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THE USE OF ISO-PARAMETRIC FINITE ELEMENTS IN ANALYSIS OF SOUND PROPAGATION NEAR A RIDGE

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INTRODUCTION

Iso-parametric finite elements were used for evaluation of sound field created by low sound source in the vicinity of a ridge. The geometry of this specific problem is illustrated in figure 1. The situation presented in the figure creates a complicated sound field in the surrounding of the ridge, especially at low frequencies. This near field effect will be treated here. The problem was already examined in the past, and the relevant sound fields were obtained analytically in previous research works, where the mathematical investigation included approximate models and assumptions [1,2,3]. Hence, it was assumed here that some special phenomena near the ridge could be overlooked, due to the complicated structure of the incident wave scattering and interference phenomena. Hence it was thought that that it might be useful to apply the aforementioned finite elements technique for investigation of the local effects.

The design of the finite elements was preceded by the formulation relevant to the acoustic problem, including the wave equation and the impedance boundary conditions. An approximate boundary condition was assumed between the investigated finite domain and the rest of the half space, where only the air impedance normal to the artificial boundary was considered. This assumption simplifies very much the analysis. However,it may cause some error. If this of boundary condition is assumed far enough from the area of interest, the error may be ignored. Hence, taking into account the limitation of the numerical technique in "covering" large areas, the boundary with the half space was put as far as possible from the ridge and the source. The iso - parametric finite element technique is a necessary link between the "classical" solution approach and the more efficient techniques of the boundary element method. Its presentation is straight forward and it yields directly the sound pressure field.

FORMULATION

Within the domain of the problem, Helmholtz equation:

$$\nabla^2 p + \left(\frac{\omega}{c_a}\right)^2 p = 0$$

should be satisfied. The boundary conditions are:
A rigid boundary:

$$\frac{\partial p}{\partial n} = 0 \; : \; S_1$$

Impedance boundary condition:

$$\frac{\partial p}{\partial n} = -i\rho_a \omega \frac{p}{Z_s} \; : \; S_2$$

Air impedance boundary condition:

$$\frac{\partial p}{\partial n} = -ikp; \; k = \frac{\omega}{\rho_a}; \; Z_s = \rho_a c_a$$

In addition, a control surface area is defined around the source, where the sound pressure, p, is specified.

APPLICATION OF VARIATIONAL DEFINITIONS FOR EXPRESSION OF THE EQUATIONS OF MOTION BOUNDARY CONDITIONS, USING THE FINITE ELEMENT METHOD .

The method consists of combining the equation of motion and boundary conditions into a descrete set of equations with descrete sound pressure unknowns. The theory begins with multiplying the equation of motion and boundary conditions by the variation of the acoustic pressure, p, and integration of these terms within the domain of the problem and along the relevant boundaries:

$$\int_A \left[\nabla^2 p + \left(\frac{\omega}{c_a}\right)^2 p \right] \delta p dA - \int_{S_1} \frac{\partial p}{\partial n} \delta p dS_1 - \int_{S_2} \left[\frac{\partial p}{\partial n} + i\rho_a \omega \frac{p}{Z_s} \right] \delta p dS_2 = 0$$

After some algebraic manipulations the following equation is obtained:

$$\delta \left\{ 0.5 \int_A \left[\nabla^2 p - \left(\frac{\omega}{c_a}\right)^2 p^2 \right] dA + 0.5 \int_{S_2} \left[i\rho_a \omega \frac{p^2}{Z_s} \right] dS_2 \right\} = 0$$

Now, using the definitions:

$$\int_A (\nabla p)^2 dA = \{p\}^T [K] \{p\}$$

$$\frac{1}{c_a^2} \int_A p^2 dA = \{p\}^T [M] \{p\}$$

$$\int_{S_2} \rho_a \frac{p^2}{Z_s} dS = \{p\}^T [D] \{p\}$$

yields in turn the condition:

$$[K] - \omega^2 [M] + i\omega [D] = 0$$

For each element it will be assumed that:

$$p = [N(x,y)]_e \{p\}_e$$

where,

$$\int_{A_e} (\nabla p)^2 dA = \{p\}^T_e [k]_e \{p\}_e$$

$$\frac{1}{c_a^2} \int_{A_e} \frac{p^2}{\omega^2} dA = \{p\}^T_e [m]_e \{p\}_e$$

$$\int_{S_2} i\rho_a \omega \frac{p^2}{Z_s} dS = i\omega \{p\}^T_e [d]_e \{p\}_e$$

and

$$[k]_e = \int_A [B]^T_e [B]_e \, dA$$

$$[m]_e = \int_A \frac{1}{c_a^2} [N]^T_e [N]_e \, dA$$

$$[d]_e = \int_{S_{2e}} \frac{p^2}{Z_s} [N_a]^T_e [N_a]_e \, dA$$

$$[B]_e = \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{bmatrix} [N]_e$$

Last functions are part of the functions Ne which belong to **the nodal** points along the element boundaries. The construction of the iso-parametric Q8 finite elements follows [4].

A typical scheme of the finite elements arrangement is shown in figure 2 and an example of results in figure 3.

Conclusively, it was proved by applying the numerical technique, that this kind of analysis is useful in certain problems of atmospheric acoustics spite of the large domain.

ACKNOWLEDGEMENT

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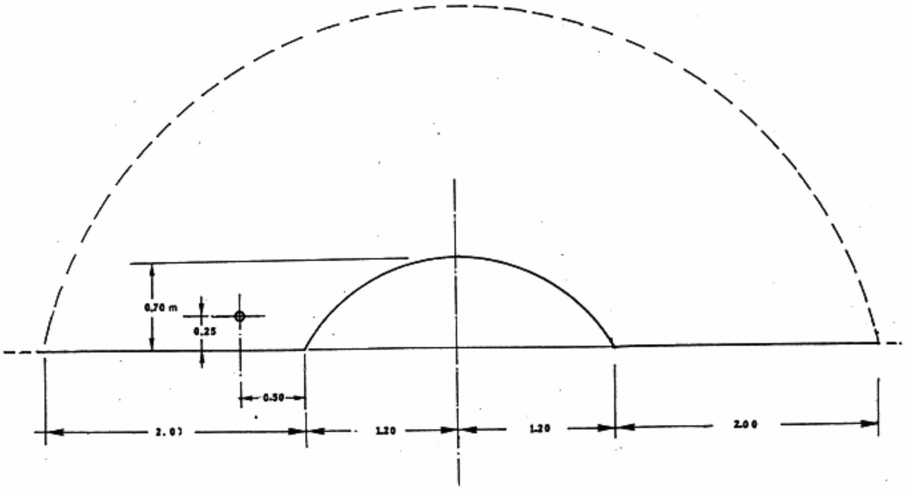

Figure 1. Configuration of the problem

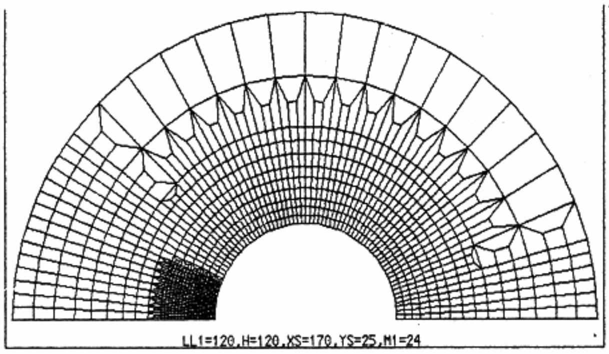

Figure 2. A typical iso parametric grid for the problem

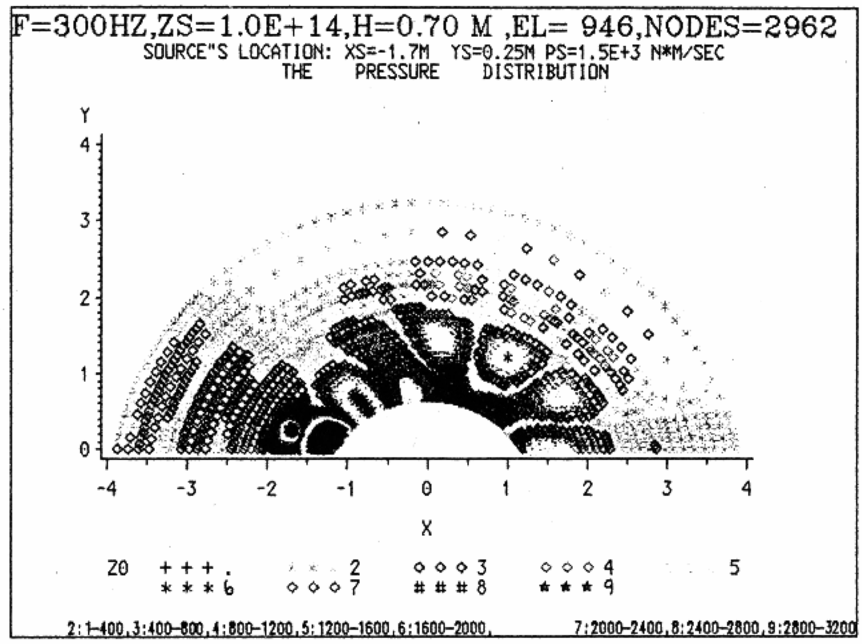

Figure 3. An illustration of results