11th International FASE Symposium
Valencia 15-17 Noviembre 94

ON THE PREDICTION OF MAXIMUM ABSORPTION AND RESONANCE SHARPNESS OF ACOUSTIC HELMHOLTZ RESONATORS

A. Moreno, C. de la Colina, F. Simón, J. Pfretzschner

Instituto de Acústica, CSIC
Serrano 144
28006 Madrid

ANTECEDENTS

When designing sound absorbers based on Helmholtz resonators, actual facilities of computers such as speed and graphycal disposals, support the idea of a predition model handling adecuate mathematical formulas. There is some oposition to classical models mainly based on tables and abacuses relating absorption to physical characteristics.

To this purpose the resistive term plays a primary rol. It completely controls the maximum obsoprtion and contributes highly to the resonance sharpness [1]. On a previous paper [2], experimental results were given for this resistive term, within the geometrical ranges of interest for usual ceramic tiles.

This paper resumes the above mentioned work in the search for a mathematical expression that represents enough acuarately the acoustic behaviour of the resistive term to design purposes. The merit of existing theories giving highly accurate values of resonance frequencies even for rather complicated geometries, is mainly due to the power of a more general method of computing resonance frequencies, due to Rayleigh [3].

When one compares formulas for the resistive term, given by various authors, substantial differences are found. Heckl [3] states a resistive term uniquely due to the radiation impedance, and takes the approach of Morse [4] for a un piston set on an infinite baffle:

$$\rho c\,(1-2\,J_1(2ka)/ka) \;\;\text{---->}\;\; \rho c\,(ka)^2/2\,, \quad \text{para } ka<<1 \qquad (1)$$

where c is the sound velocity in air, ρ is the air density, a is the radio of the resonator neck and J_1 is the Bessel function.

Ingard [5], on the contrary gives more importance to viscous dissipation within the neck, and obtains the quantitative expression:

$$2R_S(l+a)/a \qquad (2)$$

where l is the neck length, R_S is an equivalent surface resistance in the neck and whose second term, $2R_S$, comes from the contribution of resonator surfaces in the vicinity of the neck and normal to it, as computed by Nielsen [6]. According to Rayeigh [7], $R_S = \sqrt{(\eta\rho\pi f)}$, where η is the dynamic viscosity and f the frequency. Cremer [8], develops a theory similar to Ingard, and gives a suplementary formula to account for the equivalent surface resistance for narrow necks (circular necks). He further gives an abacus to bring the gap among both "types" of necks.

This work compares the experimental results with the above theories, on a wide range of sizes for cylindrical necks, with the aim of validate the more adecuate formula (s) to predict actual results, then to support a computing model to design sound absorbers based on resonators, isolated or forming assemblings to conform broad band absorbers.

EXPERIMENTAL METHOD AND RESULTS

A set of Helmholtz resonators was constructed covering geometrical ranges of interest with regard to common nature and sizes of ceramic tyles used in Spain. The following ranges were exlored $2.5<a<20$ mm, $1<l<40$ mm, $50<V<1000$ cm³.

We used a experimental arrangement based on the standing wave method, and a measuring technique using the standing wave ratio and the location of the first minimum [9]. With a rigid backing a standing wave ratio higher than *55 dB* was obtained for frequencies higher than *250 Hz*, then ensuring errors lower than *10%* for absorption coefficients higher than *0.05*. Directly measured quantities were:

- absorption coefficient in the frequency range,
- location of the first minimum (with regard to the sample surface)
- maximum value of the absorption coefficient α_{max},
- resonance frequency f_o,
- f_2-f_1 for points $\alpha_{max}/2$

By using the appropriate formulas [9] the specific impedance $Z/\rho c$ at resonance was computed, then the real part and the resonance quality factor $Q = f_o/(f_2-f_1)$.

Experimental results are represented on Figures 1 y 2, for cylindrical necks on ceramic materials. Abcissa numbers simply indicate the order number of the sample tested. Figure 1 corresponds to the resistance R of the resonator, after correction of the infuence of the standing wave tube. Figure 2 represents Q, but this time corresponding to the resonator mounted on the standing wave tube. Experimental points are grouped onto the nine sets, *e1* to *e9*, each one corresponding to a nominal neck length indicated in the figures. Within every set the neck radius takes the sequential values *3, 4.5, 6, 7.5* y *9 mm*, excepting sets *e5* (where a first point with a *2.5 mm* radius is added) and *e8* (where they are missing points for a *7.5 mm* radius on R and for radius *7.5 mm* and *9 mm* on Q).

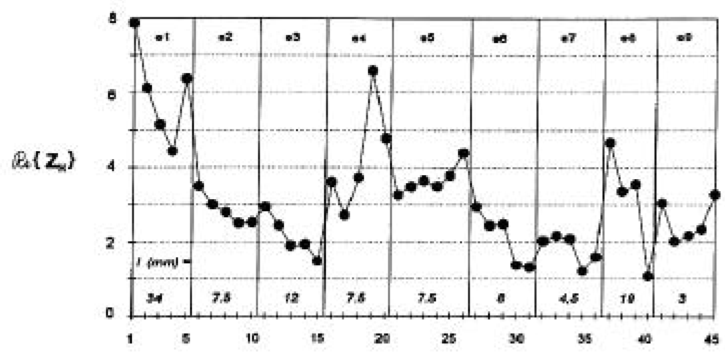

Figure 1. Measured acoustical resistance R, for 45 Helmholtz resonators.

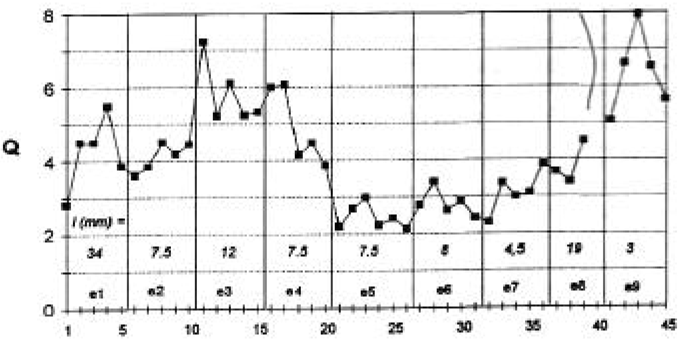

Figure 2. Measured resonance sharpness Q, for 45 Helmholtz resonators.

Most times, the frequency values to obtain resonance widths f_2-f_1, on Figure 2, required interpolations.

DISCUSSION AND CONCLUSIONS

Theoretical values afforded by the previous models have been compared with experimental results. Comparisons were performed maintaining as abscisas the sequence of cases as in Figures 1 and 2, and equal Y-logarithmic coordinates. In this way the balance between contributions of the radiation resistance and of the internal surfacic resistence become distinguishable.

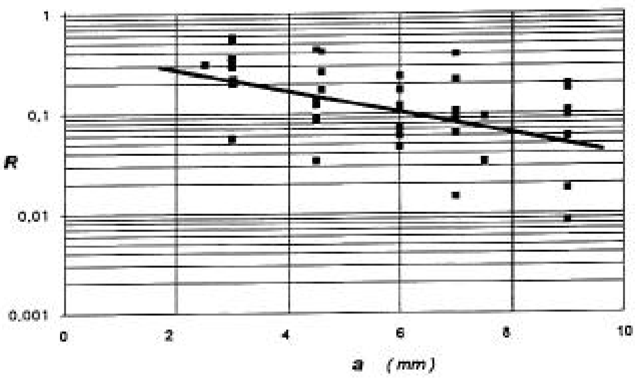

Figure 3. Dispersions of measured resistances compared to theoretical values, for 45 Helmholtz resonators.

Figure 3 gives the dispersions of experimental results, for R, with regard to the best theoretical fit as a function of a. Dispersions remain bounded to rather low values. It is significant that dispersions decrease as the neck radius a increases, in relation to errors in mechanizing of samples, that followed the above indicated variation.

We found that the best fit among theoretical and experimental results, for R,corresponds to the following conditions:

- the contribution of radiation resistance becomes worthless,
- the multiplicative factor 1 for a in the numerator of formula (2) should be used, as given by the theory, instead 2 recommende by Ingard,
- the expresion of Rayleigh (tubos menos estrechos) should be used to account for the "surfacic" resistance R_S ,
- a multiplicative constant is needed to minimize dispersions as a whole and the value of this constant computed by the least squares method was 3.9 (dispersions of Figure 2 already incorporate this correction).

When using such absorbers in common spaces like rooms, offices etc, the radiation resistance can play a significant rol, contrarily to the stanting wave tube. For such reverberant spaces a funtional dependence of the type $(ka)^2$ can be used. The expression of the radiation reistance for a piston set on a sphere, a pulsating sphere and a piston baflled by a plane have such functional dependence.

Our results supporting the Rayleigh expression for the "surfacic"resistance R_S, within the ranges indicated, can be interpreted as a confirmation of the Cremer's criteria, $a \geq 8.66/\sqrt{f}$ (*a* in mm y *f* in hertzs), to distinguish between narrow and "wide" tubes for resonator necks.

The use of the above results on a predictive model based on the equivalent circuit results very atractive to predict the acoustic absorption of either isolated or associated resonators. The complete absorption curve in the frequency range results very precise and extremely easy to obtain. When more complicated associations are to be considered some optimisations with regard to sharpness and positions of new resonances should be achived and our present work deals with this problem. Comparative results for two isolated and serial coupled resonators are respectively presented in Figures 3 and 4.

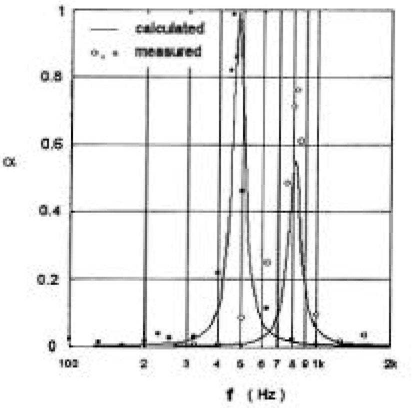

Figure 4. Predicted and measured absorption for two isolated resonators

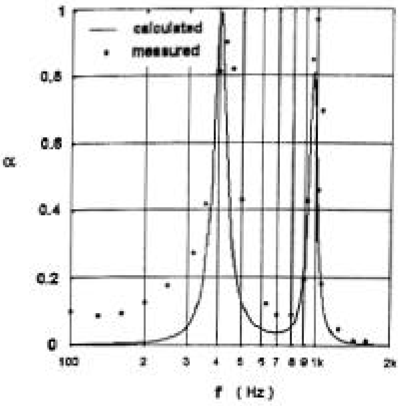

Figure 5. Predicted and measured absorption for two serial coupled resonators

REFERENCES

[1] C. W Kosten in *Technical Aspects of Sound*, Ed. E. G. Richardson, Elsevier Publishing Company, amsterdam, 1953, Chapter 4, *Behaviour of Absorbing Materials*.
[2] A. Moreno et al *Término resitivo y absorción en resonadores H*. Resúmenes XXIV Bienal RSEF, Vol. I, (1993).
[3] M. Heckl, Chap. 20, in *Modern methods in Analytical Acoustics* by D. G. Crighton et al., Springer Verlag, London 1992.
[4] P. M. Morse, *Vibration and sound*, McGraw Hill Book Company, (New York 1948), Chap. VII.28
[5] K. U. Ingard, *On the Theory and Design of Acoustic Resonators*, J. Acoust. Soc. Amer. 25 (6), 1953, pp 1037-1061
[6] A. K. Nielsen, Trans. Danish Acad. Techn. Sci. No. 10, 1949.
[7] Lord Rayleigh, *Theory of Sound*, MacMillan and Company, Ltd., London 1940.
[8] L.L. Cremer, *Principles and Applications of Room Acoustics*, Applied Sciences (1982)
[9] A. Moreno y M.C. Pérez Landazábal *Un essai de critère de qualité des tubes destinés à la mesure de coefficients d'absorption et d'impedances acoustiques*, Acustica, 28,2,(1973),109-114

Acknowlegments.
This wok was partially supported by Project CGICYT PB91-0884, and by HISPALYT S.A. Authors thank Mr. J.L. López, C. Redondo and F. Alcantarilla their experimental support.