RETRIEVAL METHOD APPROACH FOR A DOUBLE-NEGATIVE ACOUSTIC METAMATERIAL COMPOSED BY DISCONTINUED COUPLING OF HELMHOLTZ RESONATORS

PACS: (Arial, tamaño 8, línea 8, mayúsculas, alineado izquierda).

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Palabras Clave: Acoustic Metamaterials, Retrieval Model, Bulk Modulus, Effective Density, Helmholtz Resonator.

ABSTRACT

The development of a new class of materials described by its effective physical characteristics, called metamaterials, originated a novel way to manipulate sound waves in several engineering areas. Through the scattering parameter retrieval method, the acoustic characterization of an artificially structured metamaterial can lead to the observation of unusual characteristics, different from those obtained in ordinary materials. In this paper, an effective medium description based on the retrieval method is proposed to realize an acoustic metamaterial (AMM) composed of the periodical coupling of a tunable subwavelength resonator embedded in a constricted waveguide, leading to an impedance variation and reducing the sound wave velocity. A one-dimensional periodic model is initially described through the transfer matrix method (TMM), and the effective properties are evaluated. Then, a three-dimensional numerical model of the AMM structures is proposed and the retrieval method is applied to evaluate the material parameters.

RESUMO

O desenvolvimento de uma nova classe de materiais descritos por suas características físicas efetivas, chamados metamateriais, originam uma nova forma de manipular as ondas sonoras em diversas áreas da engenharia. Através do método de recuperação dos parâmetros de dispersão, a caracterização acústica de um metamaterial artificialmente estruturado pode levar à observação de características incomuns, diferentes daquelas obtidas em materiais comuns. Neste trabalho, uma descrição do meio efetivo baseado no método de recuperação é proposto para realizar um AMM composta pelo acoplamento periódico de um ressonador de sub-comprimento de onda sintonizável acoplado em um guia de onda estreito, levando a uma variação da impedância e reduzindo a velocidade da onda sonora. Um modelo periódico unidimensional é inicialmente descrito através do método de matriz de transferência (TMM), e as propriedades efetivas são avaliadas. Em seguida, é proposto um modelo numérico tridimensional das estruturas metamateriais acústicas e o método de recuperação é aplicado para avaliar os parâmetros do metamaterial.

1. INTRODUCTION

The continuous advances in the science of materials led to the development of artificial materials, designed exclusively to deal with transmission and reflection acoustic problems obtaining dynamic and dispersive responses [1], which could lead to non-natural effective physical properties of negative order [2].

The anomalous nature of these structures is carried out due to the locally resonant responsive characteristics of small periodic acoustically structures embedded in a homogeneous matrix, in which their internal periodicity is considerably smaller than the wavelength of external excitations [3], being independent of the material medium but strictly related by their geometries and shape [1].

In this sense, an example of subwavelength locally resonant structures is the Helmholtz resonators (HR). They can lead to the development of tuned absorbers, dealing with dissipative effects at audible frequencies below 1kHz, a frequency range not possible to address directly by thin porous materials [4], and originating new approaches, such as the concept of perfect and nearly perfect acoustic absorption [5] with the critical coupling condition between two near locally resonance frequencies.

Assuming the determination of the complex effective properties in AMM, e.g. density, bulk modulus, impedance, and refractive index, used to describe wave interaction with associated media [2], it is possible to manipulate reflected and transmitted acoustic waves, promoting the development of new sound absorbers and attenuation devices.

A negative effective characteristic of a single resonator was first evidenced by Fang et al. [6], who demonstrated the negative compressibility through the inclusion of Helmholtz resonators in a duct. However, the inclusion of HRs mounted in a duct and the impressive negative or zero bulk modulus and/or effective mass density, exhibiting a double-negative effective characteristic, has also been described in other works [7], [8].

Similar to the theory of electromagnetic parameter retrieval [9], and adequation of the electric permittivity, ε, and magnetic permeability, μ, in acoustic wave propagation theory, the material medium related by K and ρ, being results of relation between the refractive index n and the wave impedance Z [9]. Thus, supported by the transfer matrix method (TMM) to describe the continuity condition in a homogeneous media, an analytical technique is employed to predict the effective characteristics in a homogeneous and geometrically symmetric medium, the S-parameters retrieval method [9], [10], which is an alternative to derive the effective properties using scattering coefficients obtained on the incident and transmitted parts of a medium [11].

In this study, due to the recent axial coupling of locally resonant structures [12]–[14], the authors present a model to predict the effective complex characteristics of these propositions. Through the analytical models, e.g. retrieval technique [9], [10] and transfer matrix model [11], [15], to investigate the effective acoustic properties achieved by the inclusion of Helmholtz resonators. A finite element model is then implemented to verify the usefulness of the models, showing a good correlation between theoretical and numerical models for material characterization. Finally, an acoustical metamaterial composed of N layers of a set of Helmholtz resonators, mounted in a discontinued duct, is proposed as a strategy to achieve impressive sound absorption characteristics in subwavelength regime.

This article is organized as follows: in section 2, the authors propose a theoretical retrieval model for complex effective properties, based on S-parameters to obtain the effective compressibility and dynamic effective mass, as well as the finite element description to obtain the scattering parameters in a FEM model. Finally, the theoretical models presented will be discussed, with the main conclusions summarized in section 4.

2. RETRIEVAL METHOD FORMULATION

2.1. S-Parameter Retrieval

In this section, the transfer matrix method (TMM) is used to describe the analytical model of planar acoustic wave propagation in an arbitrary duct and the N resonant inclusions, thus, illustrating the determination of the reflection and transmission coefficients, to mount an S-matrix.

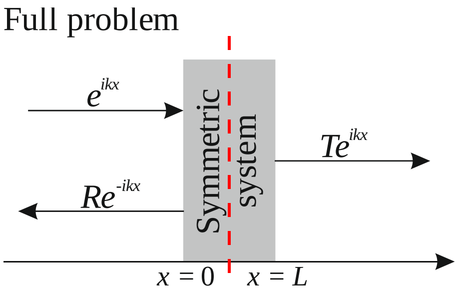


Figure 1 – Schematic diagram of the 1D transmission problem of a plane wave by a symmetric system.

Assuming the condition of continuity of sound pressure, p, and normal particle acoustic velocity, v, in a homogeneous media, through a finite acoustic system, from the beginning ($x = 0$) to the end of the system ($x = L$), with only the propagation of plane waves, the matrix $\mathbf{T}$, can be defined. The relationship between incident sound pressure and particle velocity from an initial moment, p_1 and v_1, respectively, to the final moment p_2 and v_2, is written as [8], [16]:

$$\mathbf{T}\begin{bmatrix} p_1 \\ v_1 \end{bmatrix}_{x=0} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} p_2 \\ v_2 \end{bmatrix} = \mathbf{T}\begin{bmatrix} p_2 \\ v_2 \end{bmatrix}_{x=L}. \tag{1}$$

For a symmetric and homogeneous system, where the reflection coefficient is independent of the wave propagation direction, then the following conditions need to be satisfied, $T_{11} = T_{22}$ and $T_{11}T_{22} - T_{21}T_{12} = 1$ [15], [17]. Thus, considering the TMM as a series of multiplications to express the continuity condition of the planar wave, to represent a realistic isentropic system, Figure 1. The derivation of reflection and transmission of the incident plane wave can be related, to the following equations [8], [15]:

$$T = \frac{2e^{jkL}}{T_{11} + \frac{T_{12}}{Z} + ZT_{21} + T_{22}}, \tag{2}$$

$$R = \frac{T_{11} + \frac{T_{12}}{Z} - ZT_{21} - T_{22}}{T_{11} + \frac{T_{12}}{Z} + ZT_{21} + T_{22}}. \tag{3}$$

Once obtained the matrix T, the S-matrix relates the incoming amplitudes to the outgoing waves, where the scattering elements are defined by $S_{11} = S_{22} = R$ and $S_{12} = S_{21} = T$ which defines the relationship between the waves' amplitudes resulting in the matrix [9], [17]:

$$\mathbf{S}\begin{bmatrix} p_1 \\ v_1 \end{bmatrix}_{x=0} = \begin{bmatrix} R & T \\ T & R \end{bmatrix} \begin{bmatrix} p_2 \\ v_2 \end{bmatrix} = \mathbf{S}\begin{bmatrix} p_2 \\ v_2 \end{bmatrix}_{x=L}. \tag{4}$$

Using the extracted scattering matrix (S), the effective acoustic impedance of the system, Z_{eff}, and refractive index, n, have been calculated using the mathematical relation, below written [8], [9]:

$$Z_{\text{eff}} = \pm\sqrt{\frac{(1+S_{11})^2 - S_{21}^2}{(1-S_{11})^2 - S_{21}^2}}, \quad (5)$$

$$n = \frac{\pm\cos^{-1}\left[\frac{1}{2S_{21}}(1 - S_{11}^2 + S_{21}^2)\right] + 2\pi m}{kL}. \quad (6)$$

The effective properties of acoustic metamaterial, such as effective mass density and effective bulk modulus ratio, can be estimated using the mathematical relations [8], [9]:

$$K_{\text{eff}} = n/Z_{\text{eff}}, \quad (7)$$

$$\rho_{\text{eff}} = k_{\text{eff}} Z_{\text{eff}}/\omega, \quad (8)$$

where $k_{eff} = \arcsin(\sqrt{-T_{12}/T_{21}})/L$, is the effective wavenumber.

2.2. Finite Element Model Description

To assess the correctness of the analytical approach a finite element model (FEM) has been implemented using the commercial software COMSOL Multiphysics, to reproduce the standardized method for measurement of normal incidence sound transmission [18], represented in Figure 2, thus, the scattering matrix can be mounted to obtain the effective complex characteristics of a locally resonant inclusion [10].

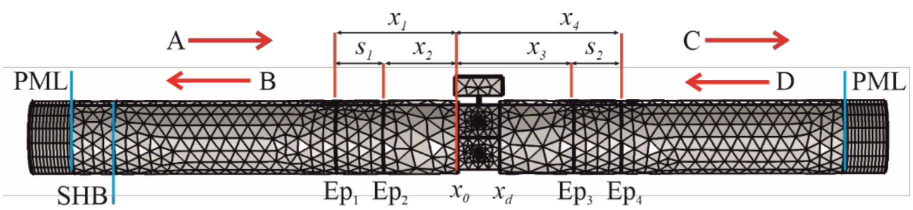


Figure 2 – A finite element model for obtaining the acoustical effective properties of a symmetric resonant inclusion.

In Figure 2, to retrieve the effective constitutive parameters, the model assumes the periodicity condition imposed in a two-port model, under the excitation of an incident plane wave of unitary value at normal incidence originating in a sound hard boundary (SHB), on the left side of the model, with the waveguide and all surfaces being considered as perfectly rigid, and the perfectly matched layer (PML) being admitted at both ends of the waveguide, to mitigate subsequent reflections. Two evaluation points, Ep_1 and Ep_2, on the left side of the waveguide, are placed distancing S_1, with x_1 and x_2 corresponding to the distance between the evaluation points and point x_0. On the right side of the waveguide, the additional points, Ep_3 and Ep_4, with distance S_2 between them, with x_3 and x_4 representing the distances from these probes to point x_0.

Assuming the complex sound pressures at the four probes, Ep_1, Ep_2, Ep_3 and Ep_4, the respective complex amplitudes of the positive and negative parts of a plane wave, are expressed by;

$$A = \mathrm{j}\frac{H_{1,\text{ref}}e^{-\mathrm{j}kl_1} - H_{2,\text{ref}}e^{-\mathrm{j}k(l_1+s_1)}}{2\sin ks_1},$$
$$B = \mathrm{j}\frac{H_{2,\text{ref}}e^{+\mathrm{j}k(l_1+s_1)} - H_{1,\text{ref}}e^{+\mathrm{j}kl_1}}{2\sin ks_1}, \quad (9)$$

$$C = \mathrm{j}\frac{H_{3,\mathrm{ref}}\mathrm{e}^{+\mathrm{j}k(l_2+s_2)} - H_{4,\mathrm{ref}}\mathrm{e}^{+\mathrm{j}kl_2}}{2\sin ks_2},$$
$$D = \mathrm{j}\frac{H_{4,\mathrm{ref}}\mathrm{e}^{-\mathrm{j}kl_2} - H_{3,\mathrm{ref}}\mathrm{e}^{-\mathrm{j}k(l_2+s_2)}}{2\sin ks_2},$$

where k is the wavenumber in the background medium; ω is the angular frequency; $e^{-\mathrm{j}\omega t}$ represents the harmonic temporal dependence, x_1, x_2, x_3 and x_4 indicate the corresponding distances of four probes relative to the respective reference planes in the two ports.

Using the continuity condition, the boundary conditions in the FEM model can be expressed by:

$$\begin{aligned} p_0 &= A + B, \\ p_d &= Ce^{-\mathrm{j}kL} + De^{+\mathrm{j}kL}, \\ u_0 &= (A - B)/Z_0, \\ u_d &= (Ce^{-\mathrm{j}kL} - De^{+\mathrm{j}kL})/Z_0. \end{aligned} \quad (10)$$

where p_0 and p_d are the pressures, u_0 and u_d express the particle velocities, at locations x_1 and x_d, in Figure 2, respectively; Z_0 is the impedance of the background medium. Then, the transfer matrix of the effective two-port network can be written as:

$$T = \begin{bmatrix} \frac{p_d u_d + p_0 u_0}{p_0 u_d + p_d u_0} & \frac{p_0^2 - p_d^2}{p_0 u_d + p_d u_0} \\ \frac{u_0^2 - u_d^2}{p_0 u_d + p_d u_0} & \frac{p_d u_d + p_0 u_0}{p_0 u_d + p_d u_0} \end{bmatrix}. \quad (11)$$

Thus, through the retrieval inverse technique, it is possible to obtain the effective complex characteristics of a homogeneous and adiabatic system, getting the effective refractive index and effective impedance, as shown in Eq. 5 and Eq. 6. Finally, the effective mass density and bulk modulus can be further determined, by the Eq. 7 and Eq. 8.

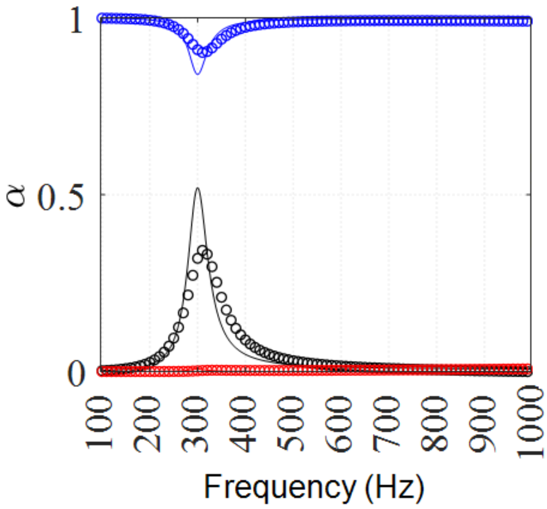
3. RESULTS

3.1. Effective characteristics of a Helmholtz resonator in a symmetric system

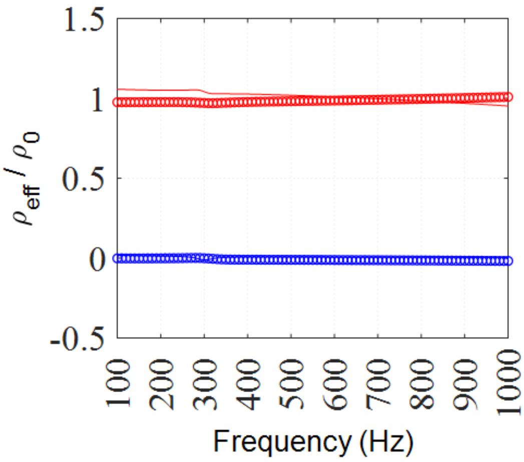
For an analysis of the complex effective characteristic, a typical acoustic system based on a single inclusion of an HR with a rectangular cross-section mounted in a duct is proposed. The inner width section of the neck, w_n = 1 mm and the respective length of the neck l_n = 5 mm; the cavity of the resonator, the inner width section is w_c = 22.5 mm, with their respective length l_c = 8.7 mm. The unit cell length of the metamaterial is L = 20 mm, with the waveguide diameter S_w = 35 mm. Thus, for this resonator geometry, the resonance frequency is given by approximately f_r = 300 Hz.

Figure 3 (a) presents the results of sound absorption (black line) for the single target unit, obtained with the numerical model, and the analytical results, with values of the absorption coefficient reaching α=0.5. In this figure, values are compared between the numerical prediction and the experimental results, with good agreement. The respective values of reflection (red line and markers) and transmission (blue line and markers) are obtained. Other variations presented outside the range of interest may correspond to the mathematical considerations in the used theoretical model, due to the thermal and viscous dissipation inside the boundary layer.

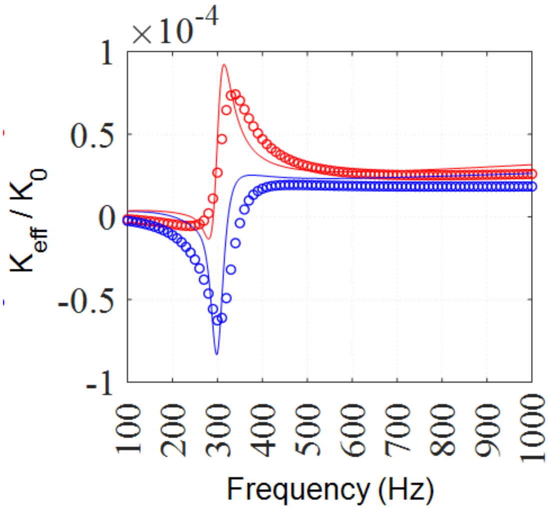
The obtained effective bulk modulus of inclusion of a single HR is shown in Figure 3 (b), between f = 250 and 400 Hz. The markers represent the FEM results, and the solid lines correspond to the values obtained through the retrieval models using Eq. 7 and Eq. 8. As expected, the negative bulk modulus of a locally resonant inclusion is observed in the real part (red line and markers). Additionally, Figure 3 (c), exhibits the results for the effective density of the fluid volume with a Helmholtz resonator, normalized by the ambient air density, for the case of a single resonator, the effective density is nearly constant over the considered frequency. Overall, a good agreement between the experimental and analytical results can be observed.



(a)



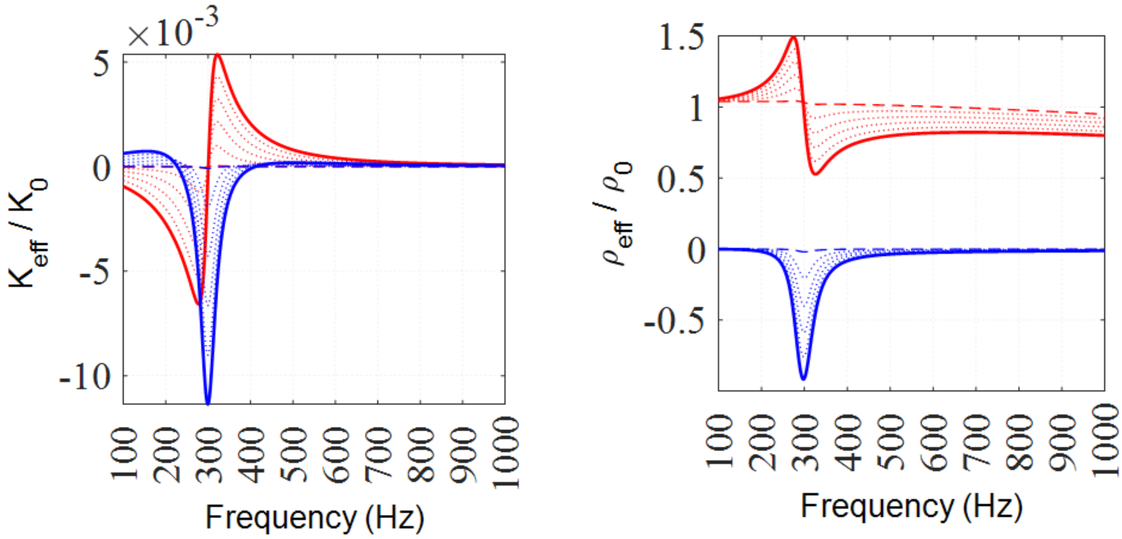
(b)



(c)

Figure 3 –In subfigures (a), (b), and (c), the Transfer Matrix Method results are represented by the solid line, and the finite element method results are represented by markers. (a) $|T|^2$(blue curves), $|R|^2$(red curves) and α (black curves), represent, respectively, the sound transmission, reflection, and absorption, for a unit HR located in a circular duct. In subfigures (b) and (c), the effective density and effective bulk modulus, respectively, depict both the real and imaginary parts of the complex effective characteristics of the symmetric system.

Next, the inclusion of N Helmholtz resonators has been investigated. The corresponding scattering matrix was estimated and applied to the retrieval model and the respective effective properties were obtained, as evidenced in Figure 4. The first observations of the parallel inclusions were observed as a single negative metamaterial. However, in Figure 4 (b), the density is observed approaching zero when a greater number of HRs is proposed. From examining the inclusions, it can be observed that the real part of effective bulk modulus, Figure 4 (a) for all inclusions is negative as expected.



(a) (b)

Figure 4 –In subfigures (a) and (b), the Transfer Matrix Method results are represented by the solid line, and the Finite Element Method results are represented by markers. The effective density and effective bulk modulus, respectively, in subfigures (a) and (b), depict both the real and imaginary parts of the complex effective characteristics of the symmetric system.

Here, evidence has been found that the AMM based in periodical locally resonant structures occupy conceptually a special possibility between effective media and photonic crystals, with the S-parameters retrieval techniques as a simple and efficient method to characterize metamaterials having symmetric unit cells.

3.2. Sound absorption performance of a discontinued double-negative asymmetric system proposition

In this subsection, the sound absorption performance by periodical coupling of the N-hexagonal tunable unit embedded in a constricted waveguide is addressed. To reach a wide frequency range per meta unit, the grouping of resonant structures with six resonance frequencies per unit cell is analyzed.

Considering the parallel arrangement of N Helmholtz resonators, grouped in the constricted waveguide, can be analyzed theoretically by the characteristic effective impedance of the AMM, and derived through the equivalent mass density $\rho_i^{[n]}$ and the equivalent bulk modulus $\kappa_i^{[n]}$of the narrow duct, as presented by [12].

In contrast to what was shown in subsection 2.1, here the analysis of proposed AMM layers is done by considering a rigid backed condition, and then the normal incidence reflection coefficient for the hard backing case is defined as [19]:

$$R = \frac{T_{11} - Z_0 T_{21}}{T_{11} + Z_0 T_{21}}. \qquad (12)$$

The predicted sound absorption for the AMM, considering a set of two unit cells with different resonant frequencies, at first, f_1, f_2 , f_3, f_4, f_5 and f_6, tuned at 350, 400, 450, 500, 550, and 600

Hz; the second, f_6, f_7, f_8, f_9, f_{10}, f_{11} and f_{12}, tuned at 650, 700, 750, 800,850, and 900Hz, respectively, each cell with a different waveguide cross-section is shown in Figure 5.

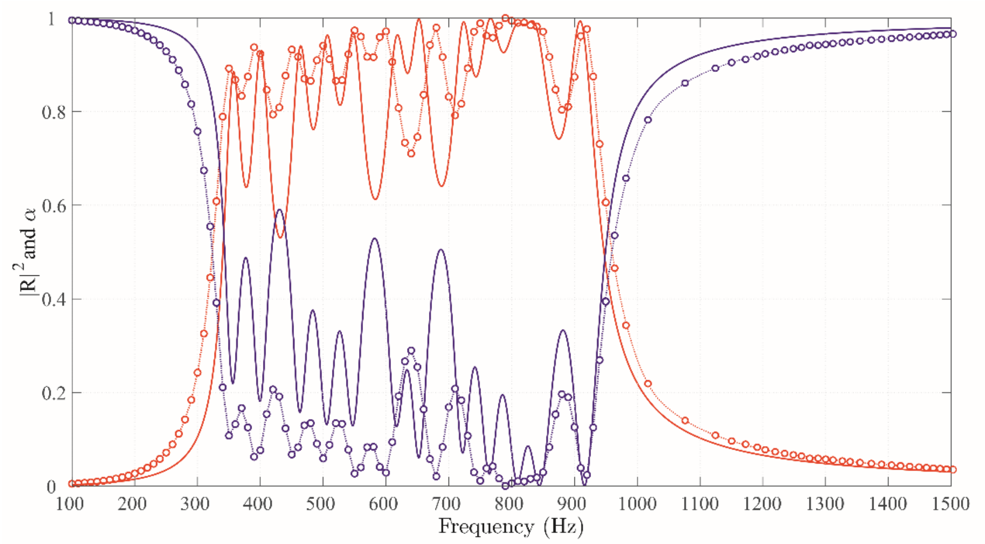


Figure 5 – The spectrum of sound absorption for the coupling of two meta unit cells; the red line corresponds to the sound absorption coefficient; the blue line corresponds to the reflectance. The dashed lines are the FEM results; the solid lines are the respective TMM analytical results.

The solid line represents the sound absorption for the proposed meta-unit calculated by the TMM. As shown, near-perfect absorption is observed in almost all the broadband; however, it is also evident that the peak attenuation only happens at the frequencies to which the system has been tuned, and a significant drop is seen between the resonant frequencies.

A FEM simulation is used to investigate the sound absorption capacity of the serial coupling of two proposed meta-structures. From the theoretical and numerical models, a good agreement can be observed, although a visible deviation is seen probably due to the TMM approach used here not including the end corrections between the discontinuity of each waveguide.

4. CONCLUSIONS

This paper proposes a simple approach to analyzing the complex effective characteristics of the N axial locally resonant inclusions. A retrieval method for obtaining the effective properties of a metamaterial with negative bulk modulus and effective density tending to zero is investigated from the reflection and transmittance coefficients.

The S-parameter retrieval model was used to characterize a symmetric proposition, nevertheless, here the model does not contain an expanded analysis for asymmetric unit cells. A modified retrieval approach leads to a unique value of the refractive index, but due to the dependence of the wave propagation direction, two generally distinct values for reflection and transmission lead to a two-wave impedance.

As expected, it was found that the inclusion of a series of HR can lead to double negativity. Additionally, a finite element procedure was used to verify the accuracy of the analytical model, and the agreement between both approaches showed that the acoustic effective properties of the inclusions can be predicted using the proposed models.

In future works, the aim will be to study the application of the proposed S-parameter method to investigate several acoustic metamaterial designs to expand the analysis to nonhomogeneous and asymmetric structures and evaluate the results to complement an experimental approach. It is expected that the present study can serve as an important contribution to the research and development of AMM with applications in sound attenuator systems that enable airflow in diverse applications in engineering.

Acknowledgments

This work is financed by national funds through FCT - Foundation for Science and Technology, under grant agreement UI/BD/150864/2021 attributed to the 1st author, and by FCT/MCTES through national funds (PIDDAC) under the R&D Unit Institute for Sustainability and Innovation in Structural Engineering (ISISE), under reference UIDB / 04029/2020.

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