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SOUND RADIATION OF A TOROIDAL CONSTANT BEAMWIDTH TRANSDUCER

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Palabras Clave: Sound radiation, Constant beamwidth transducer, Toroidal source.

ABSTRACT.

Most directional acoustic transducers and arrays have frequency-dependent beamwidths. When the wavelength is large compared to the size of the transducer, the width of the beam becomes wider for a fixed aperture of a piston or line array. As a result, the spectral content of the transmitted or received signals varies with position in the beam, and the properties of the acoustic system depend on the relative orientation of the transmitter and receiver. A constant bandwidth transducer (CBT) is a sound source whose radiation pattern (beamwidth) is not frequency-dependent. In this work, a continuous toroidal source is proposed that presents a constant bandwidth both in the horizontal and vertical planes.

RESUMEN.

La mayoría de los arreglos y transductores acústicos direccionales presentan anchos de haz que dependen de la frecuencia. Cuando la longitud de onda es grande en comparación con el tamaño del transductor, el ancho del haz se hace más amplio para una apertura fija de un pistón o arreglo lineal. Como resultado, el contenido espectral de las señales emitidas o recibidas varia con la posición en el haz y, las propiedades del sistema acústico dependen de la orientación relativa del transmisor y el receptor. Un transductor de ancho de banda constante es una fuente sonora cuyo patrón de radiación (ancho de haz) es independiente de la frecuencia. En este trabajo se propone una fuente continua toroidal que presenta un ancho de banda constante tanto en el plano horizontal como el vertical.

1. INTRODUCCIÓN

The radiation pattern of most acoustic sources is frequency dependent. The typical size of the transducer respect to the wavelength governs the sound emission. The amplitude and phase of the acoustic field depend on the relative orientation of the transmitter and the receiver. This type of source provides an acoustic output whose spectral content does not vary with direction. Based on the work proposed in [1] for underwater transducers, Keele extended the concept of the Constant Beamwitdth Transducer (CBT) to arrays of loudspeakers to control the directivity characteristics and the minimization of the secondary lobes [2]. Over more than 25 years of research and development, Keele has developed multiple types of CBT loudspeaker arrays like a straight-line array using signal delays [3], a circular-wedge array [4].

The linear CBT has been extended to two-dimensional active surfaces. A circular-arc is generated by axially revolving the line array towards the central axe [2, 5, 6]. The resultant spherical CBT presents axial symmetry and is characterized by a single parameter: the radius of the sphere. In recent years, the theoretical background of CBT arrays has been developed for circular array [7] and a cylindrical CBT array is proposed in [8] by extruding a linear array along the perpendicular axe of its plane. The theoretical far-field acoustic beam pattern is derived for an infinitely long cylindrical array. Constant beam pattern is achieved in the plane normal to the cylinder generatrix.

To achieve constant beamwidth in two perpendicular planes a geometry with two degrees of freedom must be considered. A tore is defined by two curved lines with different bend radius in both horizontal and vertical directions. If both radii are equal, the torus converges to a sphere. If any of them tends to infinity, it converges to a cylinder. An acoustic toroidal source may provide different angular coverage in both planes.

A toroidal cap with a 2D cosine shading (2DCS) is proposed to provide a constant broadband in both the horizontal and vertical planes. The sound radiation in the far-field emitted by the structure is analyzed with a parametric study.

1.1. Sound radiation

We consider the problem of the external sound radiation generated by an active toroidal cap. Figure 1 shows the torus with major radius a and minor radius b (a≥b). The ratio $\sigma = a/b$ is known as the aspect ratio of the torus. The vibrating surface is limited to the range of angular values of α in the Horizontal Plane, HP (XY) defined by the horizontal aperture α_0, $[-\alpha_0 \le \alpha \le \alpha_0]$, and β in the Vertical Plane, VP (XZ) defined by vertical aperture β_0, $[-\beta_0 \le \beta \le \beta_0]$. Let's consider a point source Q of the active toroidal cap vibrating with radial velocity v, harmonic dependency $e^{j\omega t}$ and normal to the torus surface dS. The radiated pressure at any point O in the space is the sum of pressure contributions all differential surface elements dS defined at the active surface of the toroidal cap S according to the Rayleigh integral:

$$p(r, \theta, \phi) = \frac{j\omega\rho_0}{2\pi}\int_S v(\alpha, \beta)\frac{e^{-jkR}}{R}dS \qquad (1)$$

where k is the wave number, ρ_0 is the density of air, ω is the angular frequency. The active surface of the toroidal cap is defined by the velocity distribution v(α, β) and it is called the shading function []. This function determines the radiation pattern of the toroidal cap. If the velocity distribution is a single complete Legendre polynomial [1] or a cosine function [6] of α (or β), then, the beam pattern is not frequency dependent in the plane XY (or XZ).

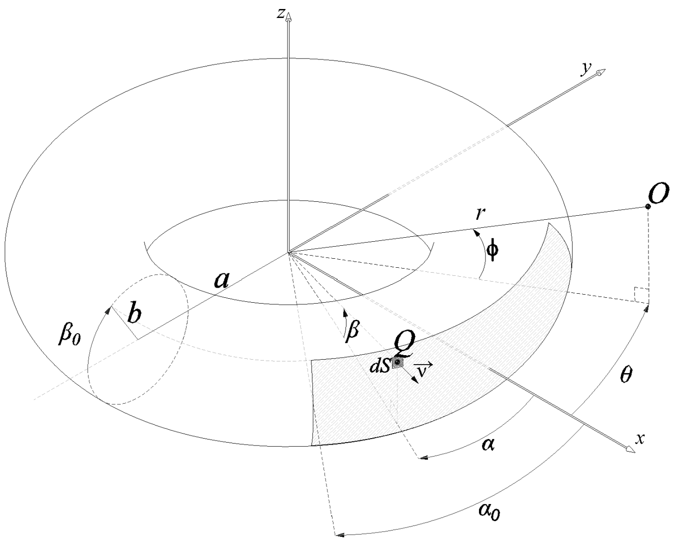

Figure 1 – Toroidal acoustic continuous source (gray). The torus is defined by its major radius ***a*** and minor radius b. The point Q is an elementary source of the active torus and O is an observation point.

To achieve a CBT in both planes XY and XZ, it is necessary to modulate the velocity distribution in both directions appropriately and independently. Here we define a 2D cosine shading (2DCS) function that is an extension to 2D dimensions of the one proposed in [1, 3, 6]:

$$v(\alpha, \beta) = v_0 \cos cos\left(\frac{\pi}{2}\cdot\frac{\alpha}{\alpha_0}\right)cos\left(\frac{\pi}{2}\cdot\frac{\beta}{\beta_0}\right) \tag{2}$$

where v_0 is the maximum velocity. The radiation pattern of the toroidal cap with this 2DCS can be obtained by replacing equation (2) into equation (1) and evaluating the integral for all the points Q at the observation point O.

2. PARAMETRIC STUDY

2.1. Sound radiation pattern

In Fig. 2, the sound radiation pattern of the toroidal cap is represented for the apertures α_0 = 60º and β_0 = 60º and aspect ratio σ = 1 for an equal shading (ES) shown in Fig. 2a and 2c, and a 2DCS (Fig. 2b and 2d). The sound pressure level is normalized to the maximum and plotted with the angle dependency in the vertical axis. The horizontal axis represents the non-dimensional magnitude defined by the wavenumber and the total radius of the torus: k·(a+b).

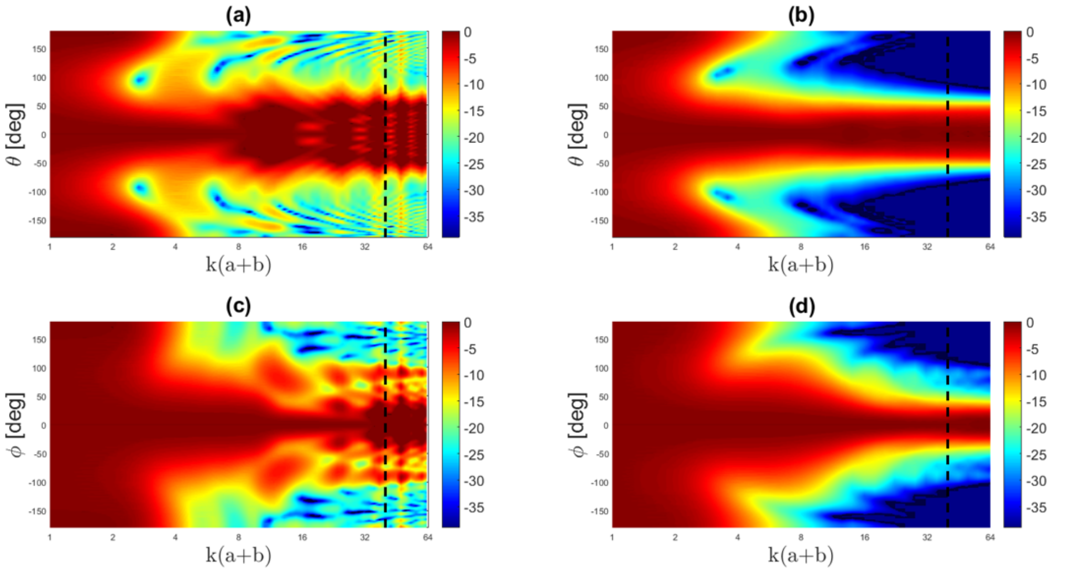

Figure 2 –Representation of torus directivity, with the a) HP with ES, b) HP with the 2DCS, c) VP with ES and d) VP with the 2DCS. For all cases: α = 60º, β = 60º and σ = 1.

The radiation pattern of the toroidal cap with ES is very frequency dependent in both planes HP and VP as observed in Fig. 2a) and 2c), respectively. Significant variations of the SPL can be observed both in the space (angle) and the frequency spectrum. High amplitude secondary lobes are present and significant variations of the SPL can be observed in the whole range of emission. However, when a 2DCS is applied to the toroidal cap, the frequency dependency of the beam is severely reduced as seen in Fig. 2b) for the HP and Fig. 2d) for the VP. In both planes, the lobes are eliminated as a result of the shading, and the spatial boundaries of the beam are very clearly defined above the value k·(a+b)= 40. For a typical array with a surface of 2.5 m^2 this means that the device would be frequency independent above f=150 Hz.

In Fig. 3 at k (a + b) = 40, the angular profiles of the two shadings are contrasted. The ES radiation pattern presents SPL oscillations higher than 6dB around the frontal emission for the HP and 3dB for the VP, as seen in Fig3 a) and Fig. 3b), respectively. Secondary lobes are present at all angles and the quality of the beam is very poor. On the contrary, when the 2DCS is applied to the toroidal cap, the radiation beam is very clean, and the secondary lobes have more than 40dB of reduction with respect to the frontal beam. At this frequency, the horizontal beamwidth (±6dB) is 100° and the vertical beamwidth (±6dB) is 60°. As the radiation pattern is frequency independent in both planes HP and VP, we can state that a toroidal cap with 2DCS is a 2D Constant Beamwidth Transducer (2DCBT)

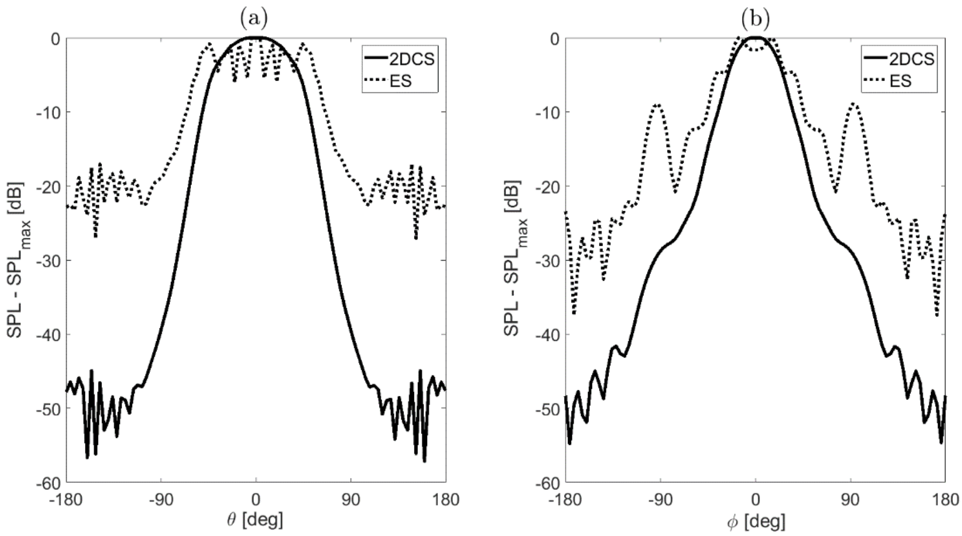

Figure 3 – Normalized Sound Pressure Level of the toroidal cap with 2DCS and ES at k·(a+b) = 40 in the a) HP and b) VP

2.2. Geometry

The dimensionless parameter k·a relates the size of toroidal structure with its angular frequency of radiation. The effect of k·a variation in directivity function is illustrated in Fig. 4. The radiation pattern becomes directional when the major radius of toroidal cap is higher than the wavelength (k·a ≤ 1) and keeps the same front directivity in all bandwidths in HP, as seen in Fig. 4a). Differences in VP are observed for higher frequencies in the lateral radiation (above 60º), and it is determined by the height and vertical curvature of the toroidal cap, as shown in Fig. 4b). The application of 2DCS achieves to minimize the frequency dependence of the radiation pattern at the frontal direction and avoids lateral lobes as well.

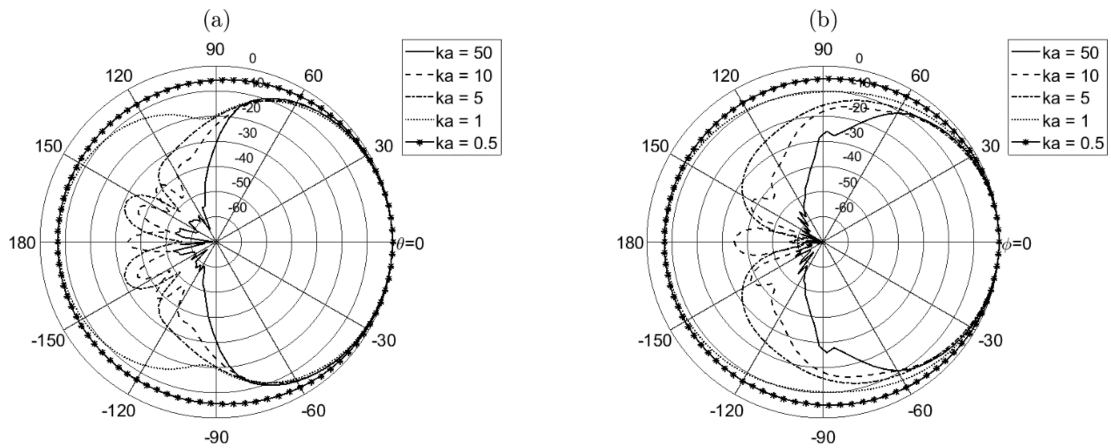

Figure 4 – Directivity pattern of the toroidal cap with 2DCS with α_0 = 90°; β_0 = 90°; σ = 1.15 with respect to the parameter k·a, in the a) HP and b) VP.

2.3. Aspect ratio

The aspect ratio σ = a / b is related to the flatness of the toroidal structure. When σ < 1, the structure is no longer toroidal (the inner hole of the tore disappears) and tends to be spherical when σ → 0. On the contrary, the higher is σ, the thinner is the toroidal cap. Fig. 5 shows directivity function in both planes with 2DCS and for different values of σ. The directivity in HP shown in Fig 5a) is shown to be independent of the geometry (σ). This is not the case in VP where they differ more for larger angles, as observed in Fig. 5b).

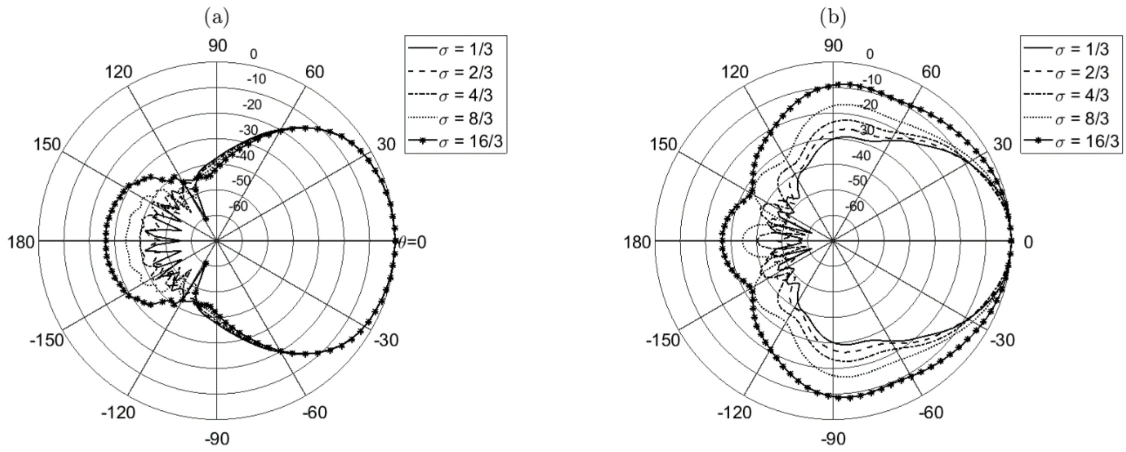

Figure 5 – Directivity pattern of the toroidal cap with 2DCS with α_0 = 60°; β_0 = 40°; k·(a + b) = 30 in the a) HP and b) VP

3. CONCLUSIONS

A 2D Constant Beamwidth Transducer is proposed based on a 2D Cosinus Shading of a toroidal cap. The radiation pattern in the Horizontal Plane and the Vertical Plane are presenting a non-frequency dependent behavior with the 2DSC and a 30dB reduction of the secondary lobes with respect to an Equal Shading. A parametric study is performed to evaluate the influence of the flatness of the torus on the directivity diagrams for different frequencies. This continuous radiating structure is the first step to analyze discrete arrays of loudspeakers in a 2D matrix configuration.

4. AKNOLEDGEMENTS

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