ALTERATION OF THE STABILITY OF THE FXLMS ALGORITHM CAUSED BY THE USE OF SYNCHRONOUS SAMPLING IN ACTIVE NOISE CONTROL IN DUCTS

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ABSTRACT

We present, in this paper, a feedforward control strategy in a cylindrical duct to reduce the noise produced by an axial fan. We have had to realize a synchronous control to follow the rotation frequency drifts of the fan. Consequently, the different identification filters are implemented with variable sampling rates producing frequency and phase response errors**.** We are interested in the variation domain of the fan rotation speed which guarantees the FxLMS algorithm stability. We have foreseen the maximum sampling interval with a sinusoidal source. We present simulation and experimental results.

INTRODUCTION

Active Noise Control was invented by Paul Lueg, german physicist, in the 30's whose he presented the first patent in 1936 based on the superposition principle **[1] [2]**. In the 50th, the first adaptive algorithms appeared **[3]** but it had to wait 80th and Digital Signal Processor to realize the control in real time. In the case of ducts, notably, it is possible to reduce noise pollution using propagative wave properties that is to say, their modal structure. In our case, we consider a cylindrical duct with an axial fan which produces a two component noise that is to say a broadband noise and a tonal noise which emerges widely. We are interested in the reduction of the tonal noise for the two first frequencies f_0 and $2f_0$ corresponding to the fundamental and first harmonic of the noise associated to the rotation frequency of the fan.

1-THEORETICAL STUDY

1-1 Independant Modal Control Strategy

The first stage of noise reduction study produced by an axial fan has been carried out by L. TARDY **[4][5]**. He has succeed in the noise reduction using the modal structure of the propagated noise in the duct. He proceeded an adaptive control of the two first mode (0,0) and (1,0) with the implementation of the FxLMS algorithm in its classical and notch versions. The Independent Modal Control has been used. This strategy considers the propagative modes as independent. Then, an unique F.I.R filter W is associated to each mode. Because of modes in cylindrical duct are turning modes and to be able to detect (1,0) mode, we have had to decompose this mode in two orthogonal components which are coupled components. This has been done with secondary sources which produces counter-noise (modal decomposition) with residual error microphones (modal decomposition) **[6]**. Moreover, the global strategy uses a feedforward control whose reference signal is given by a Hall effect sensor placed on the fan. Then, the reference signal is unique and the system is a Single-Input - Multi-Output system **[6]**. A simulated study has been done by Chan **[7]** in the case of Multi-input Multi-output system to compare different control strategies without computation time problem.

<u>1-2 Synchronous Sampling</u>

To reduce the noise produced by the fan, the FxLMS algorithm has been implemented with synchronous sampling to follow the rotation frequency drifts of the fan. But some perturbations appeared. In effect, the use of FxLMS algorithm when we want to control multi-frequency signals is perturbed by the presence of time-varying terms **[8]**. Glover **[8]** has demonstrated that the algorithm does not converge to the Wiener-Hopf solution. There are different solutions to resolve this problem. Elliott **[9]** has observed that with synchronous sampling, the adaptation process is linear and invariant with time when there is only one frequency to control. The system is linear time-invariant when the order of the filter is an integer multiple of half the number of samples per cycle. Then with a notch controller, we must have four samples per cycle **[10]**. However, time-varying terms make the system converge to a dynamic solution if we only use two coefficients per frequency to control. Glover **[8]** shew that to reduce influence of time-varying we have to increase the order of the FIR adaptive filter. Clark and Gibbs **[10]** proposed a solution for higher-harmonic control. Higher-harmonic are synthetised from the fundamental and a separate reference signal is available for each adaptive filter independently. This has been done in the HLMS algorithm. Oversampling is also used to have at least 4 samples of the reference signal per cycle **[11]**. Then, Notch version of the FxLMS algorithm converge to the Wiener-Hopf solution without coefficient ondulations of the adaptive filter because of the orthogonality of the samples x(n) and x(n-1). So it is possible to employ FxLMS algorithm in its Notch version to control tonal noise, if different reference signals are oversampled. The synchronous control is then possible with a limited computation time.
Nevertheless, if the sample rate has so much important variations, the FxLMS algorithm diverge despite of all the precautions which have been previously evoked. The problem is based on the implementation of the identification filters at a specified sample rate to an other sample rate if there is no on line identification. When a digital filter (FIR or IIR) is used at different sample rates, errors appear in its phase and magnitude responses. It can be explained as a complex coefficient digital filter **[12]**. The phase and magnitude responses are shifted with a z-variable modification of the z-domain transfer function as shown in (2):

$$z \rightarrow z * e^{jw\Delta T} \quad \Leftrightarrow \quad H(z) \rightarrow H(z * e^{jw\Delta T}) \qquad (1)$$

Fig. 1 and 2 give an example of the the errors produced on the responses of a 5-order FIR digital filter for different sample rates.

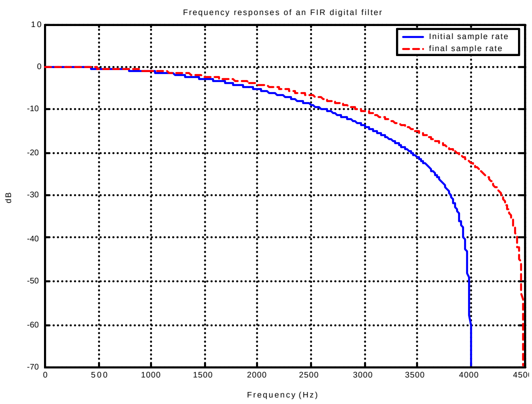

Fig.1- Frequency responses

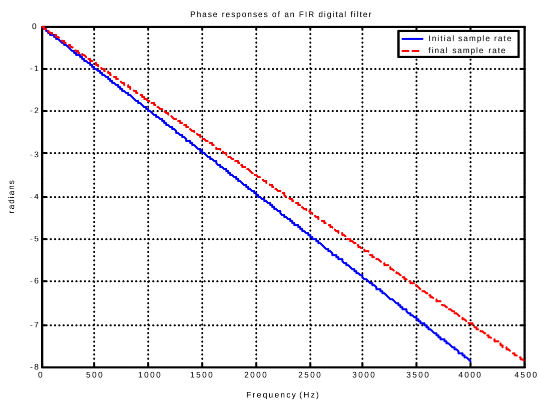

Fig.2- phase responses

As we can see on these figures, using different sample rate introduces an additional error between the real transfer function and the identification filters. This can have two consequences

on the FxLMS algorithm. The values of the control step are limited by the maximum eigenvalue of the filtered reference signal autocorrelation matrix. Snyder and Hansen **[13]** have shown that the maximum value of the convergence coefficient is altered by the error on the magnitude estimation. Moreover, the FxLMS algorithm can be made stable if the phase error is less than 90° (in absolute value) **[14]**. FxLMS algorithm implementation with synchronous sampling to control tonal noise can provoke algorithm divergence because of the increasement of the error in the estimation of the transfer function or with a fixed step-size which becomes too important.

<u>1-2 Prevision Of Divergence</u>

As it has been explain in the previous sections, variable sample rate can make FxLMS algorithm to diverge. Nevertheless, it can be possible to foresee divergence by using secondary real transfer function properties and so, identification transfer functions. The real transfer function between secondary sources, which produce counter-noise, and error microphones, which measure residual error, can be assimilated to a notch filter because of the analogic filters which isolates frequencies to control. This property is going to be exploited to try to establish the stability frequency domain of the FxLMS algorithm in the case of an identification with N coefficients. Zeros are principally complex-conjugated and near of the unit circle.

In a first time, let consider a digital filter with only 2 complex-conjugated zeros $Z_1=\exp(-i\theta)$ and $Z_1^*=\exp(i\theta)$ situated on the unit circle. The associated z-domain transfer function is given by with $z = \exp(i\omega T)$, $F=1/T$ sample rate :

$$H(z) = (1-z_1 z^{-1})(1-z_1^* z^{-1}) = 2\exp(-i\omega T)[\cos\omega T-\cos\theta] \qquad (2)$$

Now, consider another digital filter whose zeros are near of the unit circle that is to say $Z_2=\alpha\exp(-i\theta)$ and $Z_2^*=\alpha\exp(i\theta)$ with $\alpha^2 = 1+\varepsilon$ and $\varepsilon \ll 1$ so $\alpha \approx 1+ \varepsilon/2$. As precedently, the z-domain transfer function can be written as :

$$\hat{H}(z)=(1-z_2 z^{-1})(1-z_2^* z^{-1})=2\exp(-i\omega T)[\cos\omega T-\cos\theta][1+\frac{\varepsilon}{2}-i\frac{\varepsilon}{2}\frac{\sin\omega T}{\cos\omega T-\cos\theta}] \qquad (3)$$

$$=H(z)[1+\frac{\varepsilon}{2}-i\frac{\varepsilon}{2}\frac{\sin\omega T}{\cos\omega T-\cos\theta}]$$

The general idea is to express phase differences between transfer functions whose zeros are on the unit circle or near the unit circle for the different sample rates with a simple relation as :

$$\arg\hat{H}(z')-\arg\hat{H}(z)=k(\omega)[\arg H(z')-\arg H(z)] \qquad (4)$$

This is possible if we exploit relations (2) and (3), for two sample rates $F=1/T$ and $F+\Delta F=1/(T+\Delta T)$, by writing the transfer function ratios with $\cos\omega T \approx \cos\omega(T+\Delta T)$:

$$\frac{H(z)}{H(z')}=\left[\frac{\cos\omega T-\cos\theta}{\cos\omega(T+\Delta T)-\cos\theta}\right]\exp(i\omega\Delta T)\approx\exp(i\omega\Delta T) \qquad (5)$$

and

$$\frac{\hat{H}(z')}{\hat{H}(z)}=\frac{H(z')}{H(z)}\left[1-j\frac{\varepsilon}{2}\frac{\omega\Delta T\cos\omega T}{\left(1+\frac{\varepsilon}{2}\right)(\cos\omega T-\cos\theta)-i\frac{\varepsilon}{2}\sin\omega T}\right] \qquad (6)$$

Relation (6) expresses the ratio of transfer functions whose zeros are near of the unit circle in function of the ratio of transfer functions whose zeros are on the unit circle with a corrective term. This corrective term can be developed as a function of $\omega\Delta T$. At first order in ε :

$$C(\omega)=1-j\frac{\varepsilon}{2}\frac{\omega\Delta T\cos\omega T}{\left(1+\frac{\varepsilon}{2}\right)(\cos\omega T-\cos\theta)-i\frac{\varepsilon}{2}\sin\omega T}\approx\frac{1+\varepsilon-i\frac{\varepsilon}{2}\frac{\omega\Delta T\cos\omega T}{\cos\omega T-\cos\theta}}{1+\varepsilon} \qquad (7)$$

Combining expressions (6) and (7), taking argument with hypothesis that $\arctan x \approx x$, we have :

$$\arg \hat{H}(z') - \arg \hat{H}(z) = \arg H(z') - \arg H(z) + \arg C(w)$$
$$\approx -\left[1 + \frac{\frac{e}{2}\cos wT}{(1+e)(\cos wT - \cos q)} \right] w\Delta T \qquad (8)$$

With relation (8), we have found k(ω) which satisfies relation (5). This relation can be easily generalised for an N-order digital filter with N/2 complex-conjugated zeros. If k(ω) is plotted, fig.3, then we can observe this variable is a constant on more or less significant intervals in function of the digital filter order. If the study domain is known, it is possible to choose k(ω) = k to simplify the problem.

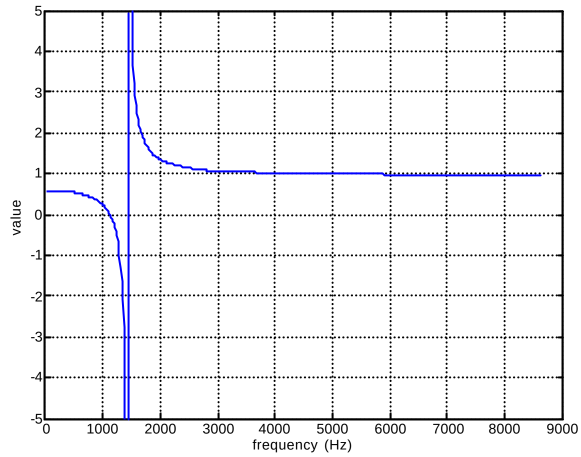

Fig.3- k(ω) corrective term for 2 complex-conjugated zeros

After having determined a simple relation between phase variation for a transfer function whose zeros are near of the unit circle and phase variation for a transfer function whose zeros are on the unit circle, it is now possible to express phase difference between real transfer function and identification transfer function for a given sample rate :

$$\arg H_{real}(z') - \arg H_{ident}(z') = \arg H_{real}(z') - \arg H_{ident}(z) - \left[\arg H_{ident}(z') - \arg H_{ident}(z)\right] \quad (9)$$

with variables z and z' for initial and final sample rates, H_{eal} and H_{ident} represent z-domain real and identification transfer functions which have zeros near of the unit circle. (9) can be simplified by using (8); by considering that the initial identification is perfect, that is to say $H_{real}(z)$ = $H_{ident}(z)$, and by supposing that real transfer function is the same for any sample rates.
So, we finally obtain :

$$\arg H_{real}(z') - \arg H_{ident}(z') = k\frac{N}{2} w\Delta T \quad \text{, N is identification order} \qquad (10)$$

Nevertheless, anti-aliasing and reconstruction filters have transfer functions which vary with sample rate. These phase response modifications can be assimilated as a p-order pure delay. Then, equation (10) can be completed :

$$\arg H_{real}(z') - \arg H_{ident}(z') = -wp\Delta T + k\frac{N}{2} w\Delta T \quad \text{, p is acquisition order p < N} \qquad (11)$$

FxLMS algorithm stability conditions implicate that (11) must be equal to ± Π/2 at divergence limits. Then it is possible to foresee the algorithm divergence as :

$$\pm\frac{p}{2} = \left[k\frac{N}{2} - p\right] w\Delta T \qquad (12)$$

This is equivalent to solve the 2-order following equation :

$$\pm\frac{1}{8F^2}\left[k\frac{N}{2}-p\right]\Delta F^2 \pm\frac{1}{8F}\left[k\frac{N}{2}-p\right]\Delta F-\frac{1}{4}=0 \quad (13)$$

ΔF is sample rate variation and F initial sample rate. Solutions of this equation are given by :

$$\Delta F=-\frac{F}{2}\pm F\sqrt{\frac{1}{4}\pm\frac{2}{\left[k\frac{N}{2}-p\right]}} \quad (14)$$

2-EXPERIMENTAL VERIFICATIONS

In a first time, we wanted to verify theoretical study by controlling noise produced by a compression chamber in a cylindrical duct using notch FxLMS algorithm version. We only consider the (0,0) mode, we obtained similar results for (1,0) mode. This experimental verification had to reproduce fan frequency drift by modulating the frequency of the signal which fed the compression chamber until FxLMS algorithm diverged. An example of the coefficient variations for a continuous sample rate evolution is given in fig.4. An initial off-line identification is realized with 100-order digital FIR filters. The control has been effected twice : with a large and small control step to show its influence. As we can see in tab.1 and 2, value of control step can make diverge FxLMS algorithm with a phase error between real and identification transfer functions different from $\pm\pi/2$. Reduce step size guarantees divergence for phase difference of $\pm\pi/2$. There exist some differences between frequency intervals given by theoretical relations and experimental results. The reason is that identification phase is not perfect, initial phase error is different of zero as shown in fig.5.

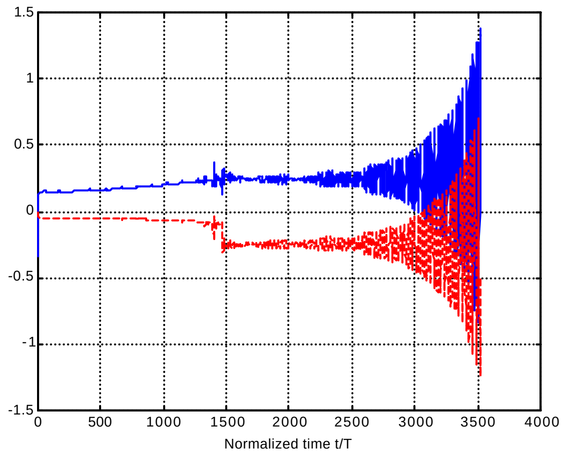

Fig.4- Control coefficient variations

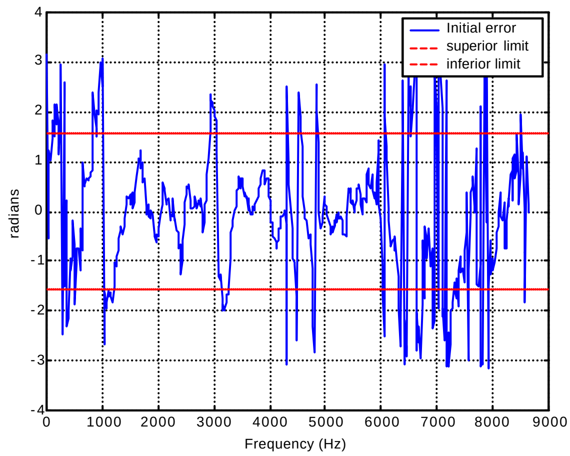

Fig.5- Identification error variations

Control step µ	$\Delta F_{exp} > 0$	$\Delta F_{exp} < 0$	$\Delta F_{theor.} > 0$	$\Delta F_{theor.} < 0$
0.01	**504 Hz**	**-492 Hz**	**586 Hz**	**-630 Hz**
0.00005	**648 Hz**	**-576 Hz**	**553 Hz**	**-638 Hz**

Tab.1- Frequency limits given by FxLMS algorithm stability

Control step µ	$\Delta\theta_{theor.}$ ($\Delta F > 0$)	$\Delta\theta_{exp.}$ ($\Delta F > 0$)	$\Delta\theta_{theor.}$ ($\Delta F < 0$)	$\Delta\theta_{exp.}$ ($\Delta F < 0$)
0.01	**-90°**	**-64°**	**90**	**95°**
0.00005	**-90°**	**-88°**	**90**	**97°**

Tab.2- angular limits given by FxLMS algorithm stability

3-CONCLUSION AND PERSPECTIVES

We have found a theoretical expression to foresee divergence of the FxLMS algorithm in its Notch version with an N-order identification FIR filter. Experimental results are coherent with theory but differences exist because of initial error between real and identification transfert functions. The solution would be to have FIR filters with an order more important but we are limited by computation time. Now, we are interesting in the real case with axial fan. We are working on the divergence prevision with variable air flow which modifies phase response of the real transfert function.

BIBLIOGRAPHICAL REFERENCES

[1] D. Guicking, "On the invention of active noise control by Paul Lueg", J. Acoust. Soc. Am, may 1990, 87(5).

[2] P.A. Nelson and S.J. Elliott, "Active control of sound", Academic Press, 1992.

[3] S. Haykin, "Adaptive filter theory", Prentice Hall Inter. Editions, 1996, third edition.

[4] L. Tardy , "Contrôle actif du bruit de ventilateur en conduit dans le domaine aéronautique", Ph.D. thesis, Université Paul Sabatier, Toulouse, France, December 2000.

[5] L.Tardy, E. Gonneau, J.P. Guilhot, "Design and implementation of algorithms for active control of multi-modal noise in ducts", Internoise 2000 : the 29th International Congress and Exhibition on Noise Control Engineering, Nice, August 2000.

[6] J. Hernandez, L. Tardy, E. Gonneau, J.P. Guilhot, "Modal Active Noise Control in Ducts : implementation of different algorithms", Internoise 2001, The Hague Holland, August 2001.

[7] T.M. Chan, "Active Control of Sound in Ducts", Ph.D. thesis, University of Southampton, September 1997.

[8] J.R. Glover, "Adaptive noise cancelling applied to sinusoidal intreferences", IEEE Trans. Acoust. Speech. Signal Processing , ASSP-35, 1977, p 484-490.

[9] S.J. Elliot, I.M. Stothers, P.A. Nelson, "A Multiple Error LMS Algorithm and Its Application to the Active Control of Sound and Vibration", Trans. On Speech and Signal Processing, vol. ASSP-35, N°10, October 1987, p 1423-1434.

[10] R.L. Clark, G.P. Gibbs, "A novel approach to feedforward higher-harmonic control", J. Acoust. Soc. Am., 96(2), Pt.1, August 1994, p 926-936.

[11] S.M Lee, H.J. Lee, C.H. Yoo, D.H. Youn, I.W. Cha, "An active noise control algorithm for controlling multiple sinusoids", J. Acoust. Soc. Am., 104(1), July 1998, p 248-254.

[12] M.J. Mismar, I.H. Zabalawi, "Complex Coefficients FIR Digital Filters", Circuits Systems Signal Process, vol. 13, N°5, 1994, p 591-600.

[13] S.D. Snyder, C.H. Hansen, "The Effect of Tranfer Function Estimation Errors on the Filtered-X LMS Algorithm", IEEE Trans. On Signal Processing, vol. 42, N°4, April 1994, p 950-953.

[14] S.D. Snyder, C.H. Hansen, "The influence of transducer transfer functions and acoustic time delays on the implementation of the LMS algorithm in active noise control systems", Journal of Sound and Vibration, 141(3), 1990, p 409-424.