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ALGORITHM FOR THE ATTENUATION IN SILENCERS

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INTRODUCTION

A slit shaped symmetrical silencer duct with width 2 h and locally reactive lining is considered. The attenuation Dh along the distance h is wanted. This attenuation is obtained from the propagation constant Γ by

$$Dh = 8.7\,\Gamma' h\ [dB],\quad \Gamma = \Gamma' + j\Gamma''. \tag{1}$$

Assuming a symmetrical cosine pressure distribution with distribution constant ε across the free area of the duct and fulfilling the boundary condition at the surface of the lining yields the defining equation [1]

$$\varepsilon h \cdot \tan(\varepsilon h) = jk_0 h Z_0 G_W \qquad (k_0 = \omega/c_0,\ Z_0 = \rho_0 c_0) \tag{2}$$

which is formalized to

$$\sqrt{E}\tan\sqrt{E} = jU \tag{3}$$

($U = k_0 h Z_0 G_W$, $E = (\varepsilon h)^2$, G_W = admittance of the lining).
From the solution E of this equation then

$$\Gamma h = \sqrt{E - (k_0 h)^2} \tag{4}$$

is obtained.
The problem now is to find an explicit solution for eq.(3) as an approximating formula which covers at least the range of the fundamental mode for technologically interesting linings. According to investigations in [1] the continued fractions expansion of the tangent is the only method to meet these demands.

A discussion of the permissible error of such an approximation has shown that an upper limit for the relative error of Dh is given by

$$\Delta Dh/Dh = 1/2\ [\Delta E'/E' + 2\ \Delta E''/E''] \tag{5}$$

That means for an error of 10 % in Dh an error of the same order for E' as well as for E'' is permissible.

THE CONTINUED FRACTIONS EXPANSION OF THE TANGENT

The expansion formula for the tangent is

$$\tan z = \frac{z}{1-}\,\frac{z^2}{3-}\,\frac{z^2}{5-}\,\frac{z^2}{7-}\cdots \tag{6}$$

Using this in eq.(3) and regarding only the approximations with the same order N in the nominator and denominator yields the following solutions for E:

$$E = (3 \cdot jU)/(3+jU) \tag{7}$$

$$E = 105+45jU \pm \sqrt{11025+5250jU+1605(jU)^2}/(20+2jU) \tag{8}$$

$$E^3 + C2 \cdot E^2 + C1 \cdot E + C0 = 0 \tag{9}$$

$$C2 = (-1260-210jU)/(21+jU)$$
$$C1 = (10395+4725jU)/(21+jU)$$
$$C0 = (-10395jU)/(21+jU)$$

Fig. 1a demonstrates the accuracy test for eq.(7), Fig. 2a for eq.(8). In these figures a grid of exact E-values is presupposed in the complex plane. Then the corresponding U-values are calculated from eq.(3) and used as inputs for the approximating formulae to determine the estimated values Ea. The approximation is the more accurate the more closely the lines of constant Ea', Ea'' reproduce the presupposed grid. The linear solution eq.(7) covers only a small area around the origin and is not sufficient for technical applications. The quadratic solution, eq.(8), covers all the range of the fundamental mode with exception of the immediate environment of the branch point to the first mode, and also the transition zone to the first mode is well reproduced. The accuracy tests for the solutions of order N=3 and N=4 are not shown here. They cover the area of the fundamental mode completely and a good part, respectively, the full range of the first higher mode. But it should be mentioned that the solution for N=4 requires an amount of calculation that is already comparable to that of an iterative procedure.

IMPROVEMENT OF LOW ORDER APPROXIMATIONS

The early truncation of the continued fractions expansion of the tangent does not lead necessarily to the optimum approximation with a given order of the algebraic equation for E. Eq.(7) needs an augmentation of validity range and a shift of this range to the domain of high attenuation. Eq.(8) on the other hand needs an improvement around the branch point to the first mode. This may be achieved in the following manner:
First, the nominator and denominator polynominals are transformed to product form (E-N1)·(E-N2) ... and (E-P1)·(E-P2) ... where N1 stands for Zerol and P1 for Polel and so on, and a variable K is introduced for the constant factor in front of the fraction. In the next step, these free parameters N_ν, P_ν, K are varied so as to minmize the squared error sum Q

$$Q(N_\nu,P_\nu,K) = \int^{G} \left| jU(E) - jUa(E,N_\nu,P_\nu,K) \right|^2 \cdot dE \longrightarrow \text{MIN} \tag{10}$$

in the considered area G of the E-plane. Unfortunately, this optimization procedure cannot be carried out analytically because the approximating function jUa does not have the form of a simple power series. Therefore a numerical solution for eq.(10) must be used and has to be combined with a simple strategy for searching the minimum Q in the multidimensional parameter space. Fortunately, only the respective highest Pole and Zero of the approximation differ considerably from the exact values according to eq.(3). This allows to restrict the parameter variation for eq.(7), (8) to the highest pole (complex!) and the multiplier K.

Fig. 3 illustrates this procedure: A contour plot is shown of the error sum Q in the complex P-plane for two given values of K. Drawing the minimum values of Q in the P-plane versus K in the next step leads to the absolute minimum of Q in the P',P'',K-space. Fig. 1b shows the accuracy test for a so optimized version of eq.(7) for a target area $-2 < G' < 1$; $0 < G'' < 3$.

The approximating formula is

$$E = jU\,(2.85-j0.58)/(3-j0.61+jU) \tag{11}$$

Evidently, the range of validity has been doubled! The target area $-2 < G < 1$ has been chosen because all technical linings behave at low frequencies as springs (negative E').

Fig. 4 shows the accuracy test for a version of eq.(7) taylored to measure for a "typical" porous lining with specific flow restistivity Ξ = 7000 Ns/m^4, thickness 0.2 m and half duct width 0.1 and 0.2 m. The NYQUIST plot of $E(\omega)$ for this lining remains inside the validity range of the approximate formula

$$E = jU\,(2.78-j0.6)/(2.883-j0.622+jU) \tag{12}$$

up to frequencies well beyond the frequency of the maximum of Dh indicated in the figure. That means, it is possible to calculate almost all the attenuation curve by the simple eq.(12)! (The inaccuracy at high frequencies is of little importance because in a real silencer duct the additional attenuation of higher modes at the inlet of the silencer - not included in the presented paper - is dominating).

Fig. 2b shows the accuracy test for a version of eq.(8) optimized for a target area $2 < G' < 5$; $3 < G'' < 6$ to improve the accuracy around the branch point to the first mode. The approximate formula is

$$E_{1,2} = (78.94-j5.43+jU(34.37-j2.2) \pm \sqrt{\ })/(16.1-j1.11+2jU) \tag{13}$$

with $\sqrt{\ } = \sqrt{6203-j857+jU(2887.3-j372)+(jU)^2(867.4-j130)}$.

This quadratic solution looks sufficient also for linings with resonators which reach this branch point and thus the theoretically maximum attenuation.

ASSIGNMENT OF MATHEMATIC SOLUTIONS TO PYHSICAL MODES

It is to be mentioned that the solution E corresponds doubtlessly to the fundamental mode, primarily of interest, only in the case of a linear approximating formula. For all higher approximations, some of the solutions for E may correspond to the fundamental mode. The simplest way fo find out which one then is to calculate Dh for all E solutions and to take that with the minimum Dh. This is not necessary for low frquencies where the fundamental mode has always a low but increasing attenuation whereas the first mode starts with Dh≈27 dB and decreases with frequency.

REFERENCES

[1] F.P.Mechel
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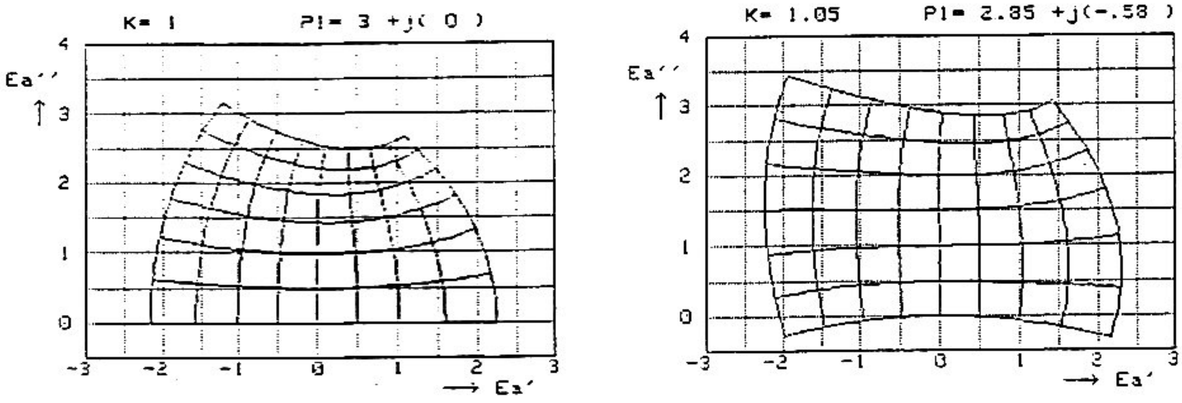
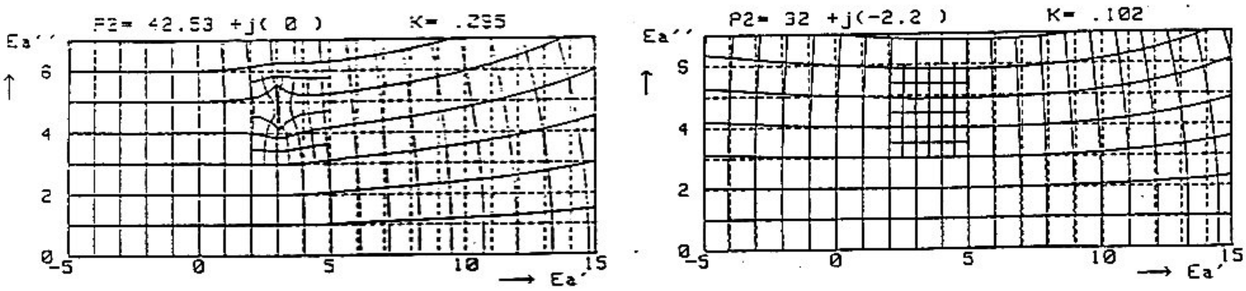


a) equ. (7)

b) equ. (11)

Figure 1: accuracy test for the linear approximation



a) equ. (8)

b) equ. (13)

Figure 2: accuracy test for the quadratic approximation

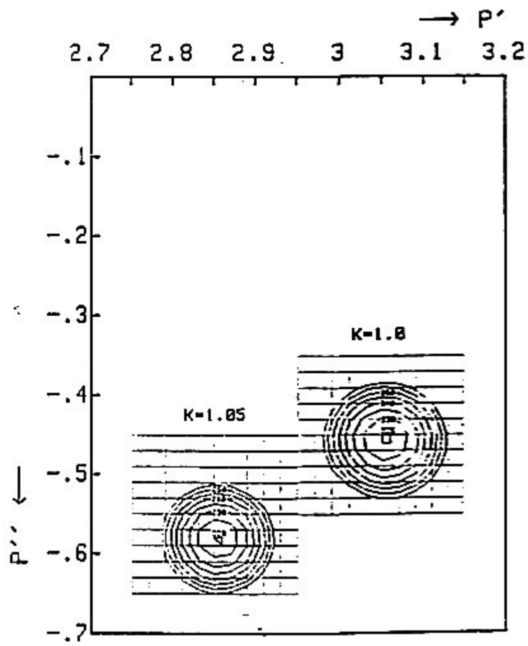


Figure 3: distribution of error sum for the linear approximation

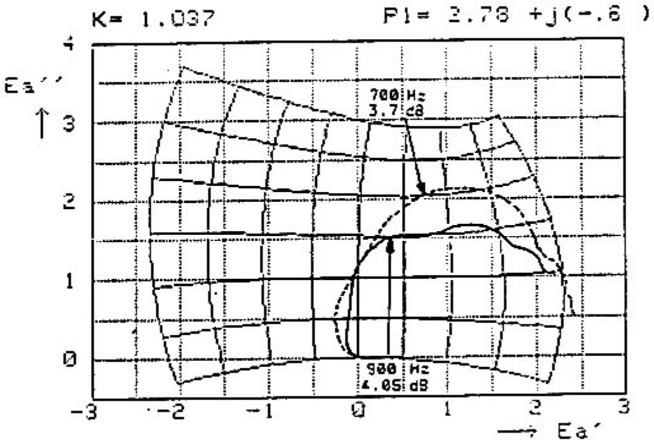


Figure 4: accuracy test for equ. (12) porous lining of thickness 0.2 m, Ξ = 7000 Ns/m^4
——— h=0.1m; ---- h=0.2m