DEFINITION AND FIRST VALIDATION OF A NEW MATHEMATICAL MODEL OF POLYESTER FIBRE MATERIALS

PACS: 43.55.Ev

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ABSTRACT

A new semi-empirical model has been developed by the authors to predict the flow resistivity, acoustic impedance and sound absorption coefficient of polyester fibre panels. Calculated results are compared with normal incidence measurements carried out using two different techniques: the transfer-function method in an impedance tube (ISO 10534-2) and the free-field impulse response method (ISO 13472-1). Measurements performed on polyester fibre materials with different densities and thicknesses are in good agreement with the predictions of the new model.

INTRODUCTION

Polyester fibre materials are an innovative class of products, quickly becoming of widespread use as sound absorber; in particular, they are more and more used to replace glass wool and rock wool where it is required to keep the air absolutely free of fibres suspected to have an influence on human health. On the other hand, scientific literature is lacking of studies on the physical and acoustical characteristics of polyester fibre materials; no specific model exists. One of the few works devoted to the acoustical behaviour of such materials [1] puts in evidence the well known models developed for glass wool and rock wool when are not well suited for polyester fibres and invites to develop new correlations for this kind of materials. It is not easy to find other studies published at international level. Therefore, the main goal of the present work was the development of a new mathematical model of flow resistivity, acoustic impedance and sound absorption coefficient of polyester fibre materials and the verification of its reliability.

THE POLYESTER FIBRE SAMPLES

The polyester fibre material investigated in the present work is distributed with the trade mark Fiberform® as panels with different density, thickness, composition and surface treatment. It is constituted by a mix of two different kind of fibres:

- fibres of polyethilenterephtalate (between 70% and 80%, depending on the material type: T or TE);
- "bicomponent" fibres constituted by a core of polyethilenterephtalate and a lining of copolyester (between 30% and 20%, depending on the material type: T or TE).

The mix of fibres is thermally treated at 150 °C in order to melt the external lining of "bicomponent" fibres and form a skeleton of thermally bound fibres.
The circular section of the fibres has a diameter between 17,9 μm and 47,8 μm (assumed mean value: 33 μm) and a mean length of 55 mm. It is worth noting that this diameter is considerably greater than the glass wool fibres diameter, usually ranging between 1 μm and 10 μm; this is one of the physical factors causing a different acoustical behaviour of the two kind of materials.
for the present work, 38 polyester fibre panels has been used, differing for density (ranging from 10 kg/m^3 to 120 kg/m^3) and thickness (ranging from 10 mm to 120 mm).

THE MATHEMATICAL MODELS

In the scientific literature, three kind of mathematical models can be found: semi-empirical, phenomenological and microstructural models. The basic idea of the present work was to define a model depending on few measurable physical parameters, in order to use it to effectively support engineering design, requiring in input only data commonly available in literature or easily measurable. Therefore, a semi-empirical model was adopted where the key parameter is the airflow resistivity of the material, a quantity which can be measured in laboratory using a dedicated device [2].
As a first step, the characteristics of the 38 polyester fibre samples were measured in the laboratory of the Engineering Department of the University of Ferrara. The flow resistance was measured using the alternate airflow method described in the EN 29053 standard [2]; the sound absorption coefficient was measured using the transfer-function method in an impedance tube described in the ISO 10534-2 standard [3].
The second step was the comparison of the measured values with the forecasting of some mathematical models for fibrous materials very often quoted in scientific works [6, 7, 8]. The results were quite poor; in particular the predicted flow resistivity values were sistematically underestimated.
The third step was the development of a new set of semi-empirical equations fitted to the measured values.

2.1 New Model for Resistivity (NMR)

The Bies-Hansen model [6] allows the calculation of airflow resistivity values starting from the apparent density of the fibrous material. Unfortunately, for polyester fibre materials it gives a large underestimation of measured values (see figure 1). Probably, this is due to the great difference between the actual diameter of polyester fibres ((33μm) and the mean diameter of glass wool fibres (1-10μm) on which the Bies-Hansen model was fitted.
The NMR model was developed keeping the basic structure of the Bies-Hansen model [6]:

$$r d^2 \rho_m^{-K1} = K2 \qquad (1)$$

and modifying the values of the coefficients K1 and K2. Here *r* is the airflow resistivity (Ns/m^4), ρ_m is the apparent density (Kg/m^3) and *d* is the mean fibre diameter (m). The new values were found by least–squares best–fitting the calculated values with equation (1) on the measured values (the assumed value for the mean diameter, *d*, was 33 μm, as indicated by the manufacturer of the materials). Table 1 reports the new values of K1 and K2 for polyester fibre materials and the Bies-Hansen values for glass wool [6].

Model	***K1***	***K2/10^{-9}***
NMR	1,404	28,302
Bies-Hansen	1,53	3,18

Table 1. values of K1 and K2 for polyester fibre materials (NMR, present work) and Bies-Hansen values for glass wool [6].

As can be seen in figure 1, the NMR model can predict with a quite good accuracy the airflow resistivity of polyester fibre materials as a function of their apparent density. The mean deviation of calculated values from measured values is 9,8%. A more detailed analysis separating the various kinds of material didn't reveal a measurable influence of the binder fibres percentage or the surface smoothing treatment.

2.2 New Model for Impedance (NMI)

The predicting model for sound absorption coefficient (at normal incidence) has been derived from the well-known Delany-Bazley model [7], as optimised for different materials by other authors [8, 9]:

$$\boldsymbol{a} = \left(\frac{2pf}{c_o}\right)\left[C5\cdot\left(\frac{\boldsymbol{r}_o f}{r}\right)^{-C6}\right] \quad (4) \qquad Z_R = \boldsymbol{r}_o c_o\left[1 + C1\cdot\left(\frac{\boldsymbol{r}_o f}{r}\right)^{-C2}\right] \quad (2)$$

$$\boldsymbol{b} = \left(\frac{2pf}{c_o}\right)\left[1 + C7\cdot\left(\frac{\boldsymbol{r}_o f}{r}\right)^{-C8}\right] \quad (5) \qquad Z_I = -\boldsymbol{r}_o c_o\left[C3\cdot\left(\frac{\boldsymbol{r}_o f}{r}\right)^{-C4}\right] \quad (3)$$

where Z_R e Z_I are the real and imaginary parts of the specific acoustic impedance Z and $\boldsymbol{a}$ and $\boldsymbol{b}$ the real and imaginary parts of the propagation constant g.
As literature lacks of a specific model of this kind for polyester fibre materials, it has been defined in the present work letting the eight coefficients C1, …, C8 in equations (2)-(5) vary and finding their optimal values by least-squares best-fitting the values of the sound absorption coefficient, calculated from the above acoustic impedance, on the measured values in the standing wave tube. The resulting model is optimised on the 38 polyester fibre samples.
As previously done for the NMR model, a more detailed analysis separating the various kinds of material has been conducted, but it didn't reveal a measurable influence of the binder fibres percentage or the surface smoothing treatment.
Table 2 shows the values of the eight coefficients of the new NMI model for polyester fibre materials compared with the values found by Delany-Bazley [7] and Dunn-Davern [8].
Table 3 shows the mean deviation between the measured values of the sound absorption coefficient and the calculated values using the NMI, the Delany-Bazley and the Dunn-Davern models.

Model	***C1***	***C2***	***C3***	***C4***	***C5***	***C6***	***C7***	***C8***
Delany-Bazley	0,057	0,754	0,087	0,732	0,189	0,595	0,098	0,700
Dunn-Davern	0,114	0,369	0,099	0,758	0,168	0,715	0,136	0,491
NMI	0,078	0,623	0,074	0,660	0,159	0,571	0,121	0,530

Table 2. Values of the eight coefficients of the new NMI model for polyester fibre materials compared with the values found by Delany-Bazley [7] and Dunn-Davern [8].

Model	***Mean deviation of sound absorption coefficient*** $\boldsymbol{a}$
Delany-Bazley	0,047
Dunn-Davern	0,039
NMI	0,031
MI	0,030

Table 3. Mean deviation between the measured values of the sound absorption coefficient and the calculated values using the NMI, the Delany-Bazley and the Dunn-Davern models.

2.3 Integrated Model (MI)

The Integrated Model MI is the final result of the study conducted on the polyester fibre materials. As seen above, the NMR model can predict the airflow resistivity as a function of the apparent density and the NMI model can give the specific acoustic impedance and the propagation constant as a function of the airflow resistivity (the panel thickness *l* is understood). Hence the sound absorption coefficient can be easily obtained using the well known formulae:

$$Z_1 = (Z_R + jZ_I) \cdot \left[\coth\,(\alpha + \mathrm{j}\beta) \cdot l\right] \qquad (6) \qquad \alpha_n = \frac{4Z_{IR} \cdot \rho_0 c_0}{|\,Z_I\,|^2 + 2\,\rho_0 c_0 Z_{IR} + \left(\rho_0 c_0\right)^2} \qquad (7)$$

The whole set of equations (1)-(7), called the Integrated Model MI, can therefore describe the acoustical characteristics of the material knowing only the apparent density and the thickness. It is very useful because polyester fibre panels are manufactured with many combinations of density and thickness and measured values of the sound absorption coefficient or the airflow resistivity are generally not available.
Figure 2 shows the noticeable difference between the sound absorption coefficient values of polyester fibre panels calculated using the MI model and other models from literature [6, 7, 8]. The evident underestimation of the sound absorption coefficient is clearly due to the underestimation of the airflow resistivity when using the Bies-Hansen model, developed for other kind of absorbers, also for polyester fibre materials.

FURTHER EXPERIMENTAL VALIDATION: FIRST RESULTS

In order to obtain an independent check of the mathematical model MI, additional measurements have been planned on polyester fibre materials not included in the original set of samples used for the development of the new model.
The first measurements have been already done on samples having an apparent density of 30, 40 and 60 kg/m^3 and a thickness ranging from 40 to 50 mm. They have been done using two different techniques: the transfer-function impedance tube method [3] and the free-field impulse method [10]. Figure 3 shows the comparison for the 60 kg/m^3, 50 mm sample: a good correlation is found between the values measured with the two different techniques and those calculated using the integrated model MI.
It should also be remarked that the free-field impulse measurements were performed conforming strictly to the ISO 13472-1 specifications: this implies that valid measurements can be obtained in the one-third octave bands between 250 Hz and 4 kHz. Further measurements are on going with different set up, allowing to get valid measurements in a broader frequency range.

CONCLUSIONS

A new semi-empirical model has been developed for predicting the flow resistivity, acoustic impedance and sound absorption coefficient of polyester fibre materials. The whole set of equations, called the Integrated Model MI, can describe the acoustical characteristics of the panels knowing only their apparent density and thickness.
The model has been developed by best-fitting the calculated values on the measured values of the relevant physical parameters for a set of 38 samples. A further validation is started, using not only the transfer-function method in an impedance tube (ISO 10534-2) but also the free-field impulse response method (ISO 13472-1).
The MI model is a simple tool that can be used by manufacturers and noise control engineers when detailed experimental information is not available.

AKNOWLEDGEMENT

The authors would like to thank ORV S.p.A., Padova, Italy, who kindly offered the Fiberform© samples used in the experiments.

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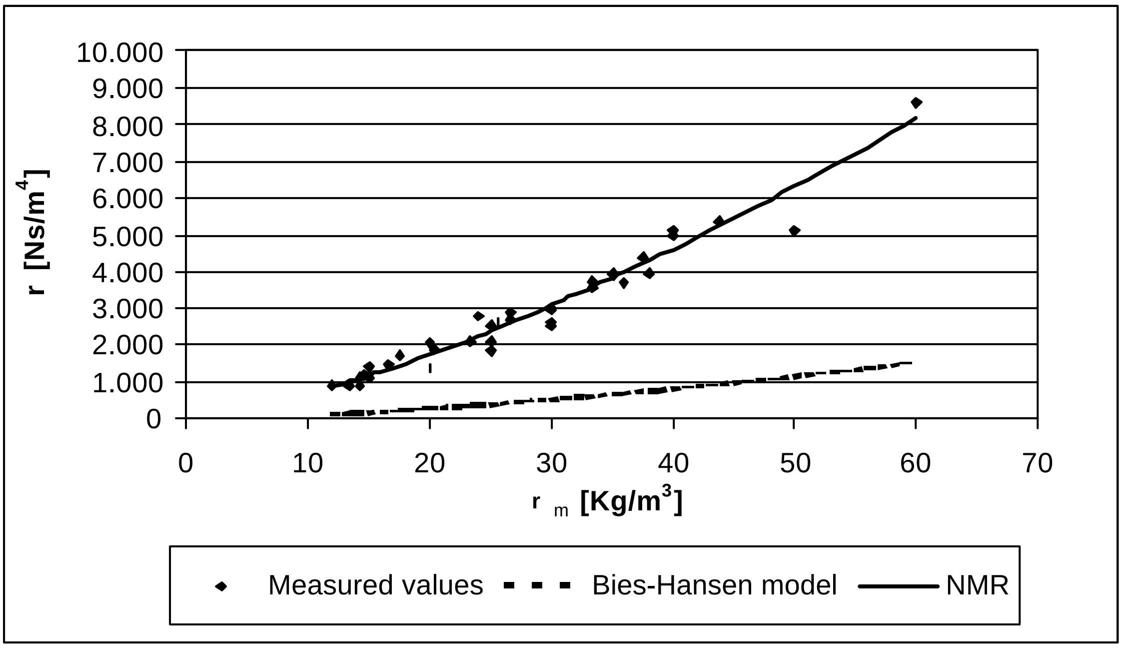

Figure 1. Airflow resistivity as a function of apparent density: comparison between measured values and predicted values using the Bies-Hansen model and the NMR model.

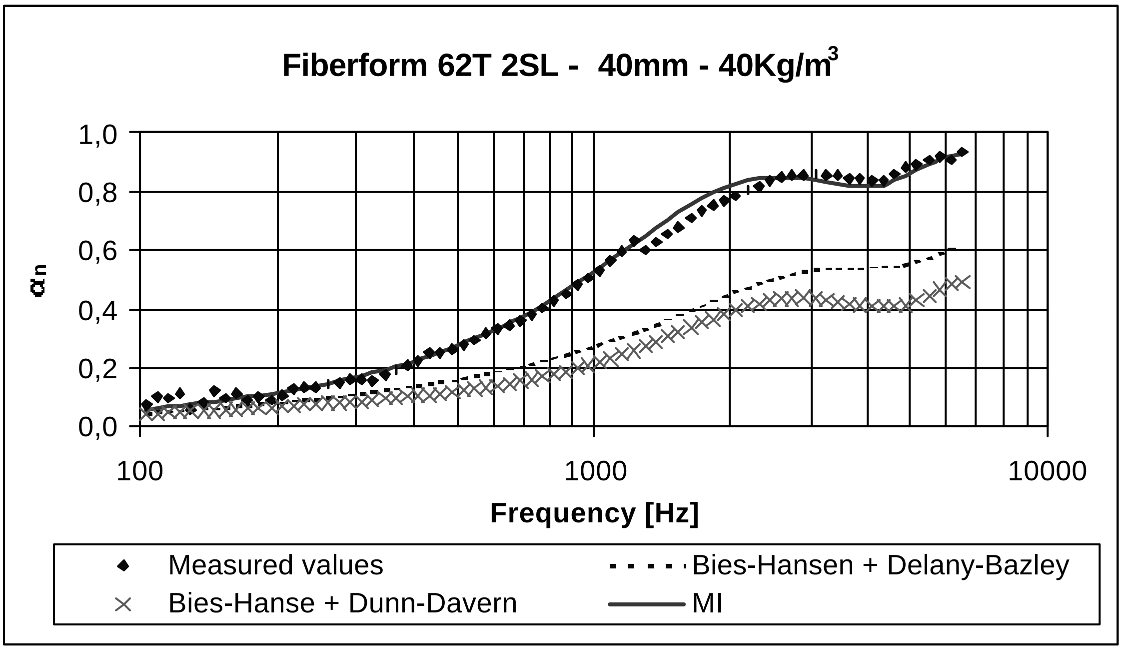

Figure 2: Sound absorption at normal incidence: comparison between measured values and predicted values from data of thickness and apparent density, using three mathematical models.

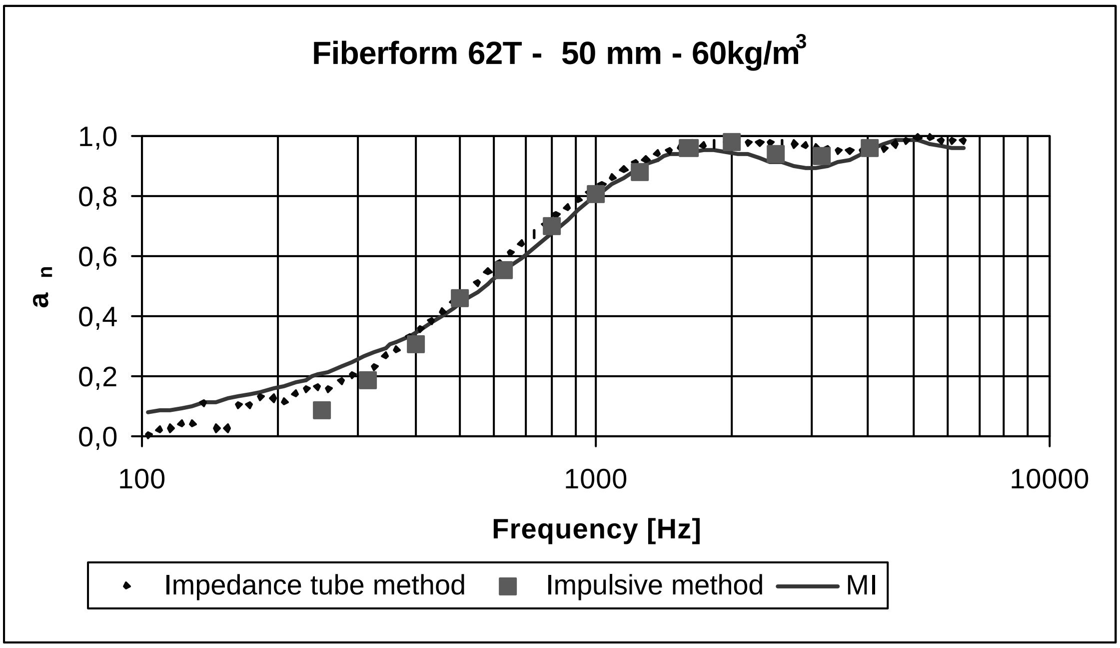

Figure 3: Sound absorption values at normal incidence: comparison between measured values using the impedance tube method and the impulse method and calculated values using the MI model.