



STUDY OF DISCRETIZATION SCHEMES FOR A HYBRID METHODOLOGY FOR THE ASSESSMENT OF GROUND-BORNE NOISE IN BUILDINGS

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ABSTRACT

In this article, a hybrid experimental/numerical methodology for the assessment of ground-borne noise and vibration in buildings to be constructed close to ground vibration sources is studied in a three-dimensional context. This methodology is based on three steps: First, the free field response due to the incident wave field induced by a particular ground vibration source is measured at the collocation points. Second, a set of virtual forces that equivalently represent the free field response induced by that incident wave field is then calculated. Finally, this set of virtual sources is applied to a structure-soil model to determine the response of the structure. In this paper, synthetic measurements are used to study different aspects related to the discretization strategies to be adopted to ensure accurate performance of the method. Specifically, two schemes for the distribution of virtual sources along the auxiliary surface are studied. Also, an exploration regarding the minimum number of collocation points that allows for a proper characterization of incident wave fields in the presence of shallow foundations is also presented. Of special relevance is the included discussion about sensor setup requirements for in situ experimental campaigns.

Keywords – Ground-borne vibration, Railway-induced noise and vibration, Building acoustics.

1. INTRODUCTION

Due to the switch of urban railway infrastructures from at-grade to underground space, railway-induced ground-borne noise and vibration is nowadays a stronger public concern than airborne noise. Many countries all around the world have established regulations (or are about to) for the maximum ground-borne vibration and re-radiated levels that can be reached in buildings nearby of a railway infrastructure. These regulations are set mainly to control the annoyance to the building inhabitants, but also to ensure the correct operation of sensitive machinery and equipment and, eventually, to avoid any building damage. One of the most common situations in which a railway-induced noise and vibration assessment study is required is when a new building is planned to be constructed near an existing and operational urban railway line. In such cases, the railway line administration or the city council usually demands a study that certifies that the maximum railway-induced ground-borne noise and vibration levels that will be achieved in the future building will comply with the applicable noise and vibration regulations. Thus, in those situations, prediction models of the railway-induced ground-borne noise and vibration levels inside buildings are required.

In order to reduce the uncertainty of full theoretical modelling strategies such as [2], hybrid modelling based on the combination between *in situ* experimental data and a theoretical model is an interesting alternative [5]. Moreover, hybrid strategies also offers benefits in terms

of computational and engineering cost compared with reliable fully theoretical models. In the framework of hybrid modelling for soil-building dynamic interaction problems, Auersch [6] presented a semi-empirical model that combines pre-calculated results obtained from detailed numerical models, a database of experimental data build from several experimental measurements and several specific analytical models. Sanayei et al. [7] proposed a hybrid approach where buildings are modelled by finite axial rods with added floor impedance obtained from infinite thin plate models, and where the incident wave field is represented by previously known column base forces or measured vibrations at the loading dock floor. More recently, López-Mendoza et al. [8] have presented a computationally efficient model based on modal superposition to predict the ground-borne railway-induced vibration levels in buildings considering SSI. The methodology is designed, specifically, for cases where the incident wave field is known, because it is previously computed numerically or because it is measured in the ground surface. This model accounts for the SSI by adding spring and damper elements to the foundation of the building model. In a more recent work, these authors presented an even faster model that uses a 3D time-domain FEM approach for the structure modelling and more elaborated spring-damper elements to account for the SSI [9]. Kuo et al. [10] presented a hybrid model that combines recorded data and numerical predictions considering the definitions proposed by the United States Federal Transit Administration [11]. In this work, the source, propagation and receiver mechanisms are uncoupled. More recently, Arcos et al. [12] presented a novel methodology for the prediction of ground-borne vibration induced by traffic in operational urban railway lines in buildings (or other structures) to be constructed in the surroundings of these railway infrastructures. The proposed approach is constructed on the basis of the method of fundamental solutions (MFS) and consists of three steps: incident wave field evaluation at a set of measurements points (collocation points), incident wave field virtualization by a set of virtual forces and building structure response determination applying the set of virtual forces to a coupled soil-structure theoretical model. Using synthetic data instead of experimental measurements on the ground surface, the methodology was numerically validated for the general problem of a two-storey building with shallow foundations at the surroundings of an underground railway line in both two-dimensional and two-and-a-half-dimensional contexts.

In the present paper, the method proposed in [12] is numerically studied in a three-dimensional (3D) framework to explore two important discretization aspects: the distribution of virtual sources as well as collocation points. In

both cases, the excitation is assumed to be a vertical harmonic point load applied on the ground surface, showing the capabilities of the methodology in a general ground vibration problem induced by an arbitrary source. Two different schemes for the distribution of virtual sources along the auxiliary surface are studied in the framework of free-field responses. Regarding collocation points, the methodology is tested using quite less collocation points than virtual forces, demonstrating a good performance when the amount of collocation points is enough to catch out the behaviour of the ground surface at the particular areas where the shallow foundations will be installed.

2. METHODOLOGY

In this section, the hybrid methodology for the prediction of railway-induced ground-borne vibration in buildings to be constructed near to urban railway lines presented in [12] is revisited. This proposed hybrid methodology poses the inverse problem of determining a virtual traction field that dynamically mimics the effect of an incident elastic wave field in the soil only considering ground surface measurements. The MFS is used in this approach to deal with this inverse problem. The MFS uses a set of virtual forces that satisfy a previously known boundary condition (or set of boundary conditions). In this particular case, the displacement field induced by an external ground vibration source at the ground surface region where the foundations of the building will be placed is the targeted boundary condition, which is evaluated in a set of collocation points. Considering an elastic half-space model of the soil, the MFS may employ its corresponding Green's functions to determine a set of virtual forces, which can be seen as a discretization of the desired virtual traction field that satisfy the boundary condition at the collocation points.

This methodology is designed considering that the boundary condition at the ground surface is obtained experimentally by a set of vibration sensors. Then, the resulting virtual forces are employed in a theoretical model that accounts for the building structure and the local subsoil to predict the vibration (and, eventually, the re-radiated noise) levels inside the new building. The term “hybrid”, when referring to this methodology, is thus employed in order to denote that it combines experimental measurements and a theoretical model. Figure 1 illustrates the three steps in which the proposed methodology is based, in where Ω represents the soil domain and $d\Omega$ the ground surface region where the building under study will be constructed. In the first step, the response of the ground surface to the incident wave induced by an external ground vibration source is evaluated at a set of sensors (collocation points) located at the domain $d\Omega$, resulting in

a discrete representation of the targeted boundary condition. The second step carries out the virtualization of the incident wave field employing a virtual traction distribution in an auxiliary surface S that encloses the building foundation systems to be constructed. This step is theoretically based on the indirect boundary integral equation for a single-layer potential in the frequency domain that defines the response of the domain enclosed by S as

$$U(\mathbf{r}) = \int_S H(\mathbf{r}, \mathbf{r}_s) T_v(\mathbf{r}_s) dS(\mathbf{r}_s), \quad (1)$$

where $U(\mathbf{r})$ represents the displacements of the soil at locations $\mathbf{r} = \{x \ y \ z\}^T$, restricted to the Ω_s domain, $T_v(\mathbf{r}_s)$ are the unknown virtual tractions along the auxiliary boundary and $H(\mathbf{r}, \mathbf{r}_s)$ represents the Green's functions of the elastic half-space that provide the response at \mathbf{r} due to a point force applied along the S domain at \mathbf{r}_s , being $\mathbf{r}_s = \{x_s \ y_s \ z_s\}^T$ the location of the dS . The MFS proposes to approximate boundary integral equations such as equation (1) using a linear combination of fundamental solutions. Thus, considering a set of virtual point sources to represent the unknown traction field in the boundary, the application of the MFS results in the following system of equations

$$\mathbf{U}_c = \mathbf{H}_{cf} \mathbf{F}_v, \quad (2)$$

where \mathbf{U}_c and \mathbf{F}_v are column vectors that collect the response at the collocation points and the unknown complex amplitudes of the virtual forces, respectively, whilst \mathbf{H}_{cf} is the receptance matrix that relates the virtual forces and the collocation points response obtained with a half-space model of the local subsoil. In the present investigation, the number of collocation points is assumed to be smaller than the number of virtual sources, turning this system of equations to be under-determined. Pseudo-inverse can be used to solve this problem, whilst incorrect source amplitudes can arise when the selection of collocation points is not carefully chosen. Results presented in the next section clearly exemplifies this issue. Once the virtual forces amplitudes are determined, the response of the building-soil system can be obtained by

$$\mathbf{U}_b = \mathbf{H}_{bf} \mathbf{F}_v, \quad (3)$$

where \mathbf{U}_b represents the response at a set of evaluation points placed in the building-soil model and \mathbf{H}_{bf} is the receptance matrix that relates the virtual forces and the evaluation points response. The building-soil model is represented by the building domain Ω_b and the local subsoil domain Ω_s , which are intersected in the domain Ω_f , representing the foundations. The \mathbf{H}_{bf} matrix is obtained employing a building-soil model specially developed for the particular building to be studied. Suitable modelling

approaches to be used in this regard are 3D FEM-PML models [13], 3D FEM-BEM approaches [10, 14], semi-analytical modelling methods for 3D soil-building systems [15, 16] or other hybrid options [17], among others.

The so-called *three-step solution* is a classical SSI method in which the problem is solved in three steps: the evaluation of the free field response, the determination of the foundation equivalent stiffness and the computation of the dynamic response of the building subjected to the base motion evaluated in the kinematic interaction step (dynamic interaction) [18]. In the context of this SSI problem solving architecture, the present approach provides various important benefits. First, the kinematic interaction is directly evaluated at the ground surface instead of at the soil-structure interface, making the procedure meshless, simple and standard for any foundation type. Furthermore, the incident wave field is here characterised by a set of virtual forces enclosing the structure which are computed in the second step of the proposed approach. Classical approaches transform the wave field into loads acting on the soil-structure interface, while in this method the virtual forces are located in the soil surrounding the structure, at S . The separation between the virtual forces and the soil-structure interface allows for controlling the effect of the structure foundations to these forces. Moreover, the virtual surface S where the forces are applied can be simply a semi-sphere (proposed in the present paper) rather than a geometrically complex soil-structure interface. Finally, the inclusion of the local subsoil as a part of the model to evaluate the building response ensures an accurate representation of the foundation stiffness.

3. OPTIMAL DISCRETIZATION STRATEGIES

In this section, the discretization strategy to deploy the hybrid method previously introduced is numerically studied at different 3D scenarios. The authors has previously studied and proposed good discretization practises for the application of this method in 2D or two-and-a-half-dimensional contexts in [12] and the present paper stands as an attempt to study the validity of the proposed schemes in a 3D point of view, which is the proper environment for the application of the method in real engineering scenarios. However, it is important to mention that the definition of the both S and $d\Omega$ domains as well as the discretization schemes for the virtual sources and the collocation points along them, respectively, is a task of significantly higher complexity in a 3D perspective than in a 2D one.

In order to develop this study, the experimental measurements on the ground surface have been replaced by

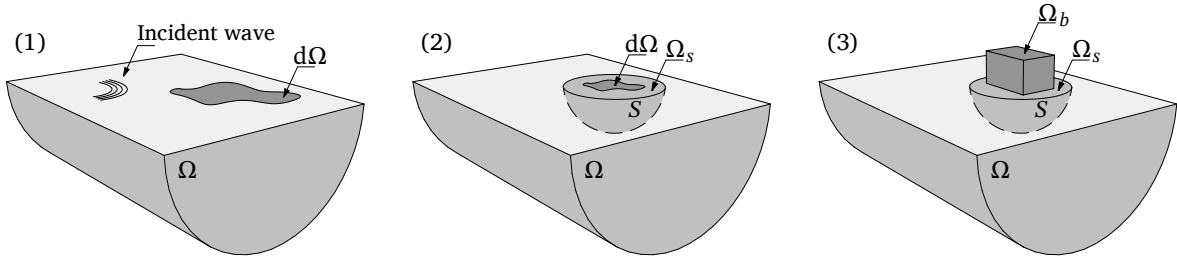


Figure 1: Schematic representation of the three steps of the proposed hybrid methodology: (1) Evaluation of the boundary condition at the ground surface where the building will be constructed, (2) virtualization of the incident wave field and (3) assessment of the building response.

synthetic data collected from the response due to a unitary harmonic vertical point load applied within the system of coordinates (x, y, z) at the point $(-4.5, 0, 0)$ m. The range of frequencies considered is up to 80 Hz. The response of the soil due to the external point load has been computed using the approach presented in [19]. The hybrid method is initially deployed considering no foundation system implemented, to study how well the method represents the free-field response of the soil depending on the distribution of virtual sources. The method is implemented considering just one collocation point at the origin of coordinates and the same point is used to evaluate its performance. For this case, the soil is assumed to be a homogeneous half-space with the following mechanical parameters: Young's modulus of 195 MPa, Poisson's ratio of 0.3, density of 2000 kg/m³ and damping of 0.04.

Thus, in this first stage, the aim is to assess the sensitivity of the method to different arrangements and distributions of virtual forces. In authors' previous study [12], virtual forces were uniformly distributed along a semicircle due to the 2D nature of the domain to be discretized. Since this distribution yielded good results in that context, it is proposed here to uniformly distribute virtual forces over a hemisphere. The distribution of N points equally spaced along the surface of a sphere is a classic problem in mathematics. In this work, two different discretization strategies that allow for a reasonably uniform distribution of the source points are studied: the first one involving Fibonacci spirals to distribute the sources and the other through an efficient distribution algorithm [20]. A illustrative view of the two distribution strategies is presented in Fig. 2.

The number of virtual forces has been determined so that their distribution over the hemisphere ensured that the distance between consecutive virtual forces is less than the wavelength of S-waves for the maximum calculation frequency, ensuring compliance with the previously established empirical rule defined in [12]. A radius of the virtual

force domain equal to two meters has been considered, resulting in a minimum of 10 virtual forces, less than what is shown in Fig. 2. In Figure 3, the vertical displacement observed at the evaluation point is presented for the reference model, where the response in the evaluation point due to the external harmonic vertical point load is directly obtained, and for the hybrid model in which both forms of distribution of virtual forces were used. The comparison of the results allows us to conclude that the optimized distribution leads to more accurate estimations of the response results obtained through the hybrid method that are closer to the numerical reference curve. Since the previous results considered a free-field situation, it is important to determine whether the conclusions can be corroborated for a situation in which there is an object in the domain that will disrupt the incident wave field.

To study the distribution of collocation points, the case of a single rigid shallow foundation embedded in the soil is adopted. The considered foundation system is embedded in the soil, which is considered to be a homogeneous half-space with the following mechanical parameters: shear wave speed of 150 m/s, Poisson's ratio of 0.49, density of 1900 kg/m³ and damping of 0.03. The shallow foundation geometry is squared from a top view, having the dimensions of 1 m side and 0.25 m depth. The centre of the shallow foundation is located at the origin of coordinates. The calculations are done considering the distribution of the virtual sources following the optimised methodology on a hemisphere of 2.6 m of radius. The amount of virtual sources considered for this case is determined in the basis of the same ideas presented before when studying the distribution of virtual sources. Two amounts are considered: 16 and 24. Regarding the amount of collocation points, three scenarios are adopted: one, four (squared distribution) and 16 (four squared distribution) collocation points distributed along the surface virtually occupied by the shallow foundation.

Results are shown in Fig. 4. It can be observed that

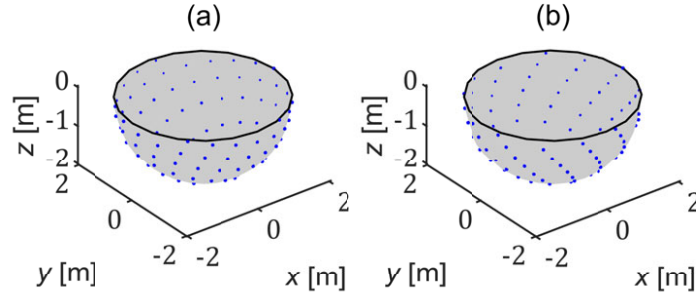


Figure 2: Schematic representation of the source distribution along the hemisphere for the Fibonacci spirals case a) and for the optimised methodology presented by Kogan b).

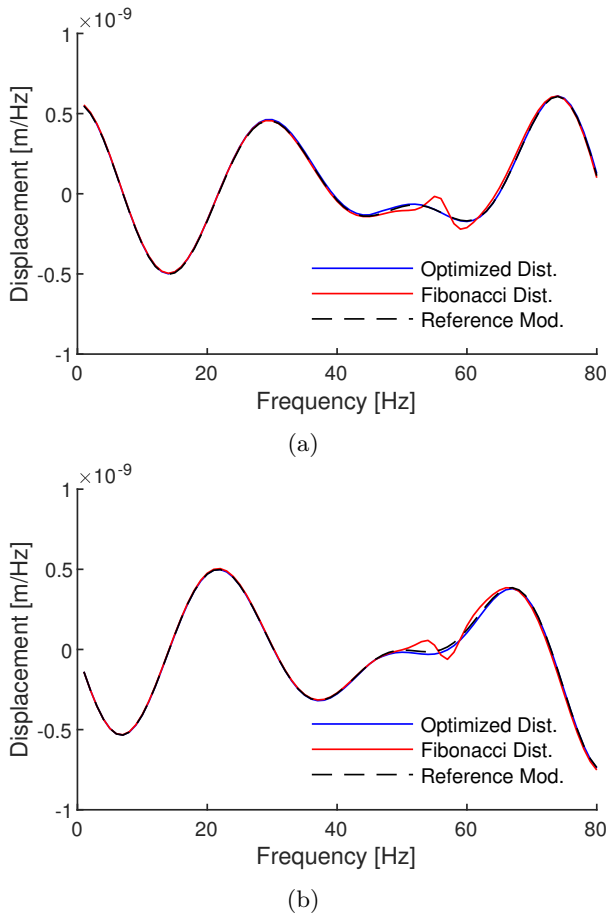


Figure 3: Real a) and imaginary b) components of the vertical displacement at the evaluation point for the reference solution and for the hybrid method applying the two different discretization strategies.

the hybrid method is not capable to reproduce accurately the response of the shallow foundations when just one collocation point per foundation is chosen. Results for this

case are significantly unstable because the system of equations appears to be strongly under-determined. However, it is found that considering just three extra collocation points per foundation highly enhances the accuracy of the method, showing a reasonable performance all along the frequency spectrum. Results obtained when 16 collocation points per foundation are adopted are found to be quite accurate, although it is important to note that a practical application of this arrangement of sensors/collocation points is largely unfeasible.

It is important to mention that these results are achieved accounting for the three components of the displacement at the collocation points and the three components of the virtual forces. Using, for example, only the vertical component for the collocation points, which could be a very promising alternative in order to reduce the amount of data acquisition work when carrying out the required ground surface measurements, leads to poor accuracy of the method. Furthermore, it is also found that the method works with multiple distributions of collocation points along ground surface enclosed by S , even they are not located at the specific regions where the foundations will be installed, as shown also in [12]. However, locating the collocation points at the virtual foundations provides a convenient trade-off between amount of collocation points and delivered accuracy.

4. CONCLUSIONS

This paper study the distretization strategies to bu used when employing the proposed hybrid methodology in a 3D context to ensure its proper operation. It is found that the tow studied distribution strategies for the virtual sources are delivering accurate results, while the optimised method reaches slightly better results. Regarding collocation points distribution, it is found that even employing significantly less collocation points than virtual sources the method is still capable to deliver accurate results in

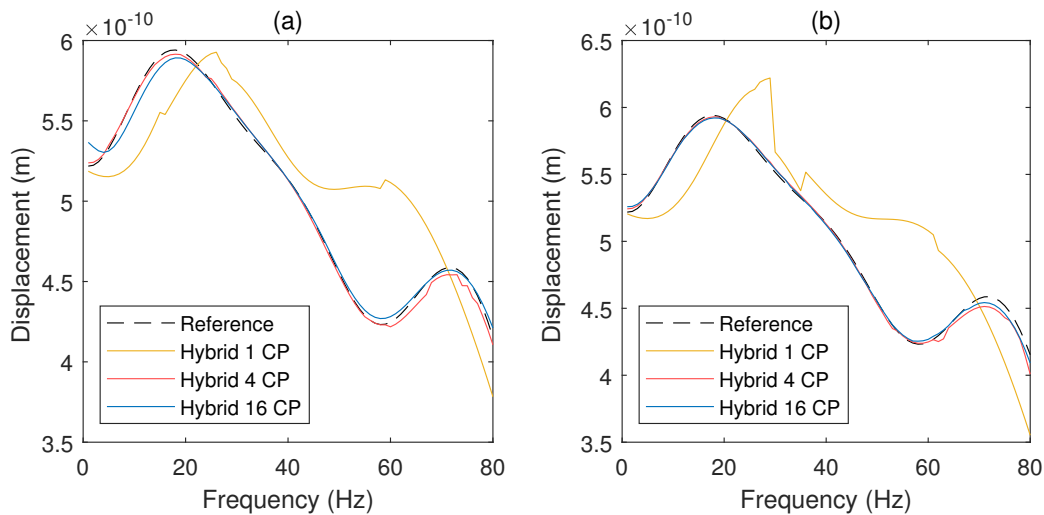


Figure 4: Vertical translational response of the foundation for the case of one foundation and considering 16 (a) and 24 (b) virtual forces. Results are shown for one, four and 16 collocation points (CP).

the frequency range of interest for ground-borne noise and vibration assessment studies. An interesting result of the present study is to show that accounting just for one tri-axial sensor per shallow foundation does not permit to reproduce correctly a particular incident wave field. Since using one tri-axial sensor per foundation is a common practice in technical assessments of noise and vibration in new buildings to be constructed near to, for example, operational railway lines, the results presented in this paper are suggesting that this usual practices should be revised. The present study also points out that the application of the proposed method for the assessment of ground vibration in new buildings to be constructed close to ground vibration sources appears to be feasible, since the results shown that the number of sensors required to reach an acceptable degree of accuracy is not prohibitive. However, when dealing with large buildings or facilities with multiple foundation systems, the proposed method may encounter practical complications due to the huge amount of vibration data that should be gathered. Non-synchronized measurements are a common solution to deal with large scale problems, but the proper operation of the proposed methodology in the presence of non-synchronized data has yet to be demonstrated. Thus, further research on the topic is advisable in order to advance towards a more practical and robust method.

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