surface area, of 1.5%. A block diagram of the experimental apparatus is shown in Fig. 2. Each pipe is installed inside the chamber, which has a radius of 24cm, and length of 20cm. At first, in order to discover the cause of the higher-order mode resonances generated, we measured the sound pressure of P_{in} by installing the microphone 2 in the center of the chamber. In this case, let P_A and P_B are the sound pressure of microphone 1 and microphone 2, respectively. Then, the relationship between P_A and P_B is

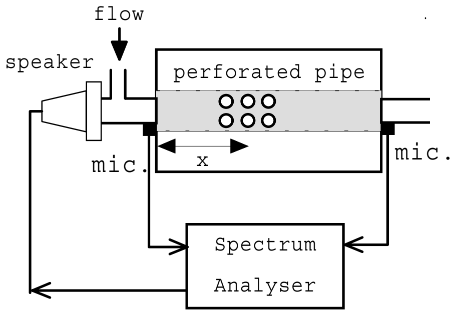

Fig. 2 Experimental apparatus

$$20\,Log\left| P_B / P_A \right| = 20\,Log\left| P_{in} / U_0 \right| - 20\,Log\left| Z_0 \sin kL_0 \right| \quad (11)$$

The second term represents the acoustic characteristic of the inlet pipe, where Z_0 is a characteristic impedance of inlet pipe, L_0 is a distance from microphone 1 to the cavity. P_{in} in the right side of the above expression is a theoretical value given in Eq. 7. Experimental results are shown in Fig. 3, where multiples of 20dB were added at each measured sound pressure level to a well-identified change of x. Symbol ● represents the resonance of the plane wave which occur when sinkL of Eq. 10 equal to zero. To compare this with the theory value of the uniformed perforated pipe, calculation of Eq. 7 when U_L=0 is made and illustrated by the vertical dot lines as shown in Fig. 3. It was found that there is no difference between the measured value and the theoretical value. The difference between them is less than 5Hz. The effects of the flow noise are shown in Fig. 4 and Fig. 5. Thick solid line and the thin dotted line of Fig. 4 are the case of V=0m/s and V=23m/s, respectively.

5. CONCLUSION

Sound propagating inside a chamber with a perforated pipe includes a plane wave component and higher-order mode component. The resonance of the latter is related to the acoustic impedance of perforated pipe. Resonances of higher-order modes (1,0), (0,1) and (2,0) were present in the experiment results for nonuniformly perforated pipe and in the calculated results for uniformly perforated pipe..

From the above experiments, there exist some differences between the resonance of non-uniformly and uniformly perforated. However, it was found that the difference between them is less than 5Hz. Therefore, our proposed analysis method for uniformly perforated pipe can also be used to forecast the resonance of nonuniformly perforated pipe. Thus, in order to find the exact resonant frequencies of nonuniformly perforated pipe, it is necessary to study the relation between the acoustic impedance and the frequency.

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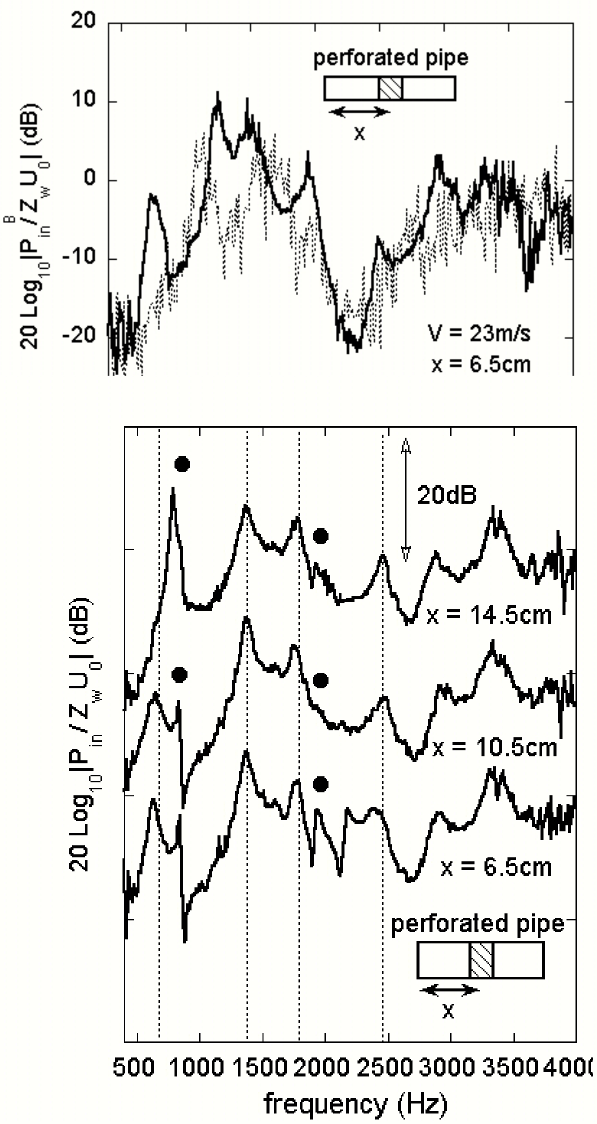

Fig. 4 Experimental results of nonuniformly perforated pipe with and without flow.