ON THE USE OF HYBRID METHODS FOR FAST ACOUSTICAL SIMULATIONS IN ENCLOSURES

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ABSTRACT

For most practical purposes in room acoustics simulation, it is sufficient to model the propagation of sound energy inside the enclosure subject to the study. The room's energy impulse response or the room's energetic transfer function forms the basis of the major room acoustics classification indexes.

Within this picture, the complex reflection characteristics of the enclosure's surfaces can be simplified through the adoption of the well-established sound energy absorption coefficient and through the introduction of the mixture of specular and ideally diffuse reflected sound components.

A hybrid method is presented in this paper. The method consists of a specular energetic ray-tracing with the adoption of the energetic transition algorithm for the lambertian components.

INTRODUCTION

The sound reflections that occur inside an enclosure are of extreme importance in the signature of the perceived acoustics. It is therefore very important to correctly model these reflections, which are composed of specular reflected components and of diffuse reflected components.

The method presented allows the correct calculation of the specular and diffuse parts of the first reflections, and can also be applied in realistic computation times in order to obtain the late part of the reverberant sound field.

METHOD

Ray Tracing

The sound field is thought to be composed of discrete energy packets that are emitted from sound sources and undergo various reflections at the boundary. The trajectories of these packets are given by linear sound rays. The rays will therefore be reflected at the boundaries, where surface absorption and geometrical reflection will take place. Air attenuation is also included in this ray tracing technique.

Ray tracing will be used only for the calculation of the specular components of the reflected packets, which are characterised by Snell's reflection law where the reflection angle equals the incidence angle:

$$R(\mathbf{x},q,f,q^{'},f^{'}) = 2s(1-a(\mathbf{x}))d(\sin^2 q - \sin^2 q^{'})d(f-f^{'} \pm p)$$
$$d(0)=1; d(k \neq 0)=0 \qquad (1)$$

These specular reflections are obtained for rigid boundary surfaces possessing dimensions that are greater than the wave length of the incident waves [1]. Any components that are not reflected into specular directions are considered to be diffusely reflected, according to Lambert's cosine law.

In any ray tracing implementation, the modelling of the sound source is important. Two alternative approaches can be implemented for the radiation of the rays from the source. The first one considers a Monte Carlo generation of the ray directions and works efficiently when the number of emitted rays is large enough to avoid statistical fluctuations. The second approach consists in the deterministically determination of the ray directions through the subdivision of the source's radiation pattern into geometrical zones [2-3].

In this method, the division is accomplished through the subdivision of an icosahedron into equilateral triangles, which will be in turn subdivided into new triangular elements [4]. This procedure is repeated iteratively until the desired number of emitted rays is achieved, as shown in figure 1:

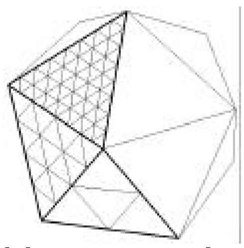

Figure 1. Icosahedron subdivision into equilateral triangles ($n = 0,\ldots,3$).

After n subdivisions the number of vertices N_V and the number of faces N_F is given by:

$$N_V = 5\left(2^{2n} - 2^n + 2\sum_{m=1}^{2^n} m\right) + 2$$
$$N_F = 20 \cdot 4^n \qquad (2)$$

The rays' emission angles are obtained through the projection of the obtained vertices over an unitary sphere.

The emitted rays are then reflected according to law (1) at the boundary. Due to surface absorption, only a part *(1-a)* of the incident energy will be reflected. In order to separate the diffuse components from the specular ones it is necessary to introduce a diffuse reflection coefficient, *d*, as proposed by several authors (see for example [5]). In this case, the ray's energy after undergoing several specular reflections at the boundary is given by:

$$E = \frac{\Pi}{N_R} D(q, f) e^{-md} \prod_i [(1-a(S_i))(1-d(S_i))] \tag{3}$$

where P is the source's energy, N_R is the number of emitted rays, $D(q, f)$ is the source's directivity function, d is the total distance travelled by the ray and m is the air attenuation coefficient, given by [6]:

$$m = 5.5 \times 10^{-4} \frac{50}{h} \left(\frac{f}{1000} \right)^{1.7} \tag{4}$$

For the modelling of the boundary surfaces, the adopted geometrical element is the triangle, since it is the one that allows a fast ray interception verification, and, in addition, it allows that any surface be represented by the union of these triangles.

The last step in a ray tracing implementation concerns the ray detection at some receiver. Several options are available, the omni directional spherical receiver being the simplest one. In this implementation, however, the receiver will be modelled as an imission disk of diameter *a/2*, surrounding the defined reception point. This disk rotates over his centre in order that the rays are always perpendicularly incident. The sound energy density captured at this disk at time *t* is then given by:

$$I_R(t) = \frac{\sum_i E_i}{p\ a^2} \tag{5}$$

where $\sum_i E_i$ is the total energy incident on the disk at instant *t*. Using this receiver model, the impulse response of the spherical region *R*, obtained through the rotation of the disk, at the end of the processing of all the rays is given by:

$$\bar{I}(t) = \frac{1}{V} \int_R \frac{e(t, \mathbf{x})}{ða^2} dv \tag{6}$$

where $V = 4ðr^3/3$ and $\boldsymbol{x}$ means the position vector inside *R*.

The suggested value for the radius of the imission disk is typically 15 cm, that is the approximate dimension of a human head. The moment of reception is the instant when the ray intercepts the disk.

The main disadvantage of the ray tracing method is the need of a large number of rays in order to correctly simulate the late part of the reverberant impulse response. This is due to the natural geometrical divergence of the rays, that grows as the propagation distance increases. This separation can become so large that the rays are no longer correctly detected. The solution is to consider a larger number of rays emitted from the source at the expense of an increased calculation effort. This consideration makes clear that the pure ray tracing is not well suited for the modelling of the late part of the echogram.

Energy Transitions

In order to model the diffuse reflected components the proposed method resorts to the technique of the energy transitions between pairs of surfaces of the enclosure. The diffuse energy exchange is determined through the geometrical form factor of the considered surface pair. This method was already presented earlier by the authors [7].

In this method, the source distributes the whole sound energy over the *M* enclosure's walls. The starting energy density over wall S_j can be determined considering spherical waves radiated from *N* sources:

$$B_{j,0} = B(S_j, 0) = \frac{1}{4pc} \sum_{l=1}^{N} \frac{\Omega_j^l \Gamma_l (J, q) \Pi_l}{S_j} \tag{7}$$

where Ω^{l}_{j} is the solid angle of surface S_j subtended at source l, P_l is the total acoustical energy of source l and $G(J,q)$ is its directivity function.

In the calculation of the solid angles of a surface subtended at some point, only the visible part of this surface will be considered. Therefore, occluding surfaces diminish the value of the solid angle.

The values $B_{j,0}$ can be stored into a row vector, called the starting vector:

$$[B]_{(0)} = \lfloor B_{1,0}, B_{2,0}, B_{3,0}, \ldots, B_{M,0} \rfloor \tag{8}$$

The method of energy transitions, in the variant of a homogeneous Markov chain of first order, considers that the enclosure walls exchange energy between themselves at constant time intervals, given by the classical transition time t:

$$t = 4V/vS \tag{9}$$

The equation for the evolution of the energy density can be written in matrix form as [7-8]:

$$[B]_{(kt)} = [\Xi]^{k}[B]_{(0)} \tag{10}$$

where k is the considered transition order. $[B]_{(kt)}$ is a M-dimensional column vector with entries

$$B_{j,k} = B(S_j, kt) \tag{11}$$

which define the energy density over S_j at time $k\,t$. The matrix X is called the transition matrix, whose entries are defined through the following relationships:

$$\begin{aligned} \Xi_{ij} &= W(S_i \rightarrow S_j)(1-a(S_i))e^{-mD(S_i,S_j)} \\ W(S_i \rightarrow S_j) &= \frac{\grave{U}_{ij}}{2\eth}\sin b;\ \grave{U}_{ij} = \frac{S_j}{D(S_i,S_j)}\left|\mathbf{u}_{ij}\cdot\mathbf{n}_j\right| \end{aligned} \tag{12}$$

with $D(S_i, S_j)=d_{ij}$ being the distance between the geometrical centres of surfaces S_i and S_j. The vector $\boldsymbol{u}_{ij}$ is an unitary vector parallel to the line connecting both geometrical centres.

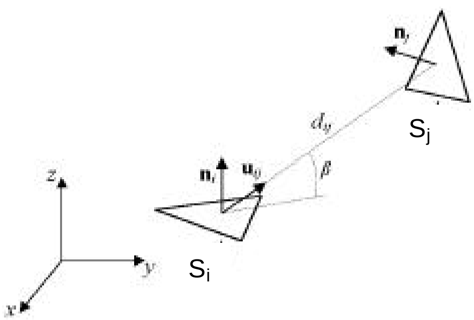

Figure. 2. Calculation of the solid angle between surface S_i and S_j

The last step in the energy transitions process is the diffuse radiation of the energy from the triangular surface elements towards the interior of the enclosure and irradiating therefore the receiver. At each transition, the elements furnish a certain amount of energy to the receiver. Summing up all contributions from all triangular elements, we find the intensity I_R at the receptor:

$$I_R(kt) = \sum_{j=1}^{M} \frac{P_{j,k}\Omega_j(1-a_j)}{p} \tag{13}$$

where Ω_j is the solid angle of the finite surface S_j subtended at the receiving point $\boldsymbol{s}_r$

Hybrid Method

The hybrid method uses a combination of the specular ray tracing and of the energy transition method in order to simulate the three most important physical phenomena of sound propagation inside an enclosure. These are the sound absorption in the air and at the boundary, the specular reflected sound components and the diffuse reflected sound components.

The calculation of the energetic impulse response of the enclosure happens in two phases. In the first phase the specular reflected components are processed, while in the second subsequent phase, the diffuse reflected components are obtained. During the frst phase, input information for the second phase is stored.

The rays emitted from the source will be followed inside the enclosure until they are detected at some receiver. When the rays hit a triangular surface element, their energy spectrum will be modified through the frequency dependent surface absorption coefficient and also through the diffuse reflection coefficient. The energy of the rays is given by expression (3). The element S_i, at which the reflection occurs at time kt_S, will store a parcel of the incident energy, namely the energy that is not reflected specularly:

$$B_i(kt_S) = B_i(kt_S) + B(1-\alpha(S_i))\delta(S_i) \tag{14}$$

where $\alpha(S_i)$ and $\delta(S_i)$ are the absorption coefficient and the diffuse reflection coefficient of element S_i respectively. t_S is the temporal resolution considered.

When the ray tracing procedure is ended, at time T, one obtains for every triangular surface element the diffuse energy that is stored for every temporal sampling interval considered. For the ensemble of the M triangular surfaces, one obtains the following diffuse energy matrix $[B_D]$:

$$[B_\mathrm{D}] = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ \vdots \\ M-2 \\ M-1 \\ M \end{array} \begin{array}{c} \begin{array}{ccccccc} t_S & 2t_S & 3t_S & \cdots & T-2t_S & T-t_S & T \end{array} \\ \begin{pmatrix} E_{1,1} & E_{1,2} & E_{1,3} & \cdots & E_{1,T-2} & E_{1,T-t_S} & E_{1,T} \\ E_{2,1} & E_{2,2} & E_{2,3} & \cdots & E_{2,T-2} & E_{2,T-t_S} & E_{2,T} \\ E_{3,1} & E_{3,2} & E_{3,3} & \cdots & E_{3,T-2} & E_{3,T-t_S} & E_{3,T} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ E_{M-2,1} & E_{M-2,2} & E_{M-2,3} & \cdots & E_{M-2,T-2t_S} & E_{M-2,T-t_S} & E_{M-2,T} \\ E_{M-1,1} & E_{M-1,2} & E_{M-1,3} & \cdots & E_{M-1,T-2t_S} & E_{M-1,T-t_S} & E_{M-1,T} \\ E_{M,1} & E_{M,2} & E_{M,3} & \cdots & E_{M,T-2t_S} & E_{M,T-t_S} & E_{M,T} \end{pmatrix} \end{array} \tag{15}$$

$[B_D]$ is changed at each ray reflection from a surface element, more precisely at the row corresponding to the considered element and at the column corresponding to the incidence instant interval. Therefore, when the first phase ends, we find a diffuse energy matrix, that represents the temporal and spatial distribution of the diffuse components and that will be processed during the second simulation phase. In this hybrid method, it is not considered that the diffuse components can be subsequently reflected in a specular way.

In the second phase, the energy transitions between the surface elements of the enclosure are then calculated on basis of the diffuse energy matrix $[B_D]$. The processing is started by exchanging the energy of all the M triangular elements from column t_S according to the procedure described in the section about the energy transition method. Every element that receives part of this energy will receive it in distinct times. Therefore, these elements will have its diffuse energy content changed at these respective times, or in other words, at the column corresponding to this precise instant:

$$\begin{array}{c} B_D \\ 1 \\ 2 \\ 3 \\ \vdots \\ M-2 \\ M-1 \\ M \end{array} \begin{array}{c} \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & \ldots \end{array} \\ \left(\begin{array}{cccccccc} 0 & E_{1,2} & E_{1,3} & E_{1,4} & E_{1,5} & E_{1,6} & E_{1,7} & \ldots \\ 0 & E_{2,2} & E_{2,3} & E_{2,4} & E_{2,5} & E_{2,6} & E_{2,7} & \ldots \\ E_{3,1} & E_{3,2} & E_{3,3} & E_{3,4} & E_{3,5} & E_{3,6} & E_{3,7} & \ldots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ldots \\ E_{M-2,1} & E_{M-2,2} & E_{M-2,3} & E_{M-2,4} & E_{M-2,5} & E_{M-2,6} & E_{M-2,7} & \ldots \\ E_{M-1,1} & E_{M-1,2} & E_{M-1,3} & E_{M-1,4} & E_{M-1,5} & E_{M-1,6} & E_{M-1,7} & \ldots \\ E_{M,1} & E_{M,2} & E_{M,3} & E_{M,4} & E_{M,5} & E_{M,6} & E_{M,7} & \ldots \end{array} \right. \end{array} \tag{16}$$

The transition of diffuse energy from surface element S_i, reflected at instant $k_i t_S$, to another surface element S_j, received at a later instant $k_j t_S$, is represented in equation (16), and is expressed by:

$$B_j(k_j t_S) = B_j(k_j t_S) + B_i(k_i t_S)\Xi_{ij} \tag{17}$$

and where

$$(k_j - k_i)t_S = D(S_i, S_j)/v \tag{18}$$

After the transition of energy from surface element S_i to all other surface elements, then the diffuse energy stored at this element at the considered time becomes zero, and therefore the entry of matrix B_D becomes equal to zero at this corresponding position. Going through the matrix B_D from initial time t_S until the end time T, then one obtains the diffuse energetic impulse response at the receiver. By summing this diffuse impulse response with the specular one, obtained during the first phase of the simulation, the total enclosure's energetic impulse response is thus obtained.

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