QUALITATIVE BRIEF INTRODUCTION TO OPERATIONAL TRANSFER PATH ANALISYS AND A TIRE NOISE APPLICATION CASE

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Rodríguez Fernández, Cástor[1]; San Millán-Castillo, Roberto[2]; De Klerk, Dennis[3]; Barten, Emiel.[3]

[1] Sound of Numbers, S.L.

15229 Ames - A Coruña

Spain

[2] E.T.S.I. Telecomunicación – Universidad Rey Juan Carlos

Camino del Molino, s/n

28943 Fuenlabrada - Madrid,

Spain

[3] Müller-BBM VAS B.V.

Zwolle,

The Netherlands

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ABSTRACT

The Operational Transfer Path Analysis (OTPA) method is an alternative to classical TPA, applied mainly in automotive and aerospace industries in a NVH framework. OTPA provides sound and vibration transmission path contribution to the sound field, requiring less resources than traditional TPA, when it is properly performed. This paper reviews its theory basis where clear similarities are found with experimental modal analysis. The method is extended with the singular value decomposition method to reduce influences of noise. Boundary conditions in practical application are a remarkable issue to consider. An analysis on tire noise is included to illustrate this method strengths.

RESUMEN

El método de análisis de caminos de transmisión operativa (OTPA) es una alternativa al TPA clásico, empleado principalmente en las industrias de la automoción y la aeronáutica dentro del marco del ruido, vibraciones y molestia (NVH). OTPA proporciona la contribución de los caminos de transmisión del sonido y las vibraciones con menores requerimientos que un TPA tradicional, siempre que se ejecute correctamente. Este artículo revisa la teoría básica en la que se manifiestan claras semejanzas con el análisis modal experimental. El método se amplía con el método de descomposición en valores singulares para reducir la influencia del ruido. Las condiciones frontera es otro asunto importante a tener en cuenta. Se incluye un análisis de ruido de neumático para ilustrar las fortalezas del método.

INTRODUCTION

Transfer Path Analysis(TPA) is and advanced and valuable methodology employed in industry when it comes to work out difficult problems in an NVH environment. Its need rises from the requirement to provide better noise and vibration products. TPA has been developed since the 60s from different points of view. Nowadays, the very theoretical first framework has been smoothed for practitioners and TPA techniques start being smart and affordable from a practical and daily basis point of view [1].

Operational transfer path analysis (OTPA), using cross talk cancelation (CTC) and singular value decomposition (SVD), is a signal processing method which finds the linearized transfer function (TF) matrix between a set of chosen input and output channels from a measurement. The in- and output relations are determined such, that the transfer functions are linearly independent with respect to each other, hence the name CTC. The resulting transfer functions can be used in a transfer path analysis (TPA), determining a source's propagation of noise and the resulting content in the response signal. The OTPA uses the singular value decomposition (SVD) algorithm to find independent principal components describing the transfer functions. In practice, the numerical operations involved often suffer from measurement noise. By rejecting smaller principal components, one reduces these influences on the TF estimates. Basically, OTPA is based on work of Bendat et al. [2]. The goal of this paper is to obtain a better understanding of the OTPA method, highlighting its capabilities and point of attention in its application. Besides, a case on tire noise OTPA analysis is included.

OTPA vs CLASSICAL TPA

Classical TPA which are based on Frequency Response Functions (FRF) measurements from different approaches can be classified as follows: A)TPA approach based on interface force [3]; B)The TPA based on matrix inversion method [4,5,6]; C)The mount stiffness TPA method [7,8]; D) The gear noise propagation (GNP) or component TPA method [9,10].

These methods basically consist of two steps. First, FRFs are determined between defined input/reference points and chosen output point. They are determined by use of impulse hammer/shaker if structural vibration is considered and/or by use of loudspeaker for air-borne. Secondly, these FRFs are combined with operational forces determined at the reference points to generate synthesized response signals. Those forces are determined in different ways depending on the method chosen. The synthesized output can thereafter be analysed, determining the contribution of each propagation path.

The OTPA method uses a one-step approach and builds a model of a structure without FRF measurements by hammer, shaker or loudspeaker. Basically, the method uses a response to response transfer function matrix, also known as transmissibility matrix when accelerators are employed, to represent the propagation paths of the structure. All signals are collected from a measurement of the operating system, so that implicitly the operating excitations are used to determine the transfer paths. Compared to the FRF approaches one can make the following remarks:

- The OTPA is very easy and fast to setup as it uses only an operational measurement. A large reduction in analysis time can therefore be achieved compared with FRF approaches. And so, operational influences are accounted for.
- Air-borne noise has a spatially complex distributed sound field on the excitation source. It is difficult to reproduce this sound field with loudspeakers, yet OTPA uses the actual excitation source to determine the TF.
- Careful design of the OTPA model of the analysed system is required.

OTPA METHOD INTRODUCTION

The OTPA method tries to find the (linearized) transfer function (TF) matrix between a chosen set of input and output quantities from a measurement. These sets of input and output can best be seen as degrees of freedom (DoF) describing the measured object's excitation (inputs) and the object's responses (output) as a linear combination of the chosen/assumed excitations. Firstly, the OTPA theory with least-squares algorithm is introduced. Consider an arbitrary linearized system model described by a set of input and output DoF, represented as:

$$\mathbf{H}(j\omega)\mathbf{x}(j\omega) = \mathbf{y}(j\omega). \tag{1}$$

, where H(jω) is the complex frequency domain transfer function matrix that links input DoF *x(jω)* signals to the output DoF signals vector *y(jω)*. In NVH problems, the measured signals are typically motions, denoted *u(jω)*, forces *f(jω)* and sound pressures *p(jω)*. The input and output vectors can thus in general be assembled from these quantities as:

$$\mathbf{x} = \begin{bmatrix} \mathbf{u}_x \\ \mathbf{f}_x \\ \mathbf{p}_x \end{bmatrix}; \quad \mathbf{y} = \begin{bmatrix} \mathbf{u}_y \\ \mathbf{f}_y \\ \mathbf{p}_y \end{bmatrix}, \qquad \mathbf{f}_x = [f_x^{(1)}, \ldots, f_x^{(l)}]^T; \quad \mathbf{f}_y = [f_y^{(1)}, \ldots f_y^{(o)}]^T,$$
$$\mathbf{p}_x = [p_x^{(1)}, \ldots, p_x^{(m)}]^T; \quad \mathbf{p}_y = [p_y^{(1)}, \ldots, p_y^{(p)}]^T, \tag{2}$$
$$\mathbf{u}_x = [u_x^{(1)}, \ldots, u_x^{(k)}]^T; \quad \mathbf{u}_y = [u_y^{(1)}, \ldots, u_y^{(n)}]^T,$$

, where the dependency on frequency is omitted for clarity. Indices *k, l, m* denote the number of input channels for the different quantities and *n, o, p* the number of output channels for different quantities, respectively. It is up to the Engineer to define the input and output sets from the measured data. Not all physical quantities have to be present in each set defined in (2), neither do the vectors have the same dimensions. In fact, usually the number of excitation channels in the input vector *x* will be larger than the number of DoF in the response/output vector. In NVH analysis a typical example is to find the transfer functions between motions measured on the driveline and the sound pressure at the driver's ear.

Typically, this matrix element property is used in Experimental Modal Analysis (EMA), where an external applied force *f* is applied (e.g. x_j), by shaker or impulse hammer, as only input DoF (e.g. $x_k = 0; k \neq j$) and the resulting responses *u* of the system are chosen as outputs (e.g. *y*). These kinds of TFs are denoted receptance frequency response function (FRF) in the literature and have the special property that their frequency peaks show the free system's eigenfrequencies. Strictly, one could thus determine a column of the TF matrix in the OTPA method also by exciting the system with only the input DoF x_j, while suppressing all other input excitations. In practice this is very hard to achieve as inputs are not only forces, but also motions, sound pressures or any kind of quantities. The determination of the transfer functions element wise will therefore often lead to very difficult, impractical and often impossible experimental setups. Analysis as such will therefore require a big expense in time and resources. To overcome this disadvantage, OTPA tries to determine all TF matrix elements from one measurement only where all excitations are at once. This determination is discussed next by first taking the transpose of (1) and writing the equation on entry level:

$$[x^{(1)}, \ldots, x^{(m)}] \begin{bmatrix} H_{11} & \ldots & H_{1n} \\ \vdots & \ddots & \vdots \\ H_{m1} & \ldots & H_{mn} \end{bmatrix} = [y^{(1)}, \ldots, y^{(n)}]. \tag{3}$$

Here *m* and *n* denote the number of in and output DoF. Taking the transpose does not allow the determination of the TF elements though. In order to do so, notice that during an operational measurement of, for example, a vehicle run-up on a dynamometer, a set of synchronized measurement blocks will be stored. In general, these sets will not have the same content, as the excitations change continuously during the measurement. If one requires, or defines, the relation between the input and output DoF as being linear(ized) and constant during the total measurement, (3) should, however, hold for each individual measurement block. One could thus extend (3) writing the equation for all measurement blocks *r*, yielding:

$$\begin{bmatrix} x_1^{(1)} & \dots & x_1^{(m)} \\ \vdots & \ddots & \vdots \\ x_r^{(1)} & \dots & x_r^{(m)} \end{bmatrix} \begin{bmatrix} H_{11} & \dots & H_{1n} \\ \vdots & \ddots & \vdots \\ H_{m1} & \dots & H_{mn} \end{bmatrix} = \begin{bmatrix} y_1^{(1)} & \dots & y_1^{(n)} \\ \vdots & \ddots & \vdots \\ y_r^{(1)} & \dots & y_r^{(n)} \end{bmatrix} - \mu. \tag{4}$$

This formulation, or system model, now requires the TFs to be linearly independent with respect to each other, hence the name cross talk cancelation. Although the input quantities might (and most often will) be coherent with respect to each other, the calculation of the transfer function matrix compensates for it. Here it is assumed that the experiment is performed such that the number of measurement blocks is bigger than the total amount of in- DoF, e.g. $r > m$. This approach makes (4) a solvable least-squares optimization problem with an additional residue μ for the content which cannot be modelled by the (chosen) set of input DoF. In general, individual observations/measurement blocks will contain a distortion due to, for example, measurement noise or additional unmeasured excitation sources that are not considered in the model.

To solve (4) one can first simplify its formulation writing it in a more compact way in matrices:

$$\mathbf{XH} + \mu = \mathbf{Y}. \tag{5}$$

The calculation needs to be performed for each individual frequency line of the FFT spectrum. Solving (5) for each frequency is now performed, in explicit sense, by pre-multiplying the equation by X^T. The TF matrix is thereafter found as:

$$\mathbf{H} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{Y} = \mathbf{X}^{+}\mathbf{Y}, \quad \Rightarrow \quad \mathbf{H} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{Y} = \mathbf{G}_{xx}^{-1}\mathbf{G}_{xy},$$

(6,7)

A more detailed mathematic explanation can be found in a previous paper [11] and proof that the OTPA method is considered purely as a least-squares calculation. It can be observed that the method is equivalent to the MIMO technique of finding FRF estimates, if the same input and output variables are chosen.

Coherence between input signals is often seen as an issue in OTPA [12,13]. However, shaker signals will be partly coherent with respect to each other in MIMO techniques as well. Indeed, as they are all connected to the test structure simultaneously, their vibrations propagate to one another and are measured by all force sensors. Furthermore, a limited number of measurement blocks is used for the MIMO calculation, rendering some remaining correlation, and thus coherence, among the excitation signals as well. Input channels coherence might well exceed 40% in practice for OTPA. In order to reduce the influences of measurement noise in such events, the OTPA algorithm is extended with a singular value decomposition to overcome this problem. OTPA typically does not determine receptance FRF, but transfer functions also known as transmissibilities, which describes the isolation of a system. Amplitude peaks and drops over a frequency do therefore not necessarily refer to resonances or anti-resonances of the system and actually represents two points modal amplitudes ratio. Then, OTPA can be enhanced by signal processing means, applying SVD. The explicit determination of the transfer function matrix *H* can cause erroneous estimates if input signals are highly coherent in combination with measurement noise. Use is therefore made of SVD, to prevent poor estimates. Indeed, matrix *X* can be expressed by a singular value decomposition as:

$$\mathbf{X} = \mathbf{U\Sigma V}^T \tag{8}$$

U is an *r x r* unitary matrix, ∑ is a *r x m* matrix with nonnegative numbers on the diagonal (as defined for a rectangular matrix) and zeros off the diagonal. V^T denotes the conjugate transpose of *V*, an *m x m* unitary matrix. The SVD is very general in the sense that it can be applied to any *r x m* matrix. A standard eigenvalue decomposition, on the other hand, can only be applied to certain classes of square matrices. Nevertheless, analytically, the SVD can be determined by an eigenvalue decomposition by the following relations:

$$\mathbf{X}^T\mathbf{X} = \mathbf{V}\boldsymbol{\Sigma}^T\mathbf{U}^T\mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^T = \mathbf{V}(\boldsymbol{\Sigma}^T\boldsymbol{\Sigma})\mathbf{V}^T, \quad (9)$$

$$\mathbf{X}\mathbf{X}^T = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^T\mathbf{V}\boldsymbol{\Sigma}^T\mathbf{U}^T = \mathbf{U}(\boldsymbol{\Sigma}^T\boldsymbol{\Sigma})\mathbf{U}^T. \quad (10)$$

Mathematical complete development can be reviewed in [11] and yields an estimate on the TF matrix *H* using the SVD method as:

$$\tilde{\mathbf{H}} = \mathbf{V}\boldsymbol{\Sigma}^{-1}\mathbf{U}^T\mathbf{Y}. \quad (11)$$

From an engineering and statistical point of view, it was found in applications that smaller singular values are mainly caused by noise influences and other external disturbances [14]. They are therefore unwanted and should be rejected. Note that the least-squares fit for the analysed measurement will be better with all singular values kept though. Yet the amount of used singular values in the TF calculation of one measurement reveals a trade-off in the resulting fit on another, similar, measurement. As the noise will be different in measurements, this cross-validation process reveals which of the smallest singular values are related to the noise influences [15,16,17]. Taking only a reduced set of singular values into account therefore improves the TF estimates in general. Details on formula are described in other referenced work [11].

OTPA PRACTICAL CONSIDERATIONS

Here the following issues are brought forth: 1) OTPA model design: source, transition and response locations;2) Quality of OTPA model using least-squares residue; 3) Variation in the structure's excitation; 4) Coherence between input/excitation signals (Discussed in the previous section)

OTPA model design: source, transition and response locations

An accurate OTPA system model essentially requires a model definition which implicitly represent the systems dynamics best, employing only operational response signals. Vehicle (driveline) components are often decoupled from other parts. Measured responses on the engine, for example, will be dominated from the engines combustion and thus implicitly characterizes the combustion itself. One could also think of responses measured on the rear axle differential or at the wheel spindle. Such responses will characterize either the internal gear noise excitation or the road input from the tires. Hence such signal can be well used as input signals/variables. Moreover, a connection point of the driveline to the bodywork contains a combination of the engine, tire, and gear noise excitation. Therefore, such kind of locations cannot be used for source characterization although tell engineers at which bodywork connection most noise is propagated.

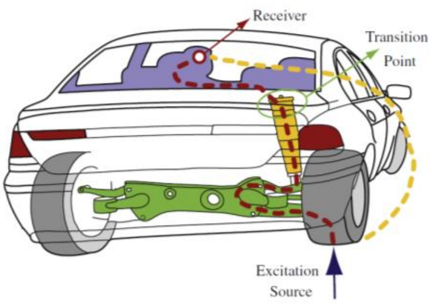

Figure 1. Different acquisition location in an example vehicle.

Locations are classified on any structure in source, transition and response positions, as schematically represented in Fig. 1 for a vehicle. One should not combine channels of different location classes in the input/excitation vector in the OTPA algorithm. It is our experience that although the true structure excitation is not measured, building models with response data on properly chosen positions as model inputs gives, for example, a good estimation of the structure's noise propagation in automotive applications [18]. If one forgets a propagation path, coherent parts are redistributed over the other signals though. To make a correct interpretation and analysis of a structure in practice, the following considerations may be used as guidelines:

- Choosing response data measured on different sources as input signal allows to some separate measured output responses.
- Choosing several responses on one excitation source as input variables allows the identification of how the source's excitation propagate into its neighbouring component(s).
- Using transition locations as OTPA inputs allow one to determine which locations transmit vibration the most. They do not indicate the origin of the source.

Quality of the OTPA model

One way to test the quality of the OTPA model is to verify if the synthesized responses are similar to the measured responses. Observed discrepancies are caused by either: input and/or output signals contain additional noise content, which is filtered by the OTPA algorithm; chosen input signals are not the only sources which contribute to the response signal; system might behave nonlinear.

The TF matrix will represent the average transfer functions during the measurement. If the TF matrix changes considerably this could indicate nonlinear system behaviour.

Variation in the structure's excitation

It is important to vary the input quantities of matrix X during the operational measurement as much as possible. Larger variation minimizes the coherence between the chosen input channels and results in a better conditioning, i.e. higher values of the lowest singular values, of the input matrix. As such, the noise influence is minimized, yielding accurate TF matrix estimates. Indeed, during the vehicle run-up excitation sources change continuously in amplitude and direction. It was noticed in [18] that care should be taken, as higher frequencies are excited by less engine orders, hence less variation might be expected at higher frequencies. The question which often rises is: what level of variation is required for an accurate OTPA. In [19], where diesel combustion is OTPA analysed, it is suggested that 10–15 dB should be sufficient to reduce noise influences.

TIRE NOISE OTPA APPLICATION DISCUSSION AND RESULTS

This section discusses a tire noise OTPA with specifications listed here:

- Vehicle Type: Volkswagen Golf 5.
- Tires: 6 different types from slicks to off-road profiles and Summer to Winter tires.
- Transmission: 2-wheel drive coast-down.
- OTPA setup: Acceleration sensors were placed at the wheel connection to the vehicle and the microphones for lead, trail and side positions. No transition location used.
    - Sources 3D ICP acc. Sensor at left and right front tire (FL/FR) hubs in global x, y, z directions (X, Y, Z). These global vehicle directions are X: longitudinal, Y: lateral, Z: vertical. 1/2 in. Microphones at left and right front tires at their lead, side and trail position (Le, Si, Tr) about 2m apart and aside from the wheel. Air-borne and structure-borne paths are denoted with (AB) or (SB), respectively.

- Receiver Driver's ear (1x) (MIC)

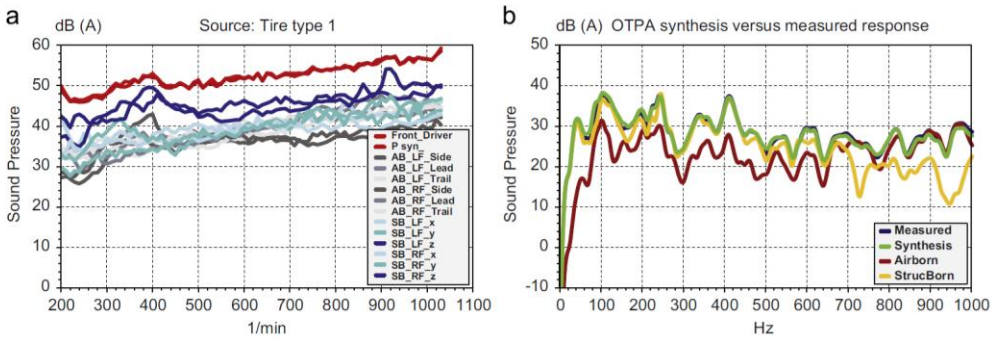

Figure. 2. (a) Microphone at the driver's ear response signal, separated in individual path contributions using the OTPA method and re-synthesis of the resulting time signal. (b) Averaged auto power spectra of the synthesized path contributions from the complete vehicle coast-down compared to the originally measured response.

One tire combination, with different tire types left and right, is analysed on the dynamometer's smooth and rough road surface. Thereafter, all 12 measurements will be compared to analyse how the OTPA results change with different configurations.

Analysis of a single measurement

After the OTPA model is calculated, one first starts with a comparison of synthesized output channels and their actually measured ones. After the transfer functions are determined from (11) in a least-squares sense, using them as FIR filters allows to synthesize the output channels in $\tilde{Y}$ with the originally measured inputs X. Fig. 2.a shows this comparison for an arbitrary tire set.

Overall sound level is depicted during the coast-down to get a good overview. The figure shows that the total synthesized sound level at the driver's ear matches very well with the original measured signal, e.g. the red curves overlap. This indicates, that on an overall level, the OTPA model is well able to describe the receiving sound pressure at the driver with the six structure-borne and six air-borne channels. The other curves in blue and grey show the individual path contributions from each of the 12 input/source channels. Observe that depending on the vehicle speed, individual path dominance varies.

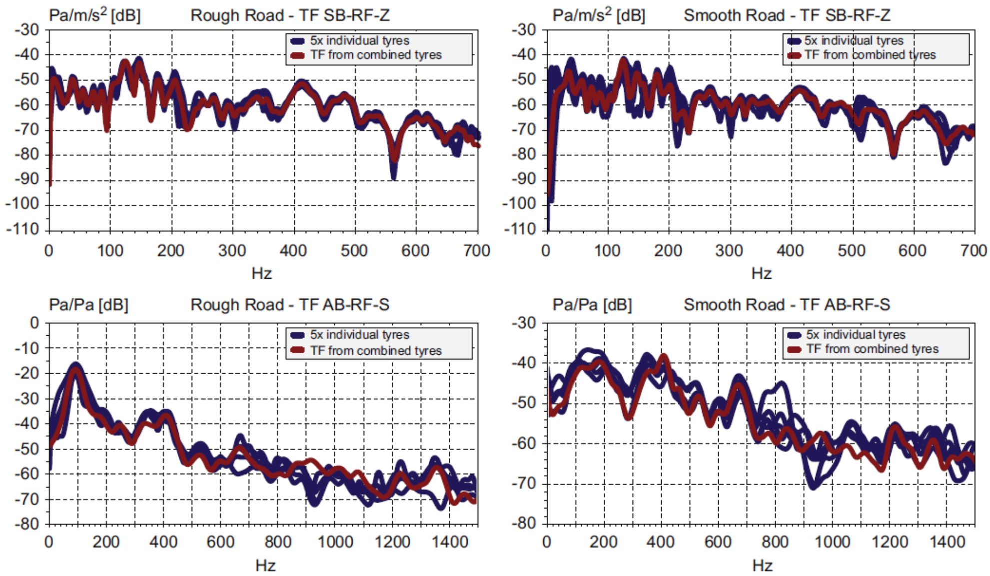

Figure 3. Transfer functions of the dominant path are quite similar for the different tyre types, especially for the structure-borne noise; larger differences are found for the air-borne path.

The structure-borne excitations in z-direction clearly have the largest overall impact on the driver's sound level and are therefore most worthwhile optimizing. It can be observed though that the left and right excitation not always yield the same contribution, which is due to the two different tires left and right. From Fig. 2a one does not have an indication what frequency range should be tackled and/or optimized. In a second step, one therefore proceeds to compare the frequency content of the synthesized and measured outputs. In order to do so, all auto power spectrum (APS) measurement blocks of the complete coast-down are averaged. This gives an overall indication of the model's fit in frequency over the complete run. As seen in Fig. 2b, the model fits the measurement well up to a frequency of about 1000Hz. At higher frequencies, not shown here for clarity, differences get into play above 1500Hz. Evaluation of the coherence between the microphones above 1500Hz also reveals a clear drop. Therefore, one can conclude that the dominant air-borne propagation path cannot be properly measured above 1500Hz, spatially, with 6 microphones next to the vehicle. Interesting to note is the different air-borne and structure-borne dominance over frequency. On average structure-borne vibration dominates up to frequencies of about 700Hz. At higher frequencies, air-borne vibration becomes the dominant one for the tested vehicle. As the highest amplitudes occur below 700Hz, the structure-borne dominance on the overall level, see Fig. 2a, is well explained.

Secondly, a parameter study is conducted with different tires and rough/soft road conditions. Fig.3 shows how the TF matrix changes. From the figure it can be seen that the transfer functions stay quite constant, although the corresponding excitation changed quite a bit. This shows that the OTPA method is capable of identifying stable results, and that an indication of all measurements simultaneously in the TF matrix calculation yields some average estimates.

CONCLUSIONS

This article shows that the OTPA algorithm, without SVD, is equivalent to the EMA technique of finding FRFs. OTPA allows for any kind of signal, like sound pressure and acceleration, for the definition of the OTPA model. As this kind of signals are responses from a physical perspective, this article introduces guidelines for a proper definition. One can group locations on a system in source, transition and response nodes. The SVD algorithm is required for an accurate transfer function matrix estimation as coherence between input signals can be quite large. It was found that measurement noise reveals itself in the smallest singular values [14].

A tire noise OTPA analysis can determine structure-borne and air-borne dominance easily. The structure-borne TFs do not depend on the tyre type and road condition much, showing that the OTPA is independent on the operational condition. Yet the parameter study also reveals that the air-borne path does show a large difference up to about 150 Hz due to the difference in directionality of the tire on smooth and rough road. A simple and affordable measurement setup revealed to draw remarkable conclusions regarding noise and vibration contribution paths. From the resources point of view, impact hammer and/or shaker FRFs and dismounting harshness sources are avoided; so a large amount of time is saved. Finally, specific operational sources constraints are included in the model, so it is a more real one than those performed with traditional TPA based on uncoupled/unforced noise and vibration sources. OTPA shows operational advantages that could benefit industry in NVH problems assessment and solution, based on TPA theory but enhanced with smart signal processing algorithm.

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