ON THE CALCULATION OF TRANSMISSIBILITY FUNCTIONS FOR VIBRO-ACOUSTIC FINITE ELEMENT MODELS

Duarte Miguel Salvado Pereira[1]
Miguel Matos Neves[2*]
Nuno Manuel Mendes Maia[2]
Hugo Filipe Diniz Policarpo[2,3]

[1] DEM, Instituto Superior Técnico, Universidade de Lisboa, Lisboa, Portugal
[2] IDMEC, Instituto Superior Técnico, Universidade de Lisboa, Lisboa, Portugal
[3] CINAV, Escola Naval, Instituto Universitário Militar, Base Naval de Lisboa, Almada, Portugal

[*] ***Autor de contacto***: *miguel.matos.neves@tecnico.ulisboa.pt*

ABSTRACT

In recent years, the interest in the research of transmissibility functions rose for several applications, like Force Identification, Operational Modal Analysis, Operational Transfer Path Analysis, etc.

In this work, the authors analyze a formulation based on the Finite Element Method (FEM) and the computational effort of two methods used to obtain the transmissibility functions for acoustic and vibro-acoustic models.

In the first method, the required receptances are extracted from the inverse of the global dynamic stiffness matrix and multiplied to obtain the transmissibility between the involved responses. In the second, the required receptances are extracted from the respective adjugates of the global dynamic stiffness matrix to obtain the transmissibility.

A review and comparison of the two methods is presented for acoustic and vibro-acoustic models. To verify and compare both methods, two simple examples are presented. The transmissibility analyses are conducted using a FEM code from where the matrices are extracted and imported to a routine, developed in Matlab environment, used to compute the transmissibility functions estimated by these two methods. The simulation times and accuracy obtained using these methods are presented and briefly discussed. The obtained results confirm the expected significant computational time reduction without loss of accuracy.

Keywords— Transmissibility functions, vibro-acoustic, finite element, computational effort, adjugate matrix.

1. INTRODUCTION

The transmissibility concept has been successfully applied to several problems. In [1, 2], the main definitions of vibrational transmissibility are presented for Multi-Degrees-of-Freedom (MDOF) mass-spring systems. These formulations were latter applied to beams [3-6]. The concept was then extended to acoustics, and used to analyse simplified aircraft interiors [7, 8]. A recent application of this concept is in estimating responses in vibro-acoustic systems [9].

The numerical analysis of transmissibility functions may be time-costly for systems with high number of Degrees of Freedom (DOFs) [7].

In this work is applied a numerical technique that uses only the necessary DOFs of the system, thereby decreasing the simulation time required to compute these transmissibilities.

Here, the transmissibility functions of structural, acoustic, and vibro-acoustic systems, are obtained using the FEM to obtain the global matrices.

The methodologies developed by previous authors [7, 9], used the entire inverse of the dynamic stiffness matrix to obtain the receptance matrix. However, only a few entries of the receptance matrix are usually required to obtain the transmissibility. In this work, the authors use the required adjugate of the dynamic stiffness matrix to obtain the necessary entries of the receptance matrix.

The methodologies proposed reduces the computational effort required to obtain the steady-state transmissibility functions in the frequency domain. The effectiveness is evaluated by the comparison of simulation elapsed and CPU times. The simulation times obtained by using the adjugate of the dynamic stiffness matrix to obtain only the necessary entries of the receptance matrix are compared with the simulation times obtained by using the entire inverse of the dynamic stiffness matrix.

The reduction of the computational effort required to conduct the transmissibility analyses is essential in applications of the transmissibility concept. In addition, e.g., source identification in acoustic and vibro-acoustic systems require a considerable amount of computational effort. Therefore, the decrease of the computational effort is really relevant.

2. THE TRANSMISSIBILITY CONCEPT

In this section the theoretical fundamentals necessary to evaluate transmissibility functions in structural, acoustic and vibro-acoustic systems are presented.

2.1. Vibrational Transmissibility

The displacement transmissibility is described in [3], and involves the definition of a set of coordinates A where the forces F_A are applied, a set of coordinates U where the responses X_U are unknown, and a set of coordinates K where the responses X_K are known. Using these coordinate sets, one may write, in terms of the receptance matrix H, the following equations.

$$X_U = H_{UA} F_A \tag{1}$$
$$X_K = H_{KA} F_A \tag{2}$$

Eliminating F_A from the previous equations, one obtains

$$X_U = H_{UA} H_{KA}^{+} X_K = T_{UK}^{A+} X_K \tag{3}$$

For the transmissibility of forces, considering a set of coordinates K of known applied loads F_K, and a set U of unknown reactions F_U , one may write

$$\begin{Bmatrix} X_K \\ X_U \\ X_C \end{Bmatrix} = \begin{bmatrix} H_{KK} & H_{KU} \\ H_{UK} & H_{UU} \\ H_{CK} & H_{CU} \end{bmatrix} \begin{Bmatrix} F_K \\ F_U \end{Bmatrix} \tag{4}$$

where X_C are the responses at the remaining coordinates. Assuming that the responses X_U are equal to zero, one may obtain the transmissibility matrix in the following manner

$$F_U = -H_{UU}^{-1} H_{UK} F_K = T_{UK} F_K \quad (5)$$

2.2. Acoustic Transmissibility

An acoustic system can be modelled in the frequency domain by the following equation

$$(K_f - \omega^2 M_f + i\omega C_f) P(\omega) = Q(\omega) \quad (6)$$

where ω is the angular frequency, K_f and M_f are respectively the global stiffness and global mass matrices of the acoustic medium (fluid), P is the acoustic pressure amplitude vector and Q is the volume accelerator vector. Equation (6) can be rewritten using the frequency response matrix [H(ω)] in the following manner.

$$P(\omega) = H(\omega) Q(\omega) \quad (7)$$

Considering the three sets of coordinates illustrated in figure 2.1, one can obtain the pressure transmissibility. The set U is the set of coordinates where the pressures may be imposed, set K is the set of coordinates where the pressures are known, and set C is the set of the remaining coordinates.

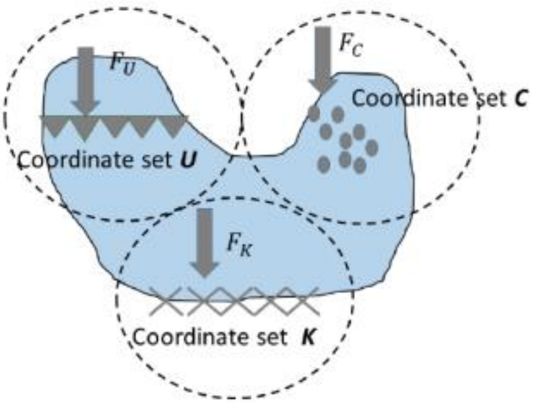

Figure 1. Schematic of the acoustic domain discretized in the sets of coordinates K, U and C [7].

Using these coordinate sets and assuming no loading in set C, one can rewrite equation (7) in the following manner

$$\begin{Bmatrix} P_K \\ P_U \\ P_C \end{Bmatrix} = \begin{bmatrix} H_{KK} & H_{KU} \\ H_{UK} & H_{UU} \\ H_{CK} & H_{CU} \end{bmatrix} \begin{Bmatrix} Q_K \\ Q_U \end{Bmatrix} \quad (8)$$

Now, solving the second line for Q_U, substituting it in the first line and considering no loads in set K (Q_K = 0), the following expression is obtained for a given pressure P_U [7]:

$$P_K = H_{KU} H_{UU}^{-1} P_U = T_{UK} P_U \quad (9)$$

which defines a pressure transmissibility matrix T_{UK}.

2.3. Vibro-Acoustic Transmissibility

To determine the transmissibility matrix in a Fluid-Structure Interaction (FSI) one must use its coupled model - indicated by ‘S’ for solid part, ‘*a*’ for acoustic fluid part and ‘C’ the interface- where the finite element equation may be given by

$$\left(\begin{bmatrix} K_S & K_C \\ 0 & K_a \end{bmatrix} + i\omega \begin{bmatrix} C_S & 0 \\ 0 & C_a \end{bmatrix} - \omega^2 \begin{bmatrix} M_S & 0 \\ -\rho_0 K_C^t & M_a \end{bmatrix} \right) \begin{Bmatrix} u_i \\ p_i \end{Bmatrix} = \begin{Bmatrix} F_{si} \\ F_{ai} \end{Bmatrix} \quad (10)$$

where K, M and C are the respective global stiffness, global mass and global damping matrices. The componentes u_i indicate nodal displacements, p_i the nodal pressures, F_{Si} the nodal loads in the solid part and F_{ai} the nodal loads in the acoustic fluid.

One may define the set of coordinates U where the displacements may be imposed, and the set of coordinates K where the pressure responses are known. Using the frequency response matrix and the described coordinate sets U and K, one may write the following equation [9].

$$\begin{Bmatrix} u_U \\ p_K \end{Bmatrix} = \begin{bmatrix} H_{UU} & H_{UK} \\ H_{KU} & H_{KK} \end{bmatrix} \begin{Bmatrix} F_U \\ F_K \end{Bmatrix} \quad (11)$$

Assuming that no forces are applied on K set ($F_K = 0$), one obtains the relation:

$$P_K = H_{KU} H_{UU}^{-1} u_U = T_{UK}^{FSI} u_U \quad (12)$$

where T_{UK}^{FSI} is the vibro-acoustic transmissibility matrix.

3. METHODOLOGIES

To determine the transmissibility functions, the authors use the Finite Element m (FE) model (using a comercial program e.g. ANSYS APDL) to define the geometries, physical properties and meshes of the model. The global matrices of the system are then extracted and imported to Matlab where the transmissibility functions are computed.

3.1. Inverse Matrix Method

Using the global matrices K, C and M extracted from the FEM analysis, one can obtain the dynamic stiffness matrix Z

$$Z(\omega) = K - \omega^2 M + i\omega C \quad (13)$$

Then, the frequency response matrix may be obtained by

$$H(\omega) = Z(\omega)^{-1} \quad (14)$$

After selecting the necessary entries of the receptance matrix, one may obtain the transmissibility matrix. Therefore, this method may be applied using the following steps:

• Import the global matrices from ANSYS to Matlab;

• Use the imported matrices to obtain the dynamic stiffness matrix *Z*;

• Create a cycle to run a certain frequency range. Within the cycle, obtain the *H* matrix by using the *H=inv(Z)* command, and the submatrices H_{UU} and H_{KU} by selecting the necessary entries of H. Then, the transmissibility matrix is computed using the respective equation (5, 9 or 12).

This method extracts the entire receptance matrix when only a few entries are needed. Therefore, one is extracting unnecessary data and using additional computational effort.

3.2. Adjugate Matrix Method

One alternative to the previous method is to use the adjugate of Z to extract only the necessary entries of *H*.

The adjugate of a certain square matrix $A_{n\times n}$ may be determined by the transpose of the co-factors matrix,

$$adj(A) = C^T = ((-1)^{i+j} M_{ij})^T \quad (15)$$

M_{ij} is the determinant of the (n−1)×(n−1) matrix that results from deleting row *j* and column *i* from *A*. The adjugate of *A* is related to its inverse in the following manner

$$A^{-1} = \frac{adj(A)}{|A|} \quad (16)$$

To obtain the entries of the adjugate matrix of Z in Matlab, one must take the following steps:

• Obtain the determinant of Z;

• Create a Zaux matrix equal to Z and remove the row j and column i from this new matrix;

• Compute the determinant of the resultant Zaux matrix;

• Use (16) to obtain the required entry of *H*;

• Repeat the process until needed entries have been computed.

Then, the transmissibility matrix is computed using the respective equation (5, 9 or 12). This calculation requires less computational time, but is subjected to posible overflows. To overcome this issue one may use a scaling of the matrix, but for large matrices one proposes to work with the logarithm of the determinant based on the LU decomposition of the matrix

$$\log(|\det(A)|) = \log(|u_{11}|) + \log(|u_{22}|) + \ldots + \log(|u_{nn}|) \quad (16)$$

to obtain the entry of the receptance matrix by

$$H_{ij} = 10^{\log(|\det(Zaux)|) - \log(|\det(Z)|)} \quad (17)$$

To keep the signal of the determinant, one needs to count the number of negative entries in the diagonal of U.

4. RESULTS AND DISCUSSION

Here one presents the results obtained using the two numerical approaches described in section 3.

4.1. Acoustic Transmissibility

This example concerns an acoustic tube similar to the one presented in [9]. The tube illustrated in Fig. 2 is 4 m long, 0.1 m wide, and is filled with an acoustic fluid. The mass density of the fluid is 1.21 kg/m^3, the boundary admittance is set to 0, and the sound speed is considered to be 344 m/s. The boundaries are rigid and reflective and no FSIs are considered. The model is constructed in ANSYS using FLUID30 FEs. The FEs have 0.04 m lengthwise . A 1 Pa pressure is imposed at one of the ends and the pressure response is measured at the mid-section of the tube along the center line.

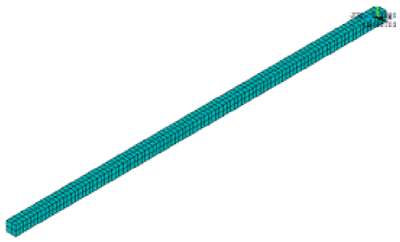

Figure 2. Schematics of the fully discretized acoustic tube.

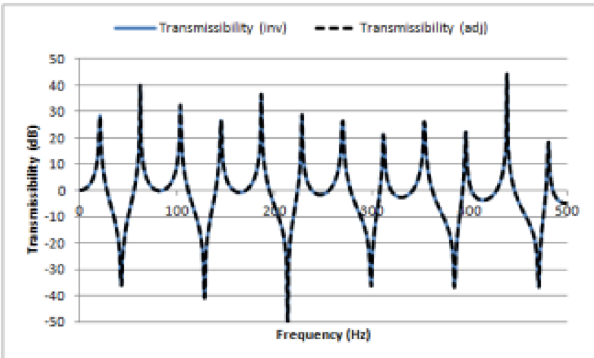

Figure 3. Acoustic transmissibilities (both methods).

Both methods present the same results (see transmissibilities in Figure3), but using the adjugate of *Z* it is posible to extract only the necessary entries (in this case only two entries are needed) of *H* instead of the entire matrix. In table 1 the simulation times needed to compute the transmissibility functions are presented .

Table 1. Times to obtain transmissibility (Z is a 909×909)

		CPU Time [s]
Adjugate(Z)	1	3.688
	2	3.679
	3	3.687
Inverse(Z)	1	424.151
	2	428.396
	3	426.517

4.2. Vibro-Acoustic Transmissibility

This example concerns a vibro-acoustic tube discretized with FLUID30 and SHELL181 FEs. The geometry of the tube is the same as in the previous example, but an elastic plate is introduced at one end of the tube. The physical properties of the acoustic fluid are the same as in the previous example, and the structural plate has a mass density of 7800 kg/m^3, a Young's modulus of 210 GPa, a Poisson coefficient of 0.3 and a thickness of 1 mm.

The system is discretized with 24 FEs per wavelength (as in [9]), and an FSI is defined in the FEs that are touching the plate according to [10]. As this model keeps the center line along the length of the tube, an 1 *N* harmonic load is imposed at the center of the plate, and the pressure response is obtained at the mid-section of the tube, along the center line. The displacement and rotation DOFs are considered to be fixed along the edges of the plate.

The transmissibility function (Fig.4) is thereby obtained between the node where the pressure response is measured, and the node where the load is imposed.

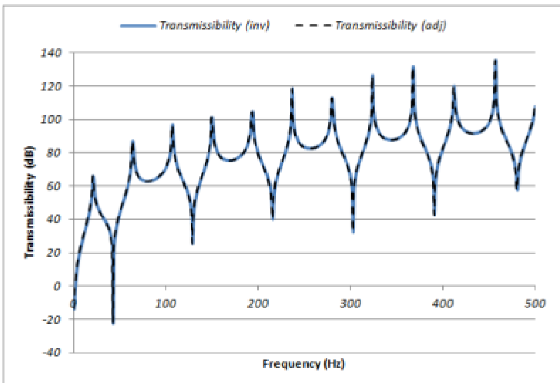

Figure 4. : Vibroacoustic transmissibilities **(**both methods**)**.

The simulation times obtained with both methods are presented in table 2. The Z matrix is a 501×501 matrix.

Table 2. Times to obtain transmissibility

		CPU Time [s]
Adjugate(Z)	1	2.072
	2	2.088
	3	2.065
Inverse(Z)	1	48.394
	2	43.390
	3	44.061

Table 2 shows that using the adjugate of Z can save a significant amount of simulation time when obtaining the transmissibility functions in a vibro-acoustic system.

5. CONCLUSIONS

The computation of transmissibility functions in acoustic and vibro-acoustic systems by using the full inverse of the dynamic stiffness matrix proves to be a time consuming and inefficient method. This approach uses unnecessary data and increases significantly the simulation time. The use of the adjugate matrix proves to be a reliable alternative to this method. It allows the user to obtain the same results, and to reduce the simulation time. In fact, by applying the method of the adjugate matrix to the problems discussed in the previous section, one was able to obtain the transmissibility functions more than twenty times faster. This is a significant improvement in the Matlab routines designed to evaluate transmissibility functions in acoustic and vibro-acoustic MDOF systems.

One major problem that arises from using the adjugate matrix to analyse transmissibilities, is the overflow in the determinants. For larger matrices, the determinants of the dynamic stiffness matrices tend to surpass the precision of Matlab. However, the methodologies described present some possible solutions to this problem.

The advances presented in terms of reducing the computational effort required to conduct acoustic and vibro-acoustic transmissibility analyses may also be extended to the field of vibrational transmissibility, in particular to the transmissibility of forces in structural systems. In addition, the methodologies presented enable the analysis of larger or more refined systems, and help in the solution of more complex problems, such as the problem of source identification in acoustic and vibro-acoustic MDOF systems.

In conclusion, this work presents a simple methodology to reduce the computational effort required to evaluate transmissibility functions in acoustic and vibro-acoustic systems, and the results presented prove its effectiveness. Furthermore, the advances achieved with this work are considered to be a relevant contribution to the field of transmissibility and, more specifically, to the field of vibro-acoustic transmissibility, which remains, to this day, relatively undeveloped.

ACNOWLEDGMENT

This work was supported by FCT, through IDMEC, under LAETA, Project UIDB/50022/2020.

6. REFERENCES

[1] M. M. Neves and N. M. M. Maia. Estimation Of Applied Force Using The Transmissibility Concept. In Proceedings of ISMA2010 Including USD2010, Pages 3887-3898, 2010.

[2] N. M. M. Maia, A. P. V. Urgueira, and R. A. B. Almeida. Whys and Wherefores of Transmissibility. In D. F. Beltran-Carbajal, editor, Vibration Analysis and Control - New Trends and Developments, chapter 10, Pages 187-216, ISBN 978-953-307-433-7, September 2011.
[3] Y. E. Lage, M. M. Neves, N. M. M. Maia, and D. Tcherniak. Force transmissibility versus displacement transmissibility. Journal of Sound and Vibration, Pages 5708-5722, DOI: 10.1016/j.jsv.2014.05.038, 2014.
[4] Y. Lage, N. Maia, M. M. Neves. Force Magnitude Reconstruction Using the Force Transmissibility Concept. Shock and Vibration, Volume 2014, Pages 1-9, DOI: 10.1155/2014/905912, 2014.
[5] N. M. M. Maia, Y. E. Lage, M. M. Neves. Recent Advances on Force Identification in Structural Dynamics. In D. F. Beltran-Carbajal, editor, Advances in Vibration and Structural Dynamics, chapter 6, Pages 103-132, ISBN 978-953-51-0845-0, DOI: 10.5772/3421, December 2012.
[6] Y. Lage, N. Maia, M. M. Neves, A. Ribeiro. A Force Identification Approach for Multiple Degree of Freedom Systems. Conference Proceedings of the Society for Experimental Mechanics, Series 4, DOI: 10.1007/978-1-4419-9831-6 7, 2010
[7] C. D. V. Guedes. Localização de Fontes Acústicas em Interiores de Avião Usando o Conceito de Transmissibilidade, Master Thesis (in Portuguese), Instituto Superior Técnico, 2016.
[8] C. D. V. Guedes and M. M. Neves. A model-based acoustic source localization using the MDOF transmissibility concept. In Proceedings of the EuroRegio 2016, June 13-15, Porto, Portugal 2016.
[9] V. M. N. Martins. Estimating Vibration, Acoustic and Vibro-Acoustic responses using Transmissibility functions, Master Thesis, Instituto Superior Tecnico, 2019.
[10] C.Howard. Coupled Structural-Acoustic Analysis Using ANSYS. Internal Report, The University of Adelaide, Department of Mechanical Engineering, Pages 1-15, 2000.