RE-VISITING BOLT'S CRITERION FOR HOMOGENEOUS DISTRIBUTION OF NORMAL FREQUENCIES IN RECTANGULAR ENCLOSURES

REFERENCIA PACS: 43.55.Gx, 43.55.Hy, 43.55.Nd

Moreno, Antonio; Ruiz, Joaquin; de la Colina, Carlos;
Instituto de Acústica, CSIC
Serano 144
28006 Madrid. España
Tel: 34 915 618 806
Fax: 34 914 117 651
E-Mail (first author):iacma15@pinar2.csic.es

ABSTRACT

The classical criterion of Bolt for homogeneous distribution of normal frequencies in rectangular enclosures is revisited. Coincidence of normal frequencies is considered a detrimental factor concerning frequency and spatial regularity in sound response of enclosures. New fundamental facts influencing normal frequency distribution are analyzed and a metric is proposed as an efficient criterion. Merit figures are compared to Bolt's results. New areas of *p,q* ratios leading to homogeneous distributions in the low frequency range are found. At high frequencies a generally better situation is observed but some values of *p* and/or *q* and some ratios *p/q* lead to clusters and "holes" of normal frequencies causing high scores of transmission irregularity.

RESUMEN

Se presenta un nuevo enfoque del criterio de Bolt sobre distribución homogénea de frecuencias propias en recintos prismáticos. La coincidencia de frecuencias propias se considera un factor clave de deterioro de la regularidad de la transmisión sonora en recintos. Se analizan las causas del espaciado homogéneo de frecuencias propias proponiendo una nueva métrica cuyos resultados se comparan con los de Bolt, localizando nuevas áreas de proporciones *p,q* con gran homogeneidad de distribución en bajas frecuencias, encontrando para frecuencias superiores que junto a una mejora generalizada, ciertos valores de p, de q, de ambos o de *p/q* conllevan agrupamientos y vacios de frecuencias propias con detrimento de la regularidad de la transmisión.

INTRODUCTION

Low scores on transmission irregularity are considered a positive acoustical factor of rooms [1]. This factor is particularly interesting in reverberation rooms where the lower frequency bands usually involve a rather reduced number of normal frequencies. Furthermore the homogeneous distribution of normal frequencies is a main factor contributing to increase diffusion a condition involved in absorption and sound power measurements as described in national and international standards.

It is well known that the total number of normal frequencies mainly depends of the volume of the room throw the power 3 of frequency under consideration. However in rectangular enclosures contributions of other geometrical factors such as surface and perimeter is recognized to play an important role in the low frequency range [2, 3].

To minimize the rather big fluctuations of sound levels inside a truncated pyramidal enclosure of 0.42 cubic meters, usual in power measurements of telephonic rings was the motivation of the present work. The starting idea of introducing a grating diffuser, conditioned the use of a rectangular enclosure and the choice of the "best" proportions.

THEORETICAL BACKGROUND

It is generally admitted that diffusion hypothesis involved in geometrical-statistical acoustics theories are satisfied in rectangular enclosures at frequencies over Schroeder's frequency. This frequency is given by the equation [4]:

$$f = 2000\sqrt{\frac{T}{V}}$$

V being the room volume in cubic meters, T the reverberation time in seconds and f the frequency in herzs. This condition involves a modal overlapping of, at least, 3. Under these conditions the frequency response relating two arbitrary fixed points in the room results quite smooth. A similar result is obtained at a fixed frequency when the position of the receiving point location varies. As indicated, under these conditions the sound field is ideally diffuse. Therefore transmission irregularity, defined as the difference between the sum of local maximum levels and the sum of local minimum levels becomes low.

Mainly because of subjective effects of noise, frequencies as low as 100 Hz are to be considered in sound absorption and power measurements in reverberant rooms. If 5 s as reverberation time and 200 m^3 as volume are admitted as usual values for reverberant rooms, the Schroeder's limiting frequency, becomes about 315 Hz. Within this 1/3 octave frequency band there is about 500 normal frequencies. At 100 Hz, there are only 16 normal frequencies and a modal overlapping of 0.6. Some authors [3], studying diffusion based on correlation analysis conclude that reasonably good diffusion is to be admitted at frequencies as low as 125 Hz, for modal overlapping about 0.2, well below the value involved in the above limiting frequency, and with a rather reduced number of normal frequencies, about 40 within the 1/3 octave band centered at 125 Hz. The shape of the room is argued to be the responsible factor of that behavior, and guidelines of some standards for a proper choice of proportions is mentioned to be respected by the rooms where experiments were conducted.

Therefore in the low frequency range a homogeneous spacing of normal frequencies becomes of most importance to minimize transmission irregularity and, given a constant volume, the influence of room proportions is the main factor to consider.

NORMAL FREQUENCY SPACING AND BOLT'S CRITERION

On a preliminary report [5] of more complete studies on normal frequency spacing statistics Bolt presented a low frequency spacing index, y_1, and a quality criterion related to the room dimension ratios. This spacing index, related to transmission irregularity but in fact a direct metric of statistical fluctuation in frequency spacing, is defined by the equation:

$$y = \frac{1}{m_b - m_a}\sum_a^b\left(\frac{d^2}{\bar{d}}\right)$$

d being the actual normal frequency spacing and $\bar{d}$ the mean theoretical spacing at the space d, between m_a and m_b, of the dimensionless frequency $\mu=\sqrt[3]{V}(f/c)$, where V is the room volume, f the frequency and c the speed of sound. Index y_1 corresponds to $m_a= 0.5$ and $m_b= 1.5$. Spacing is said to be more and more homogeneous as y approaches 1, a limit situation in which every actual spacing equals the average value spacing. As y increase actual spacing becomes more and more irregular.

The following figure represents Bolt's criterion, stated in terms of the room dimension ratios *1: X:Y*, normalized to the shortest dimension. Ratios inside the clear zone lead to "smoothest frequency response at low frequencies in small rectangular rooms" (sic, reference 5).

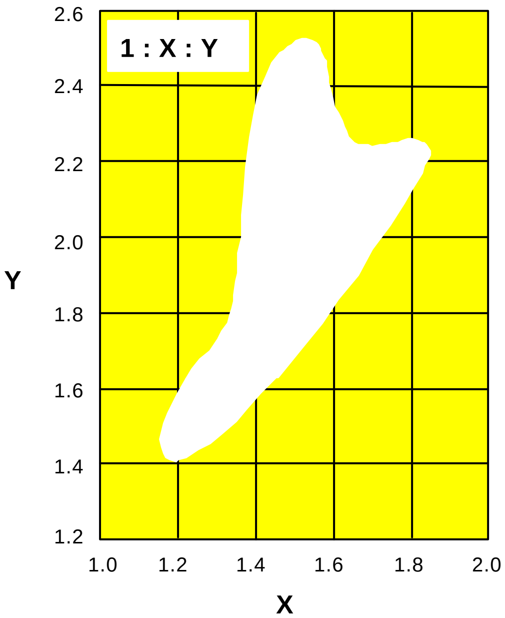

Figure 1. Bolt's criterion of dimension ratios of rectangular enclosures for homogeneous spacing of normal frequencies

The previous figure was derived from a graphic where isopleths of y_1 were computed as a function of the dimension ratios *1:p:q*, this time normalized to the longest dimension of the room. The border of clear zone of previous figure corresponds to $y_1= 1.5$. A recomputation of y_1, as a function of *p,q*, is represented in the following figure for a matrix of 91x91 elements. Interpolation, indicated by color lines, is an automatic feature of the representation program used (*J*). Blue zones correspond to low values of y_1, increasing until red zones that correspond to the highest values. Most outlines and values of this figure are nearly coinciding with Bolt's figure, but some new zones of *p,q* with high homogeneous distributions of normal frequencies, can be observed. Among them the zone around *p=0.45*, *q=0.3* is of particular relevance.

The main diagonal, *p = q*, corresponds to bad distributions. From that line some emerging cases are to be signaled *p=q=0.3* , *p=q=0.5* and the worse that corresponds to *p=q=1*, the cubic room, as it is well known.

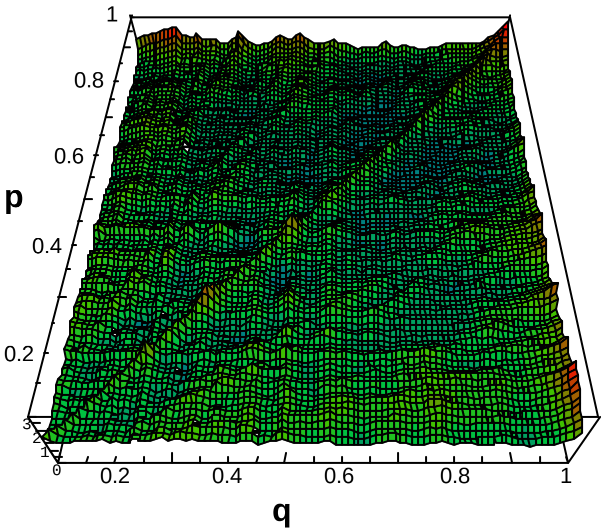

Figure 2. Low frequency spacing index $y_1(p,q)$, of rectangular enclosures.
Blue zones: highly homogeneous spacing of normal frequencies.

DISCRETE VERSUS CONTINUOUS DISTRIBUTIONS OF NORMAL FREQUENCIES

Spacing of normal frequencies of rectangular enclosures as a function of dimension ratios *p,q*, and the frequency dimensionless parameter m can be approached by the polynomial equation,

$$\frac{dm}{dn} = 1/(4pm^2 + p\frac{(p+q+pq)}{(pq)^{2/3}}m + \frac{1}{2}\frac{(1+p+q)}{(pq)^{1/3}}$$

It can easily be observed that represents a function monotonically decreasing as m increases, independently of the couple of values (p,q). Let us consider the interval $0 \le m \le 3$, that contains about 300 normal frequencies, then encompassing the frequency interval where homogeneous distribution is of particular importance. Combining all possible pairs p,q , the resulting functions dm/dn locate inside the limiting curves of the Figure 3. It means that normal frequency spacing follow quite similar variations independently of the couple (p,q), and more and more similar as m increases. Double logarithmic representation indicates an asymptotic behavior of dm/dn towards high frequencies, as can be easily seen from equation. Also the nearness with the statistics spacing found by Schroeder, for microwave cavities is evident[6].

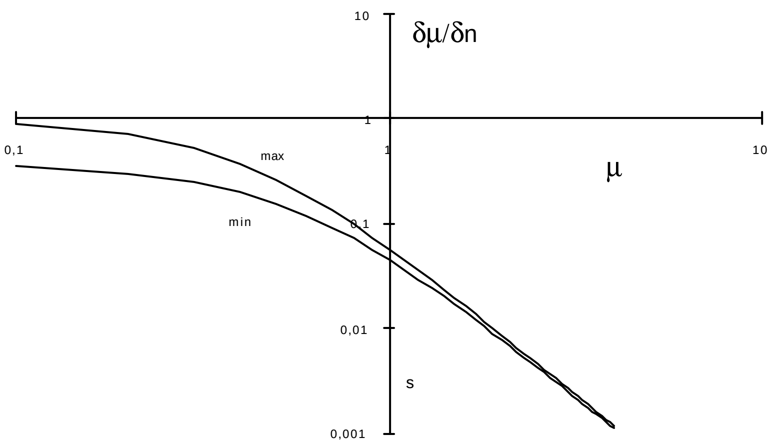

Figure 3. Continuous approach of normal frequency spacing in rectangular rooms

When dm/dn is computed using actual distributions of normal frequencies $m_{nx,ny,nz}$, discrete distributions are obtained, every distribution depending on the couples (*p,q*). However every discrete distribution tends, through *p,q*, to the associated continuous polynomial distribution. Low scores of $(dm/dn)_{nx,ny,nz}$ in an interval dm are compensated with overscores in the near frequency intervals and the polynomial results similar to the best fitting. Assuming the previous observation of equivalence among all polynomial spacings the most homogeneous spacing of normal frequencies in rectangular enclosures is obtained with the couple (*p,q*) that best follows the corresponding polynomial. A particularly well adapted metric derives from the lest squares method of fitting. If differences are normalized to the mean spacing a new index can be obtained:

$$y_p = \frac{mean(\overline{d})1}{m_b - m_a}\sum_a^b (d-\overline{d})^2$$

where $\overline{d}$ is the local mean of normal frequency spacing, corresponding to actual spacing d, and *mean(* $\overline{d}$*)* represents the mean of d over $m_b - m_a$. Defined in this way y_p is something similar to the second statistical moment of d. The lower the value of this index is the more homogeneous the spacing becomes. The best situation corresponds to $y_p=0$.

Figure 4 shows index y_{p1} (m_a= *0.5* and m_b= *1.5*). Similarity with Figure 2 for index y_1 is quite obvious. Therefore both indices are nearly equivalent but y_p indicates more clearly the relation among continuous and discrete distributions of normal modes in rectangular enclosures

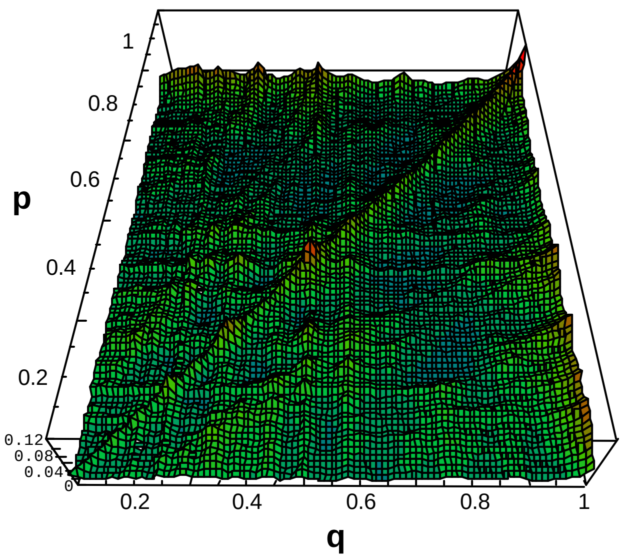

Figure 4. Like Figure 3, for the new proposed low frequency spacing index $y_{p1}(p,q)$

The influence of higher values of the frequency limits m_a and m_b , has been studied because y_{p1} only include approximately the first 25 normal frequencies (the couple *p,q* slightly influences this number). When the frequency limiting interval, one octave width, is translated towards higher frequencies different figures are obtained but some general conclusions can be derived:

a) there is a general evolution to lower values of y_p ; an evolution to more homogeneous spacing of normal frequencies, in a global sense, is expected

b) over the background indicated in a) emergent lines of relatively high values of y_p are found, that correspond to lines *p=constant*, *q=constant* or *p/q=constant*.

Conclusion b) indicates that the simple increase of frequency do not ensures homogeneous distributions of normal frequencies but straight liness correspoindeing to particular *p* and/or *q* values, lead to high scores of transmission irregularities. The most significant values are 0.5 , 0.25 for *p* or *q*, and 1, 0.5, and 0.3 for *q/p.* Quite similar results were found by Sepmeyer[7]. For these *p,q* couples probability density functions of normal frequency spacing in rooms [6] should be interpreted in a statistical sense as average descriptions of the whole of possible rooms.

Proper choices of m_a and m_b can give to the previous indices a global or local character, assumed the number of normal frequencies to be statistically significant. Of particular interest are local indices for 1/3 octave frequency bands.

REFERENCES

[1] E. C. Wente, The characteristics of sound transmission in rooms, J. Acoust. Soc. Am. 7,1935,123-126 .
[2] R. H. Bolt, Normal frequency spacing statistics, J. Acoust. Soc. Am. 19,1, 1947, 79-90.
[3] K. Bodlund, A new quantity for comparative measurements concerning the diffusion os stationary sound fields, J. Sound Viv. 44(2), 1976, 191-207.
[4] M. R. Schroeder, Frequency correlation functions of frequency responses in rooms, J. Acoust. Soc. Am. 34,1962,1819-1823.
[5] R. H. Bolt, Note on normal frequency spacing statistics for rectangular rooms, J. Acoust. Soc. Am. 18,1,1946,130-133.
[6] M. R. Schroeder, Eigenfrequenzstatistk und Anragungsstatistik in Raümen, Acustica 4, 1, 1954, 456-468.
[7] L. W. Sepmeyer, Computed frequency and angular distribution of the normal modes of vibration in rectangular rooms, J. Acoust. Soc. Am. 37,3,1965,413-423

ACKNOWLEDGEMENTS. Part of this work has been supported by the National Spanish Research Program, contract no. AMB98-1029-C04-01