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OBJECTIVE PARAMETERS AND SUBJECTIVE MEASURES RELATED TO THE ACOUSTIC QUALITY OF CONCERT HALLS

Manfred R. Schroeder
Professor (em), Drittes Physikalisches Institut
University of Goettingen, Germany

1. Theory

The first objective parameter to characterize the acoustic quality of concert halls was reverberation time, introduced by Sabine around the turn of the century at Harvard University. Sabine both defined reverberation time and made his first measurements, sounding organ pipes and himself in a barrel with the stop watch timing the sound decay [1].

Sabine also made an early theory of reverberation time. It was based on what we call the ray approximation, in other words he assumed that sound is transmitted like rays, like rays of the sun and this is actually a very good approximation as we can see in Fig. 1. Here is a little experiment that I demonstrate in Goettingen in my lectures on acoustics. There is

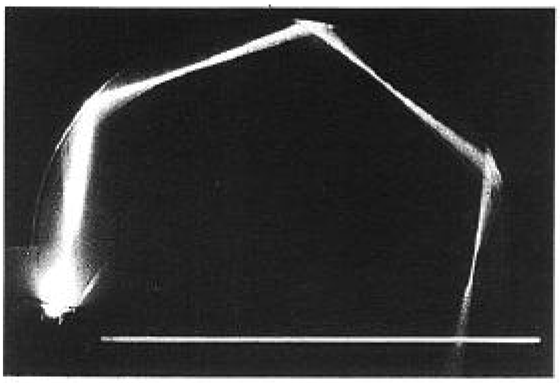

Figure 1

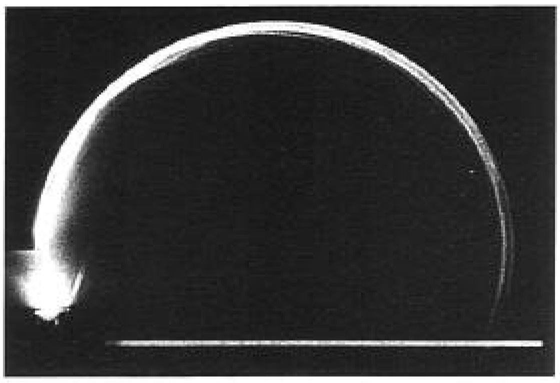

Figure 2

a curved mirror and a light source and we can see the transmission of sound in the form of rays. In fact we can even demonstrate the so called whispering gallery effect, see Fig. 2. Here we have the same set-up and see how sound rays actually go around the whispering gallery. So the ray approximation is very good for the purposes of calculating reverberation time.

The theory by Sabine, leads to this formula

$$T_{Sabine} = 13.8\frac{4V}{c\alpha S}$$

called the Sabine reverberation time formula, where V is the volume of the hall, c is the velocity of sound, α is the average absorption coefficient, and S the surface of the hall. Quite a bit later Fokker [3], Eyring [4] considered the fact that sound, of course, is absorbed at the walls in discrete steps, which leads to a different formula,

$$T_{Eyring} = 13.8\frac{4V}{-cS\ln(1-\alpha)}$$

in which α is replaced by a logarithm.

Now it has been known to acousticians for a long time that these formulas could not be quite correct; in other words that predictions based on these formulas are misleading. So Dr. Atal and I, in the nineteen sixties, started studying sound absorption by means of ray tracing, as it is called, ray tracing on the computer. This is a kind of billiard to use the most modern term from chaos theory. People always talk about billiards in this connection,

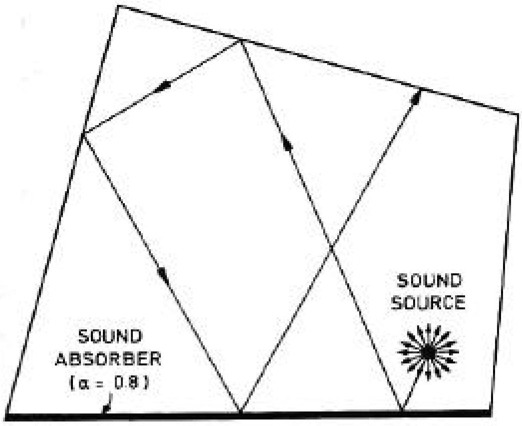

Figure 3

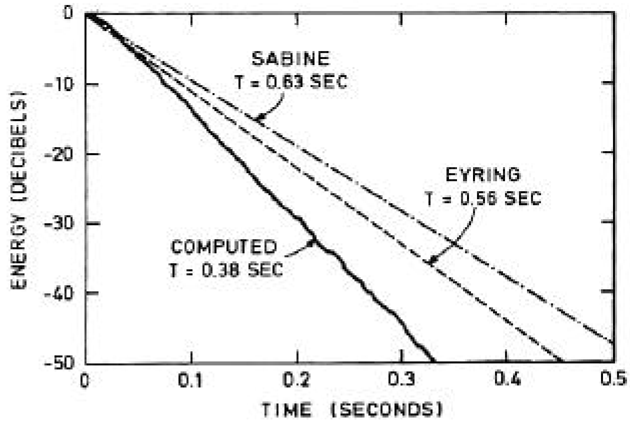

Figure 4

but we will use the more traditional term rays. In Fig. 3 we have the sound source emitting three hundred different rays in all directions. We follow a ray around this two-dimensional enclosure and every time it hits the absorber eighty percent of its energy is absorbed.

In Fig. 4 we see the results of one such ray simulation, the actual decay we found. If we compare the decay with the Sabine formula or the Eyring formula we see that the actual decay is quite a bit different and the question of course is what is the reason for this difference. One suspect parameter in the derivation of these reverberation time formulas is the mean free path. But the mean free path [2], found in ray simulations is the same as predicted, see Fig. 5.

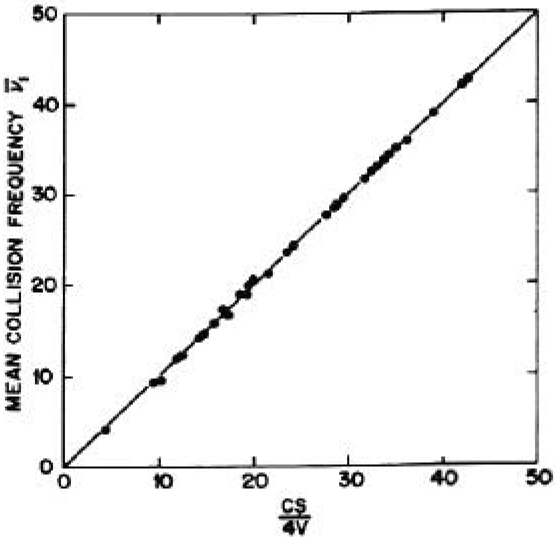

Figure 5

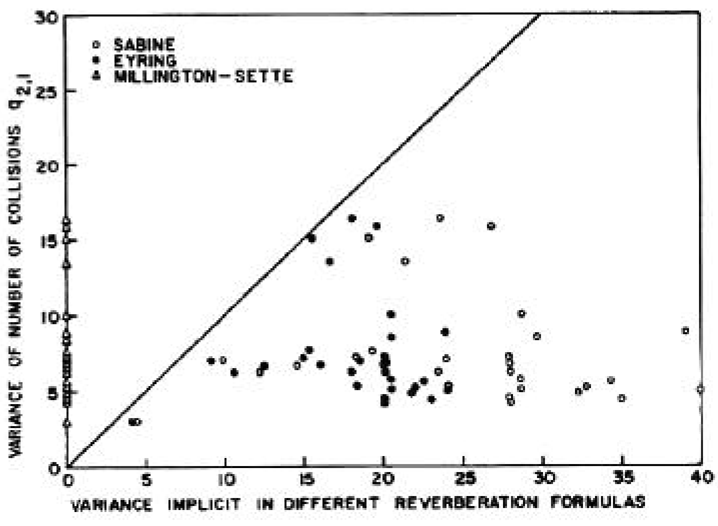

Figure 6

But something must be wrong. So we looked at the next statistical moment, the variance of the number of reflections in a given time interval as predicted by various theories and what is actually found, see Fig. 6, [9]. We see a great discrepancy between the actual statistics of the number of collisions of sound rays with the walls as predicted by various theories [7,8,11] on the one hand and what was actually found in our experiments. How do we correct these errors, how do we make a better theory ?. It seems that the only way to correct the deficiencies in the standard reverberation time formulas based on the work of Sabine, Eyring, Waetzmann, Schuster [12], Millington [5] and Sette [6] and others is to solve an integral equation as proposed by Kuttruff [10] in Germany and Joyce [13] in the US. Figure 7 shows the result of a little study that I did with a student from Columbia University, David Hackman. A very simple two-dimensional enclosure, all walls are fully

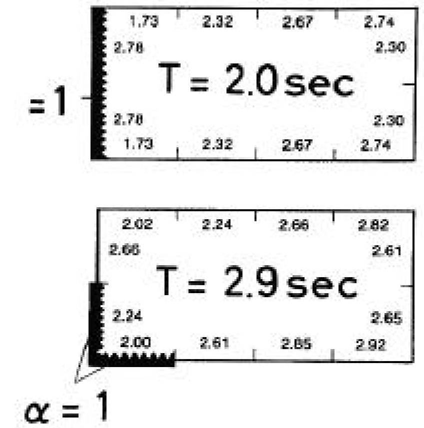

Figure 7

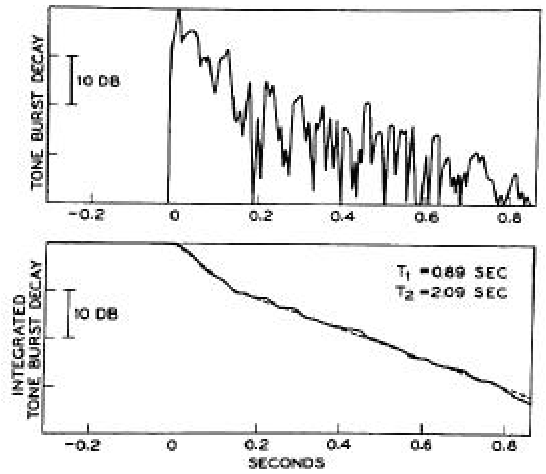

Figure 8

reflecting except one where the absorption is one hundred percent. We solved the integral equation for this space by a method suggested by Gilbert of Bell Telephone Laboratories [14, 15]. We found a reverberation time of two seconds. Then we moved one absorbing panel so we have the same total absorption. According to all these standard formulas we should find the same reverberation time, but we don't. We find a reverberation time that is forty five percent higher, 2.9 seconds instead of 2 seconds. Why is this so ?. It is actually easy to see this. The small numbers in Fig. 7 indicate the amount of energy incident on the various subsections of the walls and in the upper case we have fairly high sound energy impinging on the absorbing wall. But if we move one panel then we see that both panels now receive much lower energy because each panel sees this other panel from which no sound can come. This means that the absorption is less efficient in the lower configuration in Fig. 7 and this is why we get a higher reverberation time. In other words the location of the absorbers is very important and this is not considered by the traditional theories but of course is taken into account by integral equations.

2. Measurement

Let me say a little bit about the measurement of reverberation and sound decay [22, 23]. The pioneer was again Sabine, at Harvard, around 1900. He used a stop watch and organ pipes as the sound generator. Later another famous or preferred source of energy to excite a hall and to study the decay was white noise, or octave bands of noise or third-octave bands of noise. But every time you do this you get a different kind of decay. So in the 1960s another method was suggested based on the impulse response of the hall and an integral over the squared impulse response, see Fig. 8 [16]. In this way you get smooth decays and you can see that the initial high decay rate corresponds to reverberation time of about 0.9 seconds and the later decay with a reverberation time of about 2 seconds. This would be difficult to see in a noise decay because the decay then looks very irregular.

Recently at Goettingen we have tried to measure reverberation time and early sound energy, another important parameter, by observing music signals in the audience, because after all, you will be listening to music in a concert hall not to noise or clicks or impulses. You form your opinion on the basis of what you hear, the music. We are able to make room acoustical measurements with music signals [17]. In fact, it may be more appropriated than artificial or more physical signals. Here is an example of one of the measurements we performed at Goettingen. We measured the modulation transfer function [18], a concept that plays a large roll, as Stenneeken and Houtgast have shown, in determining intelligibility in auditoria [19]. We have a microphone on the stage that records the music and which determines the envelope, and of course on the stage the envelope fluctuates a lot. Then we have a second microphone somewhere in the audience that also records the music and also determines the envelope, and for someone in the audience area far removed from the stage the envelope fluctuations are diminished and from the degree of the remaining envelope fluctuations we can determine the reverberation time [20, 21]. This concept is what is known in vision, in television as flicker fusion and is described by the modulation transfer function. We measure the modulation transfer function between the stage and some point in the audience area. Farther away from the stage we see that the higher modulation frequencies are suppressed, they are much lower in amplitudethan they are near the stage. How well this method works we can see from the heavy lines in Fig. 9. These are carefully calculated modulation transfer functions, calculated from the precise measurement of the impulse response.

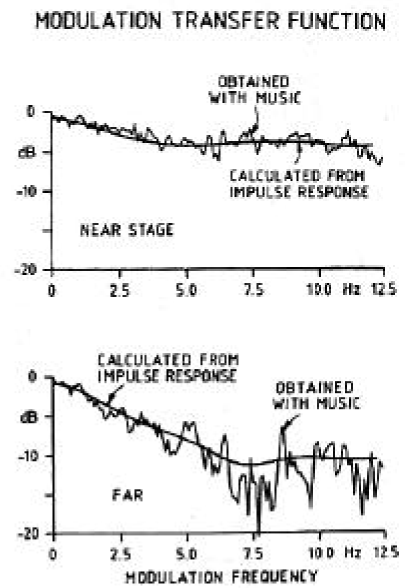

Figure 9

So we can see that just by making measurements with music we can get a pretty good idea of the effect of the reverberation. From Fig. 9 we can read both the ratio of direct to reverberant sound and reverberation time, two important parameters. Perhaps even more importantly by using music we can make these measurements during an actual performance with a full audience. This is a great advantage because many measurements in concert halls using noises and clicks are done in the absence of a full audience or no audience at all.

Now I would like to talk about some of the subjective measures that influence the acoustic quality of concert halls. It became obvious to many workers in the field that not only are there problems with the theory of reverberation and problems with the measurements of reverberation but also problems with what is really subjectively important in the reverberation process. In order to study this problem, again Dr. Atal and I, in the 1960s, studied reverberation processes by digital simulation. We simulated comb-filters and all-pass filters and we put these responses together in a appropriate way to digitally simulate various reverberation processes. One on the many things we did was to keep the initial decay constant and only after 10 dB we allowed different kinds decay rates. We found in all such experiments, even though the final reverberation time was as large as four seconds, the subjective measurements on most kinds of music are completely independent of the final decay. In other words we found that the initial part of sound decay is, by far, the most important one. This is quite understandable from basic hearing theory: in an on-going piece of music you only hear from each note the initial decay, the rest of the decay you can not hear; it is masked.

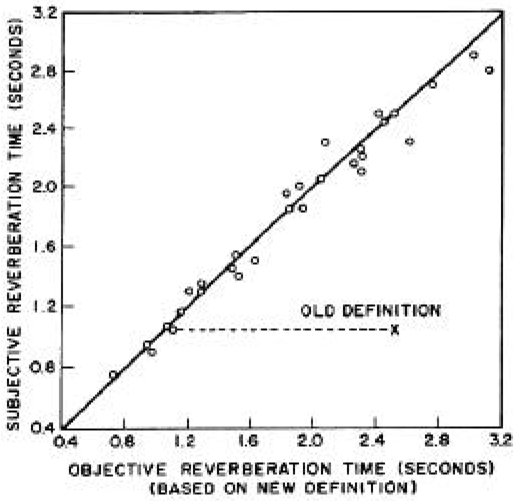

Figure 10

It is quite obvious that it is only the initial part of the decay that is important. So we introduced a new kind of objective measure of the reverberation in order to optimally match the subjective impression of reverberance [32]. It is based on the first 160 milliseconds of the decay or the first 5 decibels of the decay. Using this new definition based on the early part of the decay we found that we got a very good agreement with the

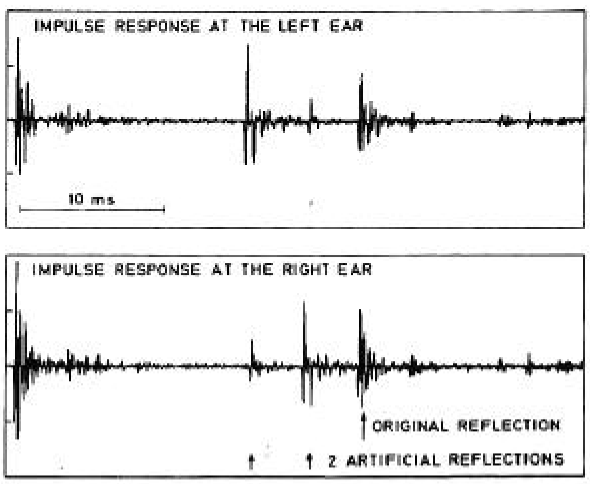

Figure 11

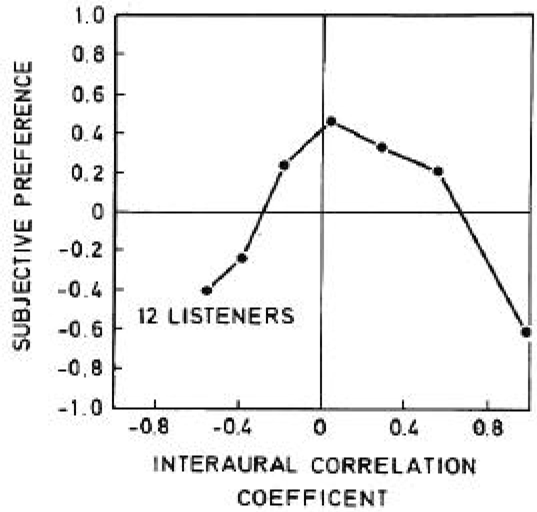

Figure 12

subjective reverberation time. We made comparisons to non-exponential decays and in

Fig. 10, in one extreme case, you see the old definition would be far off the 45 degree line. But this was based on an objective definition of the reverberation time, namely the decay between -5 and -35 dB which was the international standard and you see how misleading it can be compared to the early decay.

After this success in modeling reverberation processes, we went to full-scale digital simulation of concert halls in which on the computer one simulates the direct sound and the various early reflections with the appropriate delays and appropriate amplitudes plus the appropriate reverberation processes based on our all-pass and comb-filters [30, 31]. In fact we simulated in this manner an actual auditorium and you can make direct comparisons with the hall and we got pretty good results. One of the several uses that we put this digital simulation to was digital modifications of existing concert halls. For example if we have a modern concert hall with a low ceiling, very wide and a lack of lateral reflections and therefore a poor quality, we added on the computer, by digital simulation, additional lateral reflections, see Fig. 11. Fig. 12 shows one of the results. We measure the effect of additional lateral reflections by the interaural correlation coefficient IACC. We shall hear more about that from Prof. Ando [34] later in this meeting. But you see that as we decrease the interaural correlation the subjective preferences rises until the interaural correlation coefficient is actually zero. This is one example of the kinds of things we did with digital simulation on fairly large digital computers.

Figure 13

Figure 14

Now, of course, most of the traditional modeling in architectural acoustics was done by analog models without computers. Fig. 13 shows the reverberation chamber in

Goettingen with various diffusing elements suspended in the air.

One of the pioneers in analog modeling was Prof. Spandöck in Germany, but it was done by many others too, around the world. With scale models 1:10 we can design a concert hall by scaling down geometrically all dimensions by a factor of ten and then use frequencies ten times higher. Geometric dimensions and frequencies are in a perfect scaling relationship to each other. But for absorption in scale model studies we must simulate the proper sound absorption of people at elevated frequencies which is not simple.

Basically they are two kinds of analog studies to study open problems in architectural acoustics. In one case a spark from a spark plug from an automobile is triggered inside the scale model and the impulse response is recorded. This impulse response is then evaluated in one way or another. Or a small loudspeaker is used actually injecting music scaled by an appropriate frequency factor into the model hall. Thus you can actually, after scaling down, listen to the model.

Some of these analog studies were not done in complete halls but only on certain aspects of it. In Fig. 14 you see Professor Marshall from New Zealand studying a scale model off diffusers in his lab in Auckland. This is an example of an analog study done on one aspect, like diffusers. But many of these studies were also done in full-scale models, in actual halls. Here is a result from Professor Ando where he added extra reflections by electroacoustic means and he varied the angle of the extra reflections from 0 to 90 degrees and plotted the preference, see Fig. 15. It peaks at about 70 degrees and, sure enough, the interaural cross-correlation that I mentioned before is a minimum at this angle whereas the preference is a maximum. We see from this figure that for good concert hall quality we want to have a very low interaural cross-correlation.

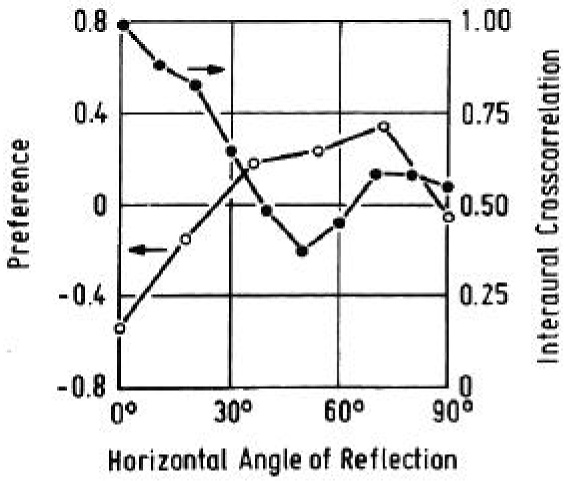

Figure 15

I have talked about reverberation time so far, early reverberation time, which was specially stressed also by Dr. Jordan [33] in Denmark and others. I have also talked a little about the importance of lateral echoes. But many other physical measures were suggested to characterize this subjective quality of concert halls. One of then was based on the frequency irregularity of a concert hall [24]. You vary the frequency very slowly from a loudspeaker and record the sound pressure, see Fig.16. Some researchers counted the number of peaks in a given frequency

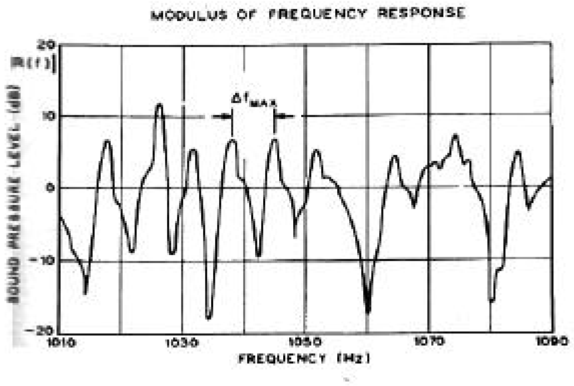

Figure 16

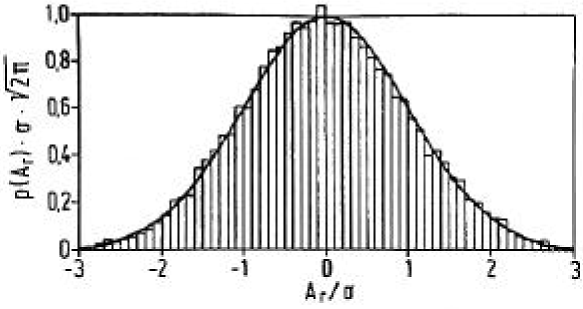

Figure 17

interval. This was supposed to be a measure of acoustic quality. I was a young student then and it seemed to me that people were looking at some kind of noise in the frequency domain. Sure enough, the underlying statistics of such responses is gaussian, see Fig. 17, and everything about such frequency responses becomes completely predictable [26-30] for frequencies above

$$f_c = 2000\sqrt{\frac{T}{V}}$$

where T is the reverberation time in seconds, and V is the volume in cubic meters. Much later I found the corresponding formula for the critical wavelength. Using the velocity of sound to convert to a critical wavelength you get a much simpler formula that is independent of units:

$$\lambda_c = \sqrt{\frac{\overline{\alpha} S}{6}}$$

where $\overline{\alpha}$ is the average absorption and S is the surface area. When you measure the surface in square meters you will get λ_c in meters. So this is a nice formula and it does not depends on the units that you are using.

aufdringlich	glorios	reich	unheimlich
aufrichtig	hallig	ruinös	verschmiert
ausgewogen	hart	schmal	verschmitzt
begeisternd	heikel	schillernd	verschmolzen
betäubend	herrlich	schön	verworren
bezaubernd	hinreißend	schrecklich	vollkommen
brillant	intim	schrill	volltönig
deutlich	jämmerlich	temperamentvoll	vorzüglich
empfindlich	kalt	trocken	wahrhaftig
erhebend	klar	überwältigend	warm
erheiternd	krankhaft	unbarmherzig	widerhallend
erschreckend	lebendig	undeutlich	wohltönend
erstaunlich	lieblich	undurchsichtig	wohltuend
glasklar	prächtig	unerbittlich	wunderbar

Figure 18

Now one kind of approach to the study of concert hall quality was by asking musicians, conductors, music critics and the audience how they like the acoustics, to describe it in some verbal terms. Professor Meyer collected a list of words that people use in describing the acoustic quality of a concert hall, see Fig. 18. Most of you may not know German. But no matter, these words make no sense. This is just a kind of "poetry" not suitable for a scientific approach to concert hall quality.

So we need more scientific methods to study this. Listeners have only two ears so two

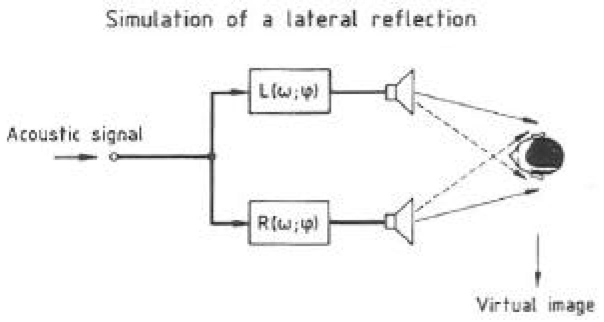

Figure 19

Figure 20

loudspeakers should be sufficient. Atal and I pursued this idea and found that indeed we could take an acoustic signal radiated from two loudspeakers and a listener would hear the sound as coming from one side, see Fig. 19. The filters contain information about sound diffraction around the human head [35]. We simulated in these filters the sound diffraction around the human head as a function of frequency and depending on the angle of incidence. So for a 90 degree incidence we used filters that corresponds to 90 degrees. And sure enough, when asking the subject where the sound comes from he or she will say to my left. But then of course the moment he turns his head the effect goes away. This only works as the subject faces forward.

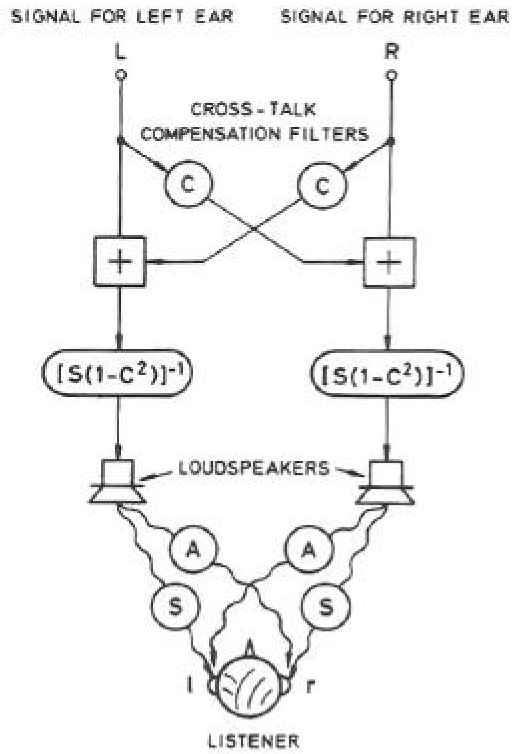

Figure 21

Now since this kind of sound field simulation works so well I asked the German Science Foundation in 1970 to fund a large study of more than twenty halls mostly in Europe but some on the US too. We decided to play a piece of music, an excerpt from the Jupiter Symphony of Mozart that the London Chamber Orchestra played in an anechoic environment. We played this from the stage of the different concert halls [40]. Others, like Wilkens [36] actually traveled with the Berlin Symphony. So they use live music but every time the music is played differently. On the other hand our radiating pure recorded music from the stage is not really what happens when a live orchestra plays live music in a given hall. Here are my three collaborators in this study: Dr. Gottlob and Dr. Siebrase and "Mr. Kunstkopf". We made these recordings in the different concert halls and then evaluated them subjectively. The listener is sitting in an anechoic environment. But of course he received sound not only from the right loudspeaker but also from the left loudspeaker and so we decided to have compensating filters which have the effect that the right signal travels only to the right and the left signal only travels to the left, see Fig. 21. With this kind of simulation we were able to "transport" the concert halls from to the anechoic chamber. I remember when I first listened to the Vienna Musikvereinssaal and the Berlin Philharmonic, two halls in which I have actually listened to live concerts. I knew some of the differences from previous listening experience. But now I could appreciate that the differences between these two halls are much greater. We presented these twenty different halls to all listeners in paired comparison tests and they had to say of each pair which one they preferred.

Then we used psychological scaling methods to construct a so-called preference space, see Fig. 22. Each vector is one of the listeners. Each symbol is one of the concert hall seats in a given concert hall. What this preference space means is the following: the projections of these points onto these different directions reproduce the individual judgments of these ten different listeners. Most of the listeners, except number 4, point to the right. So this

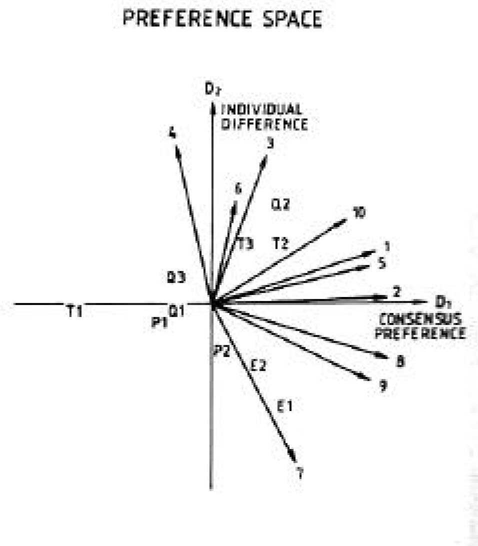

Figure 22

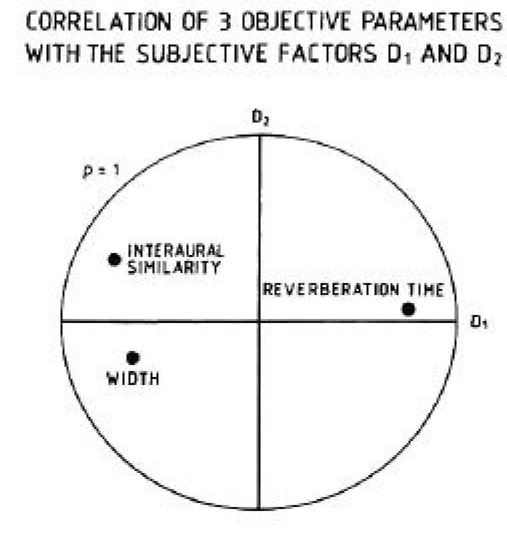

Figure 23

dimension D1 is really a kind of consensus preference. If we make a change in a concert hall that would move its point to the right, then everybody except number 4 will say it is better now. The remaining dimension reveals individual differences because different people have different musical tastes. These subjective data are then correlated with physical data.

Now we come to discuss comparing subjective data, preferences of acoustic quality with physical measurements. Figure 23 shows the correlation between these subjective dimensions, dimension 1 and dimension 2, Fig. 22. Such physical measures as reverberation time and the interaural similarity. We see that there is a negative correlation between interaural similarity and consensus preference. In other words, the more interaural similarity we have the worse it is for most listeners. This means that there is too much sound from the front arriving in the median plane, which produces two signal which are much like at the two ears and there is not enough sound coming from the left and right sides. The importance of enough lateral energy, was originally found by Barron and Marshall [38, 39] and we will hear more about this from Professor Marshall later in this meeting.

Other objective parameters that we considered are what Thiele called D, "Deutlichkeit", in other words the amount of energy in the impulse response during the first fifty or eighty milliseconds divided by the total energy. Or spectral balance, the balance between high frequencies and low frequencies in the sound. Or the initial time gap introduced by Professor Beranek and described in his book [37].

Now when I first realized the importance of lateral sound I asked myself how can we increase the amount of lateral sound in modern concert halls with low ceilings and that are very wide. These modern halls have little lateral energy and it comes very late because the side walls are so far away. We get a lot of direct sound and early sound from the low ceiling. Well we can not leave out the ceiling. So we have to do something else. I thought perhaps we could improve the quality of concert halls by adding a lot more diffusion on the ceiling and may be in other places too and I asked myself how does a surface structure have to be so that a single incident plane wave would be scattered into all kinds of different directions with more or less equal energies. I asked the question because it was already knew from the number theory [41] how to design reflection phase gratings for

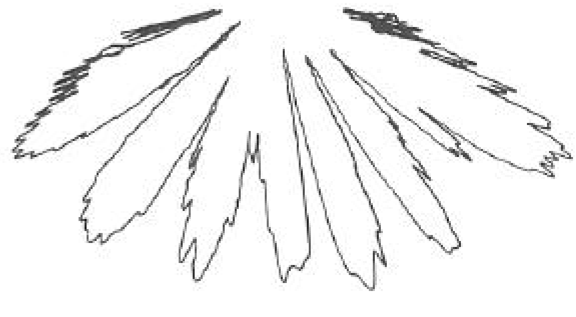

Figure 24

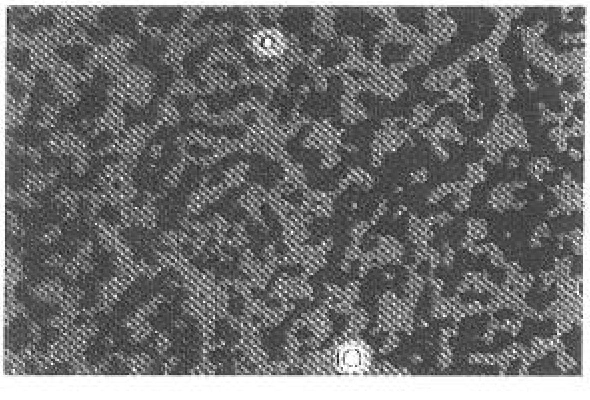

Figure 25

efficient scattering of sound. Figure 24 shows measurements actually not done with acoustic waves but microwaves. We have an incident wave that is broken up into seven different waves going into 7 different directions just as predicted by number theory. The kind of mathematics that I was using here has many many other applications for example you can construct gratings for x-rays astronomy. In Fig. 25 you see one of the results in x-ray astronomy: two sources imaged by this kind of number theoretic grating.

Figure 26 shows another application connected with the Einstein's general theory of relativity. Einstein predicted in 1915 that electromagnetic radiation would not only be bent near the sun, near a heavy star, but would also be slowed down. But how can we measure this ?. What people did, planetary scientist, was the following. They bounced radar echoes from the planet Mercury and when Mercury goes behind the sun as seen from the earth (what is called superior conjunction) the reflected radar wave has to pass by the sun and according to Einstein's theory is delayed by as much as 150 milliseconds. The measurements follow Einstein's prediction (the heavy line) very closely. I mention this because the energy that comes back to the earth from the planet Mercury in superior conjunction is 10^{-27} of the outgoing energy. A very very small amount. How can we ever make precise measurements of delays between the Earth and Mercury at such low signal-to-noise ratio ?. The answer is we use number theory and then it becomes possible.

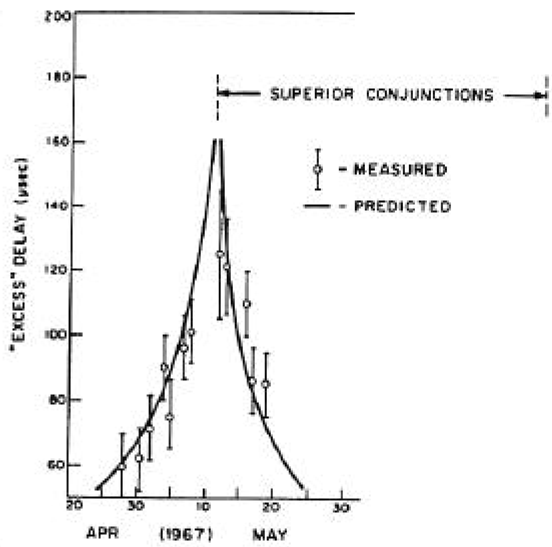

Figure 26

So there are many applications. But then later I focused on the question of how to make such gratings cover a wider range of frequencies. I suggested diffusers and reflection

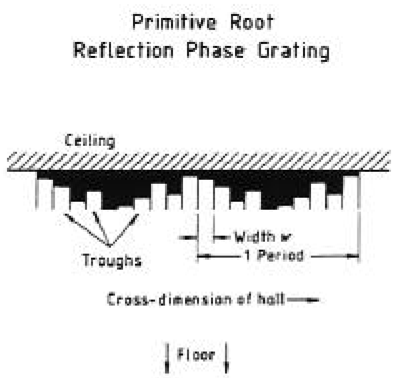

Figure 27

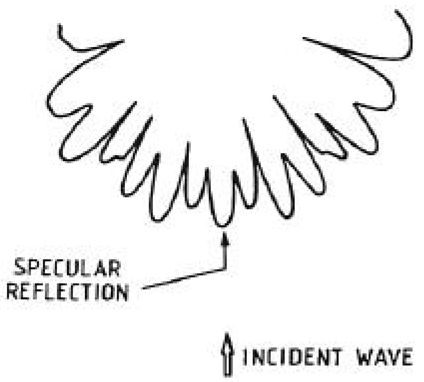

Figure 28

phase gratings, based on the quadratic residues for prime numbers such as 17, see Fig. 27 [41]. Figure 28 shows a reflection pattern measured from such a grating. We have an incident wave and many reflected waves, again more or less predicted by the theory. The first large-scale field application of diffusers was done by Marshall in Wellington Townhall in New Zealand. All of these structures were originally suggested for concert halls but they found wide-spread use in recording studios by people who make recordings for compact discs. They are also used in churches and many other places with great success [42, 43].

Another principle from number theory, primitive roots [41] gives rise to different kinds of structures which suppress the specular reflection and get even more lateral sound, see Figs. 29 and 30. All of this is described in my book on number theory in some detail.

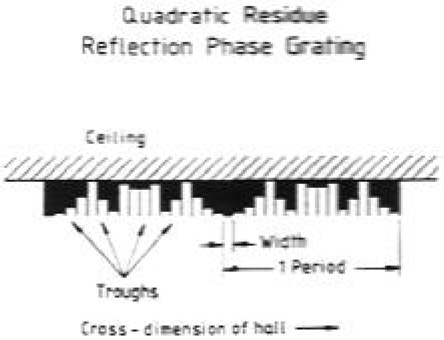

Figure 29

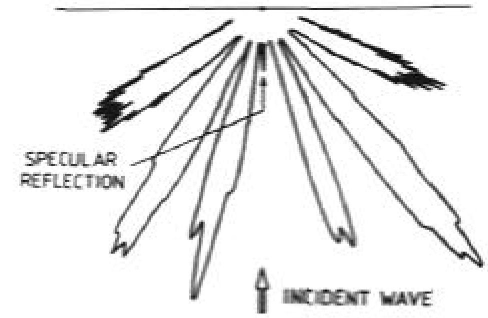

Figure 30

3. Conclusions

In conclusion let me say that we have seen a lot of progress in concert hall acoustics during the last two decades both on appropriate objective parameters and meaningful subjective measures. But there is no question that many problems remain unsolved and that much remains to be done. I hope to hear from the other speakers today and tomorrow about some of the outstanding problems, outstanding in several senses of the word.

One final question: will electroacoustics become acceptable for large halls dedicated to the traditional repertory ?. Will we see one day, in the future, halls that use sophisticated digital techniques ?. Especially for multi-purpose halls such a development seems inescapable.

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