FEM-BASED OPTIMAL DESIGN FOR THE TUNING OF *SIXXEN* INSTRUMENTS

Vincent Debut[1,2], *Rui Borges Maia*[3], and *Pedro Carneiro*[3]

[1]Instituto de Etnomusicologia, Centro de Estudos em Música e Dança, Universidade Nova de Lisboa, Lisboa, Portugal
[2]Instituto Politécnico de Castelo Branco – Escola de Artes Aplicadas, Castelo Branco, Portugal
[3]Orquestra de Câmara Portuguesa, Algès, Portugal

RESUMEN

Para su composición *Pléiades*, Iannis Xenakis imaginó un instrumento de percussion que se llama *Sixxen*. Inspirado en las sonoridades de la música da Indonesia, consta de una serie de 19 barras metálicas dispuestas según un sistema de afinación microtonal construido con intervalos musicales desiguales. Con motivo del centenario de Xenakis, el sexteto de percusión de la Orquesta de Cámara Portuguesa decidió interpretar esta composición, para la que se construyó específicamente un conjunto de seis *sixxen*. Los instrumentos diseñados se fabricaron con barras de aluminio de sección transversal en forma de U, utilizando dos geometrías diferentes. Para un perfil de barra determinado, la sintonización se logró mediante una metodología de dos pasos que incluía la actualización del modelo. Combina análisis modal 3D, técnicas de optimización y mediciones acústicas, y persigue la afinación del modo radiante dominante ajustando la longitud de la barra. Cada instrumento se afinó según una escala musical predifinida, construida con una estructura similar a una escala da Indonesia e incluyendo variaciones aleatorias. El método da lugar a instrumentos con desviaciones absolutas de afinación típicamente inferiores a 7 centésimas, es decir, inaudibles, lo que ilustra la eficacia de la metodología propuesta.

ABSTRACT

For his composition *Pléiades*, Iannis Xenakis imagined a metallophone instrument that is called *Sixxen*. Inspired by the sonorities of Indonesia music, it consists of a series of 19 metal bars arranged according to a microtonal tuning system built with uneven musical intervals. On the occasion of Xenakis' centenary, the percussion sextet from the Portuguese Chamber Orchestra decided to perform this composition, for which a set of six *sixxen* was built specifically. The designed instruments were made of aluminium bars with U-shape cross-section, using two different geometries. For a given bar profile, tuning was achieved in a two-step methodology including model updating. It combines 3D modal analysis, optimization techniques and acoustic measurements, and pursues the tuning of the dominant radiating mode by adjusting the bar length. Each instrument was tuned according to a predifined musical scale, built with a similar structure to an Indonesian scale and including random variations. The method leads to *sixxen* instruments with absolute tuning deviations typically below 7 cents, that is inaudible, illustrating the efficiency of the proposed methodology.

Keywords: optimal tuning, Finite Element modelling, bar percussion instrument, Xenakis, music acoustic.

1. INTRODUCTION

Written for a percussion sextet by Iannis Xenakis in 1978-79, *Pléiades* stands as a major work of 20th century music. Placing percussion at the centre of contemporary musical exploration, the piece requires a set of *sixxen*, an unusual percussion instrument imagined by the composer that incorporates the essence of his musical concepts. As specified in the performance notes of the score (see Figure 1), Xenakis requested six percussion instruments made of 19 metal bars arranged according to a microtonal tuning system built with uneven musical intervals, and with subtle deviations in pitch among the set of instruments. With that said, Xenakis, however, remained vague regarding the building and specific musical features of these instruments and in this way, offered freedom to musicians and instrument makers in defining their musical attributes. This has resulted in a considerable number of different interpretations of the instruments, built with different elements regarding bar shape, material and components (resonator/pedal), and leading to musical performances with large variations in tuning and timbre [1].

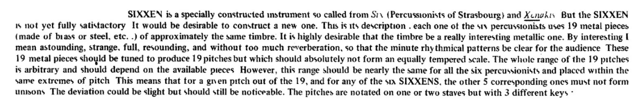
SIXXEN is a specially constructed instrument so called from *Six* (Percussionists of Strasbourg) and *Xenakis*. But the SIXXEN is not yet fully satisfactory. It would be desirable to construct a new one. This is its description: each one of the six percussionists uses 19 metal pieces (made of brass or steel, etc. .) of approximately the same timbre. It is highly desirable that the timbre be a really interesting metallic one. By interesting I mean astounding, strange, full, resounding, and without too much reverberation, so that the minute rhythmical patterns be clear for the audience. These 19 metal pieces should be tuned to produce 19 pitches but which should absolutely not form an equally tempered scale. The whole range of the 19 pitches is arbitrary and should depend on the available pieces. However, this range should be nearly the same for all the six percussionists and placed within the same extremes of pitch. This means that for a given pitch out of the 19, and for any of the six SIXXENS, the other 5 corresponding ones must not form unisons. The deviation could be slight but should still be noticeable. The pitches are notated on one or two staves but with 3 different keys:

Figure 1: Performance notes of *Pléiades*.

On the occasion of Xenakis' centenary, the percussion sextet from the Portuguese Chamber Orchestra decided to perform *Pléiades* and sought advice from the Musical Acoustics Laboratory of the Nova University for designing and tuning the six *sixxen*. Starting from readily commercially available metal bars, a tuning approach for the set of instruments was pursued. For a given

vincentdebut@fcsh.unl.pt

instrument, it aims at tuning the dominant radiating mode of each bar, following a musical scale based on precisely defined microtonal intervals. Since small detuning deviations must exist from one instrument to the next according to the composer, attention was paid to control the amount of musical beats produced when two instruments are played in unisson. Inspired by studies dealing with the optimal design of bars in mallet percussion instruments [2–5], the tuning strategy combines finite element (FE) modal analysis and optimization techniques. Moreover, because of the inherent variations in geometry and material properties present in industrially manufactured products, a two-step strategy was devised including model updating based on acoustic measurements. The present paper describes the methodology developed, starting from the definition of the musical scales up to the realization of the instruments, and discuss the developed tuning approach.

2. SELECTION OF THE BAR PROFILE AND MATERIAL

Since the sound qualities produced by idiophones are strongly influenced by the geometry and material properties of the vibrating component, the first aspect to consider before attempting any tuning was to select the bars attributes, shape and material, to be optimized. This issue, which is crucial for the sounding outcome of the performance, was addressed by the artistic director (PC) and the instrument maker (RBM), based on informal listening tests. A set of commercially available aluminium bars with different profiles and geometries was tested by striking each bar, recording their sounds and comparing their perceived qualities from a musical point of view. No acoustical descriptor was used for guiding the selection and no attempt was made to understand what make attractive the sound produced. Starting from a number of eight bars, the selection process results in the choice of two types of aluminium bars (alloy 1050) with U-shape cross-section. One for the six lowest tones of the instruments, the other for the thirteen highest tones (see Figure 2). Figure 3 shows the spectrograms obtained from the sounds of the two selected profiles obtained by impact excitation close to their center. A quick look on both plots evidences two different frequency contents and respective time evolution. The spectrogram corresponding to the lower tones shows a long duration with a large number of inharmonic partials that all have a long decay while the sound of the higher tones is shorter and clearly less complex. This will result in timbral changes along the musical scale, accompanied by a clearer sensation of pitch for the high register tones.

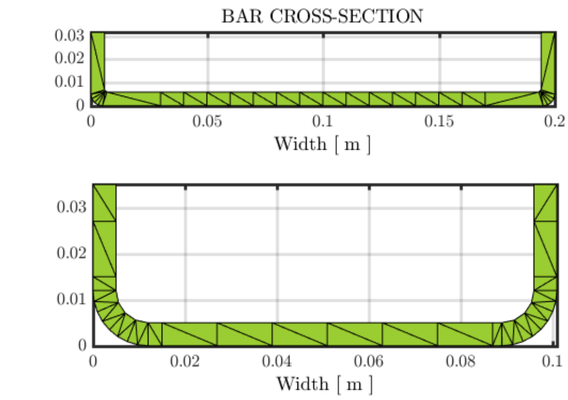

Figure 2: Cross section of the aluminium bars considered in this work for the lowest tones (top) and highest tones (bottom).

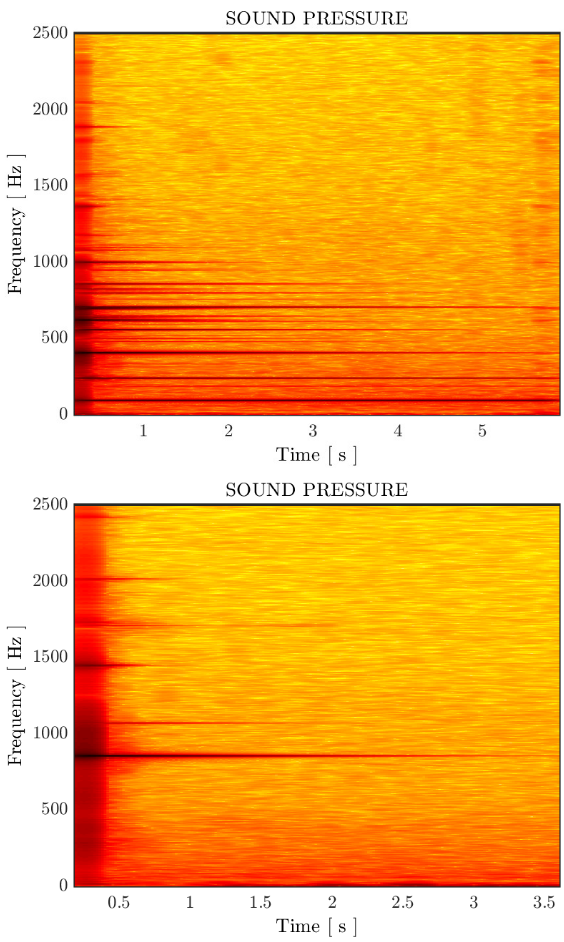

Figure 3: Spectrograms of the sound of the struck bars used for the low (top) and high registers (bottom).

3. DEFINITION OF THE MUSICAL SCALES

The instruments are tuned to the same set of notes, following an uneven microtonal scale but including slight deviations in pitch and interval size among the instruments in order to give a specific tone color to each *sixxen*. Following Xenakis' instruction, the reference scale is a non-octaviating scale, where no systematic musical patterns can be evidenced. Inspired by gamelan music, the reference 19-tone scale was calculated by algorithmic generation, based on extrapolation of a stretched *pelog* scale [6] and then compressing the entire new scale in order to fit within the predefined frequency range of the instruments, i.e. 92.48- and 932.32 Hz. The musical scales for the other five instruments were calculated by allowing small frequencies deviations around the frequencies of the reference scale, either positive or negative, that were generated by a random process. Attention was paid for avoiding crossing between the musical scales, meaning that when all instruments play the scales in unisson, the sounding scale is ascending. Moreover, the presence of sounding beats occuring when the same notes are played on different instruments was controlled. Based on informal listening tests, it was decided that the beating frequency must fall below 5 Hz, thus resulting in a range of pitch deviations of about 80 cents for the group of *sixxen*. To be specific, Table 1 presents the calculated tuning frequencies for the six instruments: these are the target frequencies for the following optimization of the bars.

Note #	*sixxen* 1	*sixxen* 2	*sixxen* 3	*sixxen* 4	*sixxen* 5	*sixxen* 6
1	92.499	94.488	94.677	91.935	90.305	90.887
2	108.968	108.466	109.845	110.402	111.253	111.269
3	113.715	114.342	111.402	116.000	115.021	111.755
4	124.073	124.667	125.247	126.271	126.015	126.362
5	148.748	147.089	148.374	146.688	146.512	150.456
6	163.396	161.525	161.556	162.283	161.737	162.741
7	198.293	196.529	198.659	197.419	197.898	198.795
8	208.352	210.360	206.611	207.395	206.082	207.981
9	230.299	231.509	229.208	232.187	232.248	229.612
10	282.586	281.270	281.362	280.946	280.815	284.451
11	313.624	312.751	311.914	315.282	312.986	314.170
12	387.568	388.901	385.278	388.574	386.054	386.804
13	408.881	410.739	409.710	410.231	407.132	406.734
14	455.386	457.675	456.820	454.798	454.776	456.220
15	566.177	564.134	567.004	568.165	566.982	563.950
16	631.944	630.918	631.236	632.765	630.367	633.225
17	788.627	787.597	790.649	787.113	790.066	790.828
18	833.788	834.681	835.658	831.925	834.877	832.311
19	932.328	931.870	932.739	933.728	930.412	934.564

Table 1: Musical scale of the six instruments.

4. OPTIMIZATION STRATEGY

Having selected the bar profiles, the next stage consists of implementing a pratical and efficient strategy for the tuning of the set of instruments. For struck percussion instruments, research has demonstrated the feasibility of precise and even difficult multi-modal tuning by devising sophisticated optimization strategies [2–5]. Implementation of these techniques to our case is relatively simple since only a single mode has to be tuned and a single parameter is involved in the geometry optimization. Two difficulties, however, emerge due to (i) the large number of bars to be tuned and (ii) the variability in geometry and/or mechanical properties of the original bar profiles. Indeed, the bars are necessarily cut from different aluminium profiles that inherently present small variations due to their industrial making process. Using the same fixed input parameters in all of our computations will hence result in differences between the actual and predicted vibratory behaviour, that then would be reflected in less-than-precise tuning in relation to the objectives. To overcome these difficulties, a two-step approach was retained. During the first stage, the bars are tuned to target frequencies slightly lower than the final objectives (40 cents) by perfoming first a global optimization and then a gradient-based optimization. Bars are then struck with a mallet and a simple spectral analysis is performed in order to identify the frequency of the dominant partials. If differences between the actual and predicted values are evidenced, improvement of the FE model is attempted manually in order to match the values, by changing the density and/or Young's modulus used in the computations. Finally, after frequency matching, a second gradient-based optimization is performed in order to converge to the final target frequencies, using the new adjusted parameters and starting from the actual length of the bar to be tuned.

In practice, the complete optimization of the group of *sixxen* was performed instrument by instrument, starting from the lowest tone and tuning the other bars following the musical scale.

4.1. FEM modal analysis

Predictions of the tuning are drawn upon a 3D finite element model of the bars, built from the geometrical measurement of their cross-section. The central idea behind FEM is to build a spatially discretized version of the structure i.e. a mesh, from which engineering analysis can be performed. Developed in the late 50s, this technique has been extensively used to tackle issues in vibration predictions, including in music acoustics [7].

The construction of the mesh of the bars starts with a series of simple geometrical measurements made using a slide gauge, assessing the width, height and thickness. Having a rough description of their cross-section, the complete surface geometry is then virtually created by extrusion along the direction perpendicular to the cross-section - corresponding to the bar length, and by using alpha shapes algorithms. From the surface mesh, the final solid 3D mesh is computed by using a classic mesh generator. All bar models are comprised of 10-node tetrahedron solid elements arranged in an unstructured mesh, for which the number of elements is adjusted according to the target frequency.

The tuning prediction of each bar is then carried out based on vibration analysis concepts. Modal computations aiming at computing the natural frequencies of the bars are performed assuming free-free boundary conditions since bars will be positionned close to the nodes of the first vertical bending mode, and using estimates of the mechanical parameters. The material density is calculated from the mass and volume of the bar, which are directly measured and estimated from the 3D model respectively. The Young's modulus and Poisson coefficient values used for starting the modal computations are the nominal known value of aluminium 1050, i.e. 71 GPa and 0.33 respectively. In practice, functions from the Matlab's PDE Toolbox were used in all the task dealing with FE computations.

Figure 4 presents the first modes computed for one of the U-shape aluminium bar. If all the modes have an inherent 3D character due to the non-uniform geometry of the bar, the mode series starts surprisingly with a torsional mode, followed by a vertical-bending mode and other higher order modes that are combinaison of bending (vertical and lateral), torsional and longituduinal motions. Of course, a mechanical approach as proposed here can only offer partial information regarding sound radiation but a quick comparison between the sounding and FEM-computed frequencies evidences that the dominant sounding frequency comes from the second mode. This seems consistent with the monopolar radiation characteristic of the second mode seen in Figure 4. This results also indicates that a classification of the modal families is essencial after each modal computation in order to assert the frequency of the major radiator to be accounted in the optimization procedure. To that end, we implement an automatic process for modal family identification following the work by Soares et al. [5].

4.2. Tuning optimization

Following the approach presented in [2], tuning is performed coupling a global and local optimization approach. The global optimization algorithm is first applied in order to search for a feasible region of solutions, and is then followed by a gradient-based optimization method in order to improve the convergence. As already mentioned, a method for automatic classification of the modes was also implemented in order to identify the first vertical bending mode which is responsable for the fundamental frequency of the sound radiated. As usual when performing

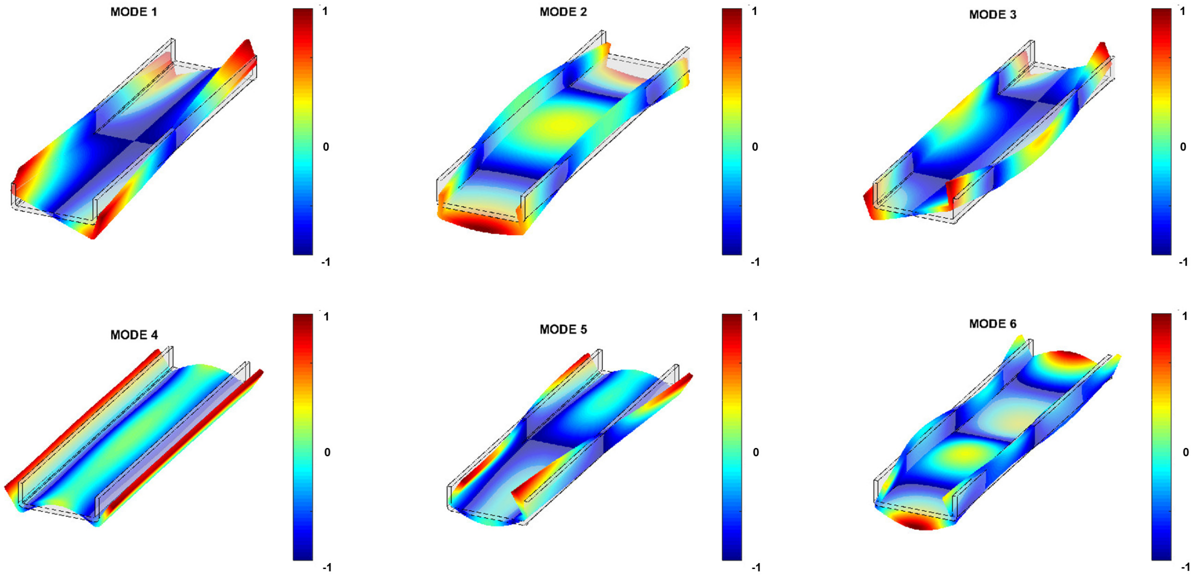

Figure 4: First six non-zero frequency modes computed by FEM for bar # 4. Modes are normalized such as $\max(|\varphi_n(\mathbf{r})|) = 1$.

stochastic optimization, the chosen global optimization method, here the simulated annealing method [8], relies on a large number of function evaluations that eventually leads to considerable computional costs. In our case, the optimization to perform remains very simple as it concerns a single variable, i.e. the length of the bar, and there was no real need, technically, of implementing such global optimization technique. With that said, the optimization consists of minimizing an objective function, here simply defines as the relative error given by:

$$\mathrm{Err}(\ell) = \left| \frac{f_c(\ell) - f_0}{f_0} \right| \tag{1}$$

where f_0 and f_c are the target and computed frequencies of the first vertical bending mode and ℓ is the length of the bar to be determined. For the lowest tone of every instrument, the search space for the bar length was in the range $3W < \ell < 1.3$ m where W is the bar width and the initial solution was set to $\ell_0 = 0.5$ m. Successive bars are then optimized sequentially, starting from the previous optimal solution and reducing the search space in order to accelerate the convergence.

Before performing all the optimizations, the developed strategy was validated. Given frequency values identified experimentally from different specimens, the tests consist of recovering the known bar lengths by performing the optimization. For the four tested cases, ranging bar lengths from 0.5 to 1.2 m, errors ess than 1% in bar length were found, thus validating the implementation of the tuning strategy.

4.3. Acoustic measurements

After each cut, acoustic measurements are performed for checking the frequency of the dominant radiating mode and eventually updating the FE model. This is achieved by strucking the bar to one of its extremities and using a small-diaphragm condenser microphone AKG SE300, placed in the close sound field, and connected to a PHOTON+ data acquisition system for frequency analysis. If necessary, model updating is performed manually by modifying the density and/or Young's modulus of the FE model based on physical reasoning until matching the actual and computed frequencies. With the new set of input parameters, the second optimization can be performed toward the final target frequencies.

5. CONSTRUCTION OF THE *SIXXEN* AND RESULTS

As described in Section 4, the construction of each instrument was achieved in two stages, ending with a final assessment of its overall tuning. In most cases, results were found in good agreement with predictions, falling within an acceptable tunability range. In other few cases however, bars were actually found out-of-tune with respect to the targets and need further manual tuning. This was no surprise because errors in the FE model and the construction process itself, make very unlikely that the final tuning is perfect. Indeed, the construction of the FE model relies on measurements and the procedure of model updating is not automatic. Certainly there were differences of detail in the bars geometry that could have been compensated by uncorrect values of the Young's modulus and density, and finally led to different modal behaviour. Moreover, bars were cut manually with a miter saw and errors in machine tolerance could have occured. As long as the bars were lower in pitch than predictions, fine adjustments were performed with a sanding machine, by removing material. For the more delicate case of bars with higher pitch, the construction process benefits from the large numbers of bars to be tuned, allowing bars from different instruments to be interchanged.

An overview of the final tuning results of the full set of instruments is given in Figures 5 and 6. Figure 5 compares the measured and target frequencies expressed in *cent*, which is the the micro-interval equal to one hundredth of a tempered semitone. Absolute tuning deviations are typically below 7 cents for all the instruments, with a maximum standard deviation of 6 cents.

Globally, each instrument shows an average tuning deviation of less than 2 cents over the musical scale. The plot in Figure 6 then shows the maximum of the beating frequency between all the instruments for each note of the musical scale. It shows that beating is in general less than 5 Hz, which is satisfactory according to the objective. It is worth noticing that beating frequencies of about 8 Hz are evidenced for some notes of the high register but this is the less-than-perfect tuning of these notes that contributes to this result. To give an illustrating view of the sound produced by a *sixxen*, Figure 7 is a spectrogram obtained by playing the musical scale on one of the built instrument. This particularly shows the expected global changes in timbre that occurs just before 2s due to the change of bar profile between bar numbers 6 and 7. Finally, Figures 8 and 9 present the final results of this project, with one of the *sixxen* and the set of instruments used during a live performance in Lisbon.

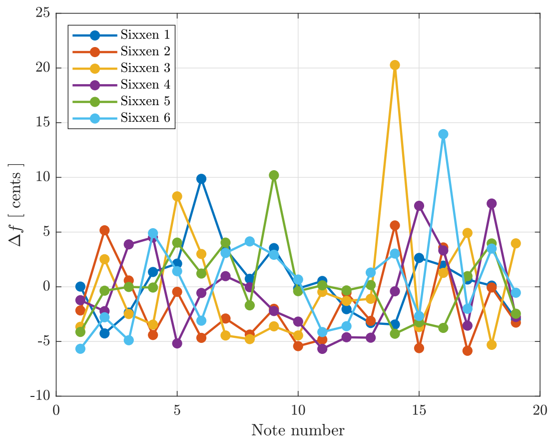

Figure 5: Tuning deviation for the set of *sixxen*.

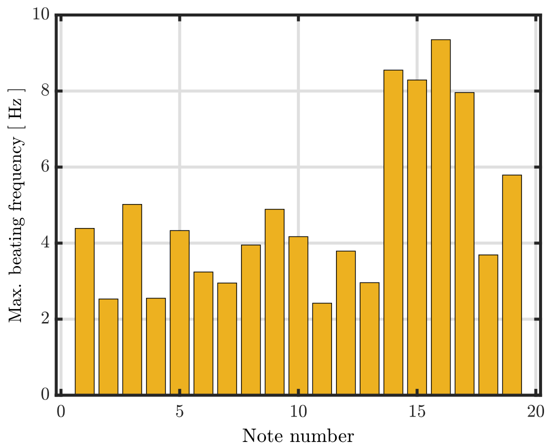

Figure 6: Maximum beating frequency among the set of *sixxen* calculated for the dominant sounding frequencies.

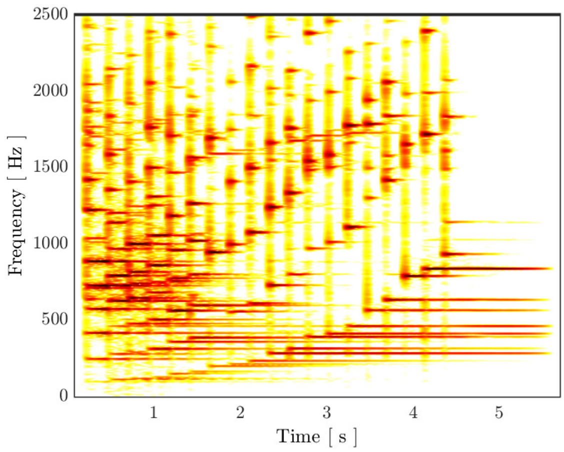

Figure 7: Spectrogram of the musical scale played on one *sixxen*.

6. CONCLUSION

Toward the performance of Xenakis' masterpiece *Pléiades* by the Portuguese Chamber Orchestra, this work presents a concrete application of techniques developed in music acoustics for tuning bar percussion instruments. The problem offered by the composer to construct a set of *sixxen* was successfully addressed by combining FE modeling and optimization procedures. From the musical point of view, specific musical scales were defined for every instrument inspired by the *pelog* scale that influenced the composer in his work. From the knowledge of the precise microtonal intervals, the bars were cut manually to optimal lengths stemming from optimization algorithms and using predictions of the bar modal behaviour computed by FE model. Overall, the approach produced totally satisfying tuning results with tuning deviations generally falling below 7 cents for the full set of instruments, which is inaudible, and with controlled musical beats between the set of instruments. It would be interesting for future research to explore the radiation properties of the bars in the instrument design, which are rather surprising, and develop optimization scheme focusing directly on the sound properties of the bars instead of accounting for their dynamical behaviour.

Figure 8: One of the six *sixxen* built.

Figure 9: Concert of *Pléiades* at the Museum of Art, Architecture and Technology (Lisbon, 27/11/2022). Credits: Bruno Vicente.

7. ACKNOWLEDGEMENTS

Our thanks to the team of the Portuguese Chamber Orchestra for their invaluable collaborations in the project.

8. REFERENCES

[1] Diederik Mark de Ceuster, "Use 19 metal pieces of approximately the same timbre," *Zeitschrift der Gesellschaft für Musiktheorie*, vol. 18/1, pp. 71–100, 2021.

[2] L. Henrique and J. Antunes, "Optimal design and physical modelling of mallet percussion instruments," *Acta acustica united with Acustica*, vol. 89, pp. 948–963, 2003.

[3] J. Petrolito and K. A. Legge, "Designing musical structures using a constrained optimization approachs," *J. Acoust. Soc. Am.*, vol. 117, pp. 384–390, 2005.

[4] D. Beaton and G. Scavone, "Optimization of marimba bar geome- try by 3D finite element analysis," in *Proceedings of International Symposium on Musical Acoustics 2019*, 2019, pp. 402–407.

[5] F. Soares, J. Antunes, and V. Debut, "Tuning of bending and torsional modes of bars used in mallet percussion instruments," *J. Acoust. Soc. Am.*, vol. 150, 2021.

[6] J. Harley, *Xenakis: his life in music*, Routledge, 1st edition, 2004.

[7] A. Chaigne and J. Kegomard, *Acoustics of musical instruments*, Springer-Verlag, New-York, 20016.

[8] P. van Laarhoven and E. Arts, *Simulated annealing: Theory and applications*, Reidel, Dordrecht, 1987.