# ESTIMATED ECHO PULSE FROM OBSTACLE CALCULATED BY FDTD FOR AERO ULTRASONIC SENSOR 

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#### Abstract

Recently, the aero-acoustical sensor is utilized in the car because it is effective to detect obstacles behind a car. The improvement of its performances, however, is literally a continuous process of trial and error. In this paper, the detection characteristic of an ultrasonic sensor is analyzed used by Finite Difference Time Domain method. The reflected pulse from an obstacle is calculated using a short pulse of 40 kHz . The amplitude and propagation time of the reflected pulse from obstacle changed with its height and outer shape.


## INTRODUCTION

Aerial back sonar of a car is very useful to detect the obstacles in the rear of an automobile. However, it is very difficult to predict receiving pulse waveforms from obstacles, because aerial back sonar systems are used in much kind of circumstances. Because of the limitation of the car design, the outer size of aerial back sonar is almost fixed. The new calculation method has been desired to predict the detection ability of sonar to obstacles. In this paper, the Finite Difference Time Domain (FDTD) method is proposed to calculate the reflection characteristics of the back sonar system. The FDTD calculation method that is common in the electromagnetic field [1] has ability to calculate an instantaneous pressure of a sound pulse. The recent development of computer system enables FDTD method to be applied in the acoustics filed although it requires the huge computer resources.[2] To confirm the validity of the FDTD method to a back sonar system, the receiving waveforms from the targets were calculated as changing target's height. A snap shot of propagation pulses from three different targets was obtained because the FDTD was capable of calculating the instantaneous sound pressure along the propagation of pulse. The large difference of echo signals was obtained for the difference of target's shapes.

## FINITE DIFFERENCE TIME DOMAIN METHOD

## Formulation of FDTD

The basic equations of the FDTD method, that is taking account of attenuation, are given as follows [3]:

$$
\begin{align*}
& -\frac{1}{\mathrm{~K}} \frac{\partial p}{\partial t}=\frac{\partial v_{x}}{\partial x}+\frac{\partial v_{y}}{\partial y}  \tag{1}\\
& -\rho \frac{\partial v_{x}}{\partial t}=\frac{\partial p}{\partial x}+\eta v_{x}  \tag{2}\\
& -\rho \frac{\partial v_{y}}{\partial t}=\frac{\partial p}{\partial y}+\eta v_{y} \tag{3}
\end{align*}
$$

where $p$ is sound pressure, $v$ is the particle velocity, $K$ is the bulk modulus, $\rho$ is the density and $t$ is time. The second part of the right hand side in eqs. (2) and (3) show an attenuation of the medium cased by absorption

$$
\begin{align*}
& \frac{\partial^{2} P}{\partial x^{2}}+\left(\frac{\omega^{2}}{c^{2}}-j \frac{\omega \eta}{\rho c^{2}}\right) P=0  \tag{4}\\
& P=P_{0} \exp \left[-j\left(\gamma_{1}-j \gamma_{2}\right) x\right] \tag{5}
\end{align*}
$$

where $P_{0}$ is the constant and $\gamma_{1}$ and $\gamma_{2}$ are the wave number and attenuation constant, respectively. The velocities of sound $c$ and resistance coefficient $\eta$ are obtained where $\omega$ is angular frequency. In this paper, the resistance coefficient that is proportional to the particle velocity is ignored because of low attenuation in air.

$$
\begin{align*}
& c=\omega / \sqrt{\gamma_{1}^{2}-\gamma_{2}^{2}}  \tag{6}\\
& \eta=\frac{2 \gamma_{1} \gamma_{2}}{\sqrt{\gamma_{1}^{2}-\gamma_{2}^{2}}} \rho c \tag{7}
\end{align*}
$$

The finite differential equations are obtained as a function of discrete positions $x, y$ in space and a discrete time $t$ as shown below. [8]

$$
\begin{align*}
& p^{n}(i, j)=p^{n-1}(i, j) \\
& \quad-C_{p}^{\left[v_{x}^{n-1 / 2}(i+1 / 2, j)-v_{x}^{n-1 / 2}(i-1 / 2, j)+v_{y}^{n-1 / 2}(i, j+1 / 2)-v_{y}^{n-1 / 2}(i, j-1 / 2)\right],} \\
& v_{y}^{n+1 / 2}(i, j+1 / 2)=C_{v 1} \cdot v_{y}^{n-1 / 2}(i, j+1 / 2)-C_{v 2}\left[p^{n}(i, j+1)-p^{n}(i, j)\right] \\
& v_{x}^{n+1 / 2}(i+1 / 2, j)=C_{v 1} \cdot v_{x}^{n-1 / 2}(i+1 / 2, j)-C_{v 2}\left[p^{n}(i+1, j)-p^{n}(i, j)\right] \tag{8}
\end{align*}
$$

where $C_{p}=c^{2} \Delta t / \Delta x$. In these equations, superscripts show the time and $i$ and $j$ are the grid-numbers in the $x$ and $y$ directions in space, respectively. For simplification, $\Delta x=\Delta y$ in this paper. FDTD requires satisfying the next Curant's equation for the stability on calculation.

$$
\begin{equation*}
c \Delta t \leq \frac{1}{\sqrt{\left(\frac{1}{\Delta x}\right)^{2}+\left(\frac{1}{\Delta y}\right)^{2}}} \tag{9}
\end{equation*}
$$

## CALCULATION RESULTS

## Amplitude of reflected pulse from the square

## target in air

The reflected sound waveforms were calculated in heterogeneous temperature that changed from 30 to 50 degrees like at the summer time. It was assumed that the sound speed changed from $349 \mathrm{~m} / \mathrm{s}$ at the ground surface to $361 \mathrm{~m} / \mathrm{s}$ at 70 cm above the earth. The square function of height was also assumed in the sound speed profile.

Back sonar placed at 50 cm above the earth projected a pulse sound of 40 kHz as shown in Fig.1.


Fig. 1 Sound filed of sensor.

Calculated sound source was assumed to be composed of 25 discrete point-sources on the calculating grids. Gaussiun weighting function was assumed to the vibrating velocity of sound source. The objective target was place at $x=1.5 \mathrm{~m}$ and the earth was rigid. For the accurate calculation, calculation increments in space $\Delta x=\Delta y=0.8 \mathrm{~mm}$ and in time $\Delta t=0$. Mur's first order absorbing boundary conditions were provided to eliminate the reflection wave from the outer boundaries of the calculation space. Figure 2 show the receiving echo signals from the target as a function of its height. The reflected pulse was composed of two pulses. The amplitude of first echo generated by the upper-left corner of the target or surface of


Fig. 2 Receiving waveforms as a function of height the target increased with target's height as shown in Fig.2. The amplitude of second pulse generated by the corner of the target and the earth was about -30 dB to the projected pulse. It had almost the same amplitude and propagation time $t=10.2 \mathrm{~ms}$, even thought the height of target was changed from 0.2 to 0.7 m . The amplitude and propagation time changed a little with temperature variation. The maximum amplitude of the first echo decreased with temperature. When the temperature was 50 degrees, traveling time to and from the target became 0.1 ms shorter than at 30 degrees. This results shows that aerial back sonar can detect the objects whose height is over 0.1 m using the second reflected pulse.

## The difference of waveforms from different object's shape

The calculation was executed for the there kind of object's shape because aerial back sonar must practically detect many types of objects. The objects were place at 1.5 m from the sound source as mentioned above. Figure 3 shows the snap shot of propagating pulses of three different objects. Not only reflected pulses but also diffracted pulses are clearly shown in this figure. The height of objects was fixed at 20 cm because we can detect the rectangle object using the second pulse. Figure 4 shows the receiving waveforms of the three objects. The first pulses of objects decreased steeply except for object1 and disappeared in the case of object 3. The amplitude of the second pulse is also very small or one hundredth in the case of objects 3 . This results shows that it is very difficult to detect the slant objects by aerial back sonar.


Fig. 3 Snapshot of the reflected pulse

## CONCLUSIONS

The FDTD method calculated the receiving waveform and propagating snap shot of aerial back sonar. It projected a 40 kHz pulse whose pulse-width was about 0.15 ms . We obtained the reflected echo signal from the target as a function of its height. Aerial back sonar detects the rectangle target whose height is over 0.1 m . The amplitude and propagation time of the reflected echo pulse changed a little with temperature variation. Te difference of receiving waveform of three types of objects was also calculated. The snap shot of sound propagation was also shown in the figure. These results show the validity of the FDTD method for estimation of receiving waveform.

## REFERENCE

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Fig. 4 Receiving pulse waveforms from three kind of object.


Fig. 5 Receiving pulse amplitude $v s$ object's height in case Object 2.

