ANALYSIS OF A SOUND SPEED MEASUREMENT MADE IN 1738

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ABSTRACT

Spanish and French scientists made a determination of the sound speed close to Quito (Ecuador) in 1738. This measurement had a special interest in that time because the proximity of the equator and the great altitude were thought would have an important influence in the sound speed. From two different places the difference in time between the perception of the light of a gun shoot and the perception of the corresponding sound was measured. In one position, Jorge Juan and Godin got 339,0 m/s for the sound speed. From the other position, Antonio de Ulloa and Bouguer measured a value of 347,8 m/s.

INTRODUCTION

The problem about the shape and the dimensions of the Earth raised in the XVIII century, as the result of the different measurements of the meridians arcs and the lengths of the pendulums. In view of the different ideas about this question, the Science Academy in Paris agreed to measure two meridian arcs in two different places very far away one from another, designing two commissions for this work. One of them would go to Lapland, and the other to Ecuador.

The chosen place for the works belonged to the viceroyalty of Peru, so that permission was asked for to Felipe V, King of Spain, to make the measurements. Felipe V acceded on condition that two Spanish scientists would accompany to the French academics. The chosen scientists were Jorge Juan (Novelda, Alicante, Spain 1713- Madrid, 1773) and Antonio Ulloa (Seville, Spain 1716- Isla de León, Spain 1795). Both of them participated very actively in the expedition and made numerous proper measurements.

The obtained scientific results were published in two books. We are specially interested on the book written by Jorge Juan entitled *"Observaciones astronomicas y phisicas hechas de orden de Su Majestad en los reynos del Perú, de las cuales se deduce la figura de la Tierra y se aplica a la navegación"*, published in Madrid in the year 1748.

In this book we can find measurements of the obliquity of the ecliptic, the latitude of different places, eclipses of Jupiter satellites, dilatation and compression of metals, barometric heights

and many more. Chapter six of this work is devoted to the sound speed. Many questions worried to the studious of the sound in the XVIII century. Jorge Juan summaries these questions numbering the questions previously raised by Derham. We have listed these questions in the Table 1.

Table 1: Different questions about the sound and its speed, proposed by M. Derham in the XVIII century.

Number	Question
1	How long moves the sound in 1 second
2	If the sound speed is higher with a gunshot, for example, with the mouth of the gun to the observer or in the opposite direction.
3	If the sound speed changes according to the state of the atmosphere, especially with the atmospheric pressure.
4	If the sound speed changes from night to day.
5	The influence of the wind in the sound speed.
6	If the sound speed changes from calm to storm weather.
7	If the crosswind delays the sound speed.
8	If the sound speed is the same in summer that in winter.
9	If the sound speed changes with the snow.
10	If the sound speed changes according to its intensity.
11	If the sound speed of a gun changes with the degrees of elevation of this.
12	If the different types of powder affect to the sound speed.
13	If the sound speed changes with the height above the Earth surface.
14	If the direction and sense of the sound speed have any influence (up to down or down to up).
15	If all the types of sounds have the same speed.
16	If the sound speed changes during its displacement.
17	If the sound speed is constant along its displacement.
18	If the sound speed is the same in different places of the Earth.
19	If the sound moves in straight line and following the Earth curvature.

The Measurement Of The Sound Speed

Being in the region of Quito, the members of the expedition, who should measure a degree of the terrestrial meridian arc, decided to start a series of experiences about the sound speed. They thought to be in a good situation to answer the previously mentioned questions, as for instance, if the sound speed changes with the height above the Earth surface (question 13 of Table 1), and if the sound speed is the same in different places of the Earth (question 18 of Table 1). Jorge Juan expressed it with the following words:

> "To the thirteen because Quito is 1517 toesas above the Sea surface, and the Mercury raises in the Barometer no more until 20 inches 1 line [...] and to the eighteen because we are almost on the Equator" (Juan 1748, 135-136).

The first trial of the expeditionary was made when M. De La Condamine and Jorge Juan went to Lima. They put a cannon of four and a half feet long and eight to nine pounds of bullet on the top of a peak called Panecillo close to Quito. They wanted to hear the shot from the hill Pumbamarca (it is passed the village called Quinche) 19300 to 19400 toesas far away. However they never heard the shot there. They thought about the influence of the wind and left the experiment.

The second trial to measure the sound speed was made at 31st of August of the 1737. Godin and Jorge Juan went up to the peak called Pambarca. They checked with an eyeglass the position of the cannon (placed on…). Both observatories saw the two detonations, but they did not hear anything. The report of Jorge Juan says:

> "It came the time of the observation, that was made very carefully, and although two different flare-ups were seen, we did not perceive any shot. As the running wind was very soft, we attribute this defect to the many peaks and deeps which there were between the positions in that place, where the sound, without any doubt, was lost reflecting in the Quebradas with more than 100 toesas depth, and the high Montes, because the Pambamarca, where we were, had 883,5 toesas of height, counted from the flat ground where it raises. (Juan 1848, pp. 136-137)

After the failures of these two trials they decided to prove with shorter distances. The third set of experiments was made at 10th of July of 1738. The cannon was in the peak called Panecillo. Godin and Jorge Juan went to the property of the Augustine fathers, placed in an extreme of the Añaquito flat. Antonio de Ulloa and Bouguer went to the Saguanche property.

They used as clocks pendulums protected against the wind. It was thought to make five shots. The three first in the direction to Ulloa and Bouguer. The fourth one in the direction to Jorge Juan and Godin. In the last shot the cannon would be placed vertically. With respect to the meteorological conditions, we only know that in the Panecillo peak and the position of Ulloa and Bouguer there was not wind. From the position of Jorge Juan and Godin the wind speed was opposite to the propagation of the sound, with an approximate value of 2 Toesas per second. Jorge Juan also comments in his report that, rigurously, the light speed should be accounted too, to evaluate the time that light took to arrive to them, but in practice, they assumed the light speed as infinitum, quoting the work of Roemer about the observation of the Jupiter satellites.

Table 2: Measured times in half seconds since the vision of the flare-up until the perception of the shot sound.

Shots	Measures of Jorge Juan	Measures of Ulloa
1	65	76
2	66,5	76,5
3	66	77
4	66	77
5	66	76

In this trial, the observatory appreciated the sound produced by the gunshots and measured the time taken by the sound to reach their positions. Table 2 shows the different obtained results. With respect to the distances between the observation points, we can say that they were measured by geometric methods. In table 3 we can see these distances and the sound speeds obtained by them.

Table 3: Distance to the Panecillo peak and sound speeds determined by the different observatory.

Observatory	Distance	Sound speed
Jorge Juan	5736 Toesas	$173^{9}/_{11}$ Toesas/second
Ulloa	6820 Toesas	$178^{46}/_{153}$ Toesas/second

In February of 1744, Carlos María de la Condamine, another member of the expedition, repeated the experiments to determine the sound speed. He was in the Couru Mountains, and from there saw the gun shots of a cannon placed in Cayena (French Guayana nowadays). He obtained a result of 183,5 toesas per second.

DISCUSSION

To have and idea about these measurements, we are going to convert the given values in the description of Jorge Juan into International System units. The length unit used by the members of the expedition was the Toesa:

1 Toesa = 1,95040 m.

In Table 4 we have put the above-described measurements in International System units. We can see that the values of the sound speed obtained by the two groups of observatory are in the expected range.

Table 4: Measured values by the expeditionary and values of the sound speed in International System units.

	Jorge Juan	Ulloa
Distance	11187,5 m	13301,7 m
Time	33,0 s	38,3 s
Sound speed	339,0 m/s	347,3 m/s

Jorge Juan does not indicate, as usual at that time, the experimental errors of his measurements. However, assuming an error of $\pm$ 1 Toesas in the length and $\pm$ 0,5 seconds in the time, we can estimate an error in the sound speed of about $\pm$ 2,5 Toesas/s or, approximately, $\pm$ 4 m/s.

With respect to the measurement made by La Condamine in 1744, after changing the old units to the System International units, a value of 357,9 m/s is obtained. It seems that these results is a bit excessive, although we do not know the details and conditions of these experiments.

Epilogue

This determination of the sound speed made at 1738, gives us an idea about the level reached by the Physics and, in general, by the Spanish science in the Eighteen-century. The Spanish scientific works were numerous covering a wide spectrum of questions from the altitude determination of peaks by barometric methods (Gallego et al., 1999) until the discover of new chemical elements as the Platinum.

Finally, we also must say that in the book of Jorge Juan there is a last chapter about the application of these results of the sound speed to the Geometry (and Geodesy) and to the navigation. The author puts special interest in the utility of the knowledge of the sound speed to made, for example, map of harbours during naval attacks or, in general, to measure distance in open spaces.

REFERENCES

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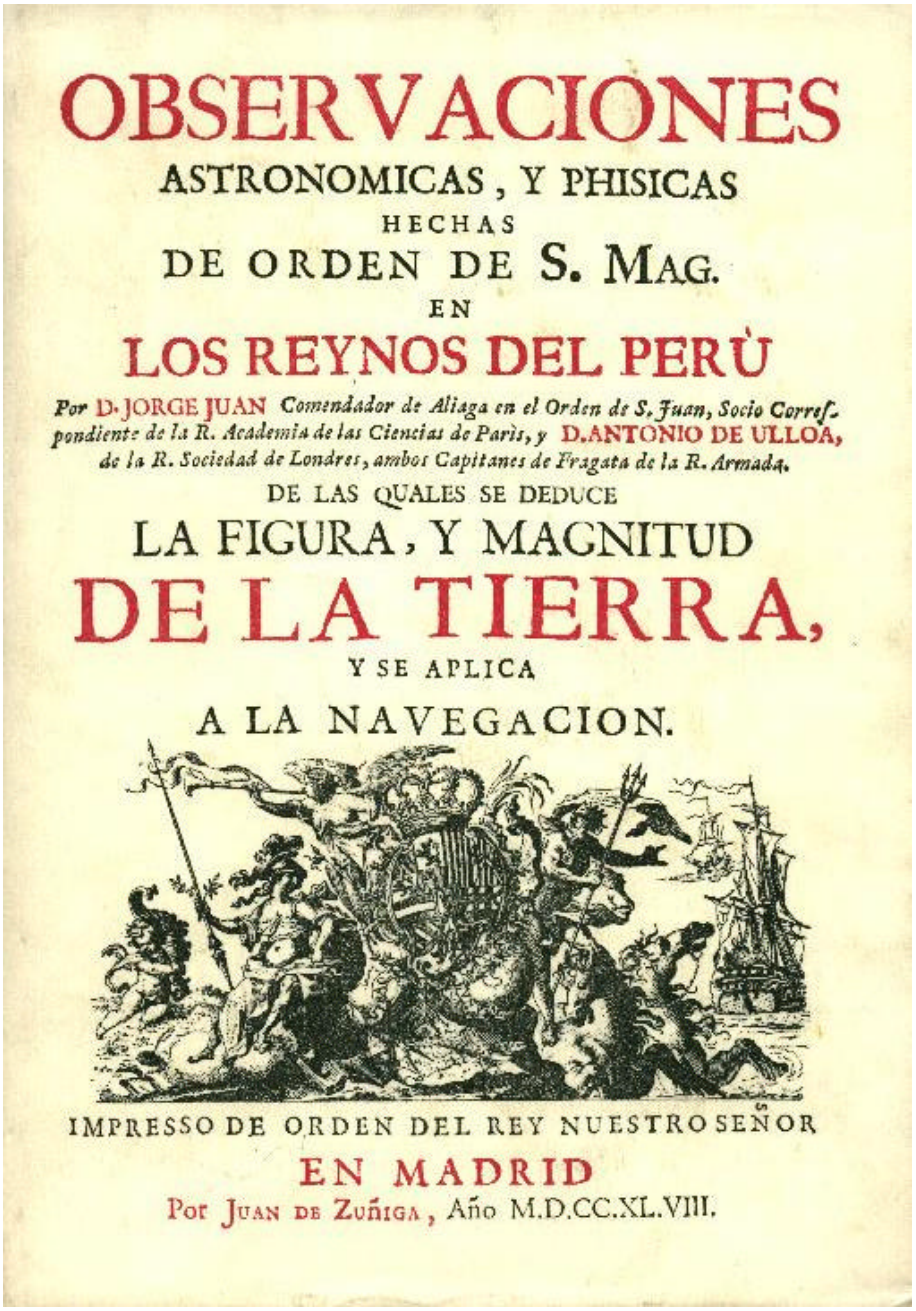

OBSERVACIONES

ASTRONOMICAS, Y PHISICAS

HECHAS

DE ORDEN DE S. MAG.

EN

LOS REYNOS DEL PERÙ

Por D. JORGE JUAN *Comendador de Aliaga en el Orden de S. Juan, Socio Correspondiente de la R. Academia de las Ciencias de Paris, y* D. ANTONIO DE ULLOA, *de la R. Sociedad de Londres, ambos Capitanes de Fragata de la R. Armada.*

DE LAS QUALES SE DEDUCE

LA FIGURA, Y MAGNITUD

DE LA TIERRA,

Y SE APLICA

A LA NAVEGACION.

IMPRESSO DE ORDEN DEL REY NUESTRO SEÑOR

EN MADRID

Por JUAN DE ZUÑIGA, Año M.D.CC.XL.VIII.

Figure 1: Front piece of the first edition of the Jorge Juan's work about the scientific expedition to Peru. Edited in Madrid in 1748.