Characterisation of the equivalent orthotropic elastic properties of CLT panels

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Abstract
Cross-laminated timber (CLT) panels are non-homogenous and non-isotropic structures, whose elastic properties are not easy to determine. The elastic, stiffness and structural resistance properties of a CLT panel can be experimentally determined using different approaches. In this study, the elastic properties of equivalent homogeneous orthotropic elastic panels, representing CLT elements with different layup configurations, are evaluated through static finite element (FE) simulations, based on the representative volume element (RVE) approach. This analysis allows investigating the influence of the number of layers, or the ratio of their thickness on the elastic behaviour of the panel. Moreover, this study aims to verify the suitability of static elastic properties of the equivalent orthotropic element in vibroacoustic analysis. The comparison of the numerically computed dispersion curves, of different propagation modes, evaluated both for the layered structures and the associated equivalent orthotropic elements showed that equivalent static elastic properties while providing a good approximation at the lowest frequencies, do not guarantee a good accuracy within the entire frequency range. Finally, the transmission loss numerically computed from layered and homogenised models of the CLT panels are also computed.

Keywords: elastic constants, compliance matrix, equivalent orthotopic solid, wavenumbers, vibroacoustics.

1 Introduction

Cross-laminated timber (CLT) elements are wood-engineered multilayer panels constituted by a certain number of layers of adjacent timber beams bonded together. Due to the timber characteristics and such a crossed layup, CLT panel are non-homogenous and non-isotropic structures, whose elastic properties are not easy to determine. Moreover, also the vibroacoustic modelling of cross-laminated timber (CLT) panels is also challenging. Each layer of these wood-engineered multilayer plate structures is made of parallel timber boards, often called lamellae, selected for a specific stress grade. CLT panels have at least three layers, which can have a different thickness, and the grain direction of the lamellae of consecutive layers is orientated orthogonally. Timber is an anisotropic material, generally treated in engineering as an elastic orthotropic solid; CLT plates, also due to their layup which orthogonally alternates the orientation of adjoining layers, often exhibit an orthotropic behaviour. For engineering purposes, it is pretty common to consider a CLT panel as an equivalent orthotropic homogeneous panel [1], whose elastic properties depend on the elastic characteristics of the timber beams, the plate's layup configuration (e.g. the number of layers), and the ratio of the layers' thickness. Nevertheless, at the moment, there is not a standardisation of the CLT manufacturing process, causing a high variability of panels' characteristics, since each producer developed different construction rules, which define for example the total plate thickness associated with a certain number of layers, the odd to even layers thickness ratio, or many other factors which can influence the dynamic behaviour of the structure. The elastic characteristics of CLT elements, provided in the material data-sheet, refer to the timber boards constituting the different layers, which are selected according to their stress grade. The elastic, stiffness and structural resistance properties of a CLT panel can be experimentally determined using different approaches, such as the γ-method, the k-method or the shear analogy method [2, 3]. In this study, to investigate the influence of the layup of the panel and the ratio of the layers' thickness on the global elastic behaviour of the CLT panel, the elastic properties of an equivalent homogeneous orthotropic elastic panel were evaluated through static finite element (FE) simulations, based on the representative volume element (RVE) approach. CLT elements with a different number of layers and a different ratio of their thickness were considered, modelled in the commercial software Comsol Multiphysics as multilayer orthotropic structures.

FE approaches can also be useful to investigate the vibroacoustic performance of CLT structures. Yang et al. [4] recently proposed an interesting numerical approach, based on the wave and finite element (WFE) method [5], which allows for accurate vibroacoustic characterisation of CLT structures, and to compute their sound TL including the effect of higher-order propagating modes. To verify the suitability of static elastic properties of the equivalent orthotropic element for vibroacoustic analysis, a similar approach, based on FE analysis, was used to characterise the wavenumbers propagating in different CLT panels. CLT elements were modelled both as multilayer and equivalent homogeneous structures. The comparison of the numerically computed dispersion curves, of different propagation modes, evaluated both for the layered structures and the associated equivalent orthotropic elements showed that equivalent static elastic properties while providing a good approximation at the lowest frequencies, do not guarantee a good accuracy within the entire frequency range. Consistent conclusions were highlighted by comparing the transmission (TL) computed from FE simulations based on the unit cell approach [6, 7, 8], of the multilayer and equivalent orthotropic plates.

The next section briefly introduces the FE-RVE approaches which were used to determine the equivalent orthotropic elastic constants, to compute the wavenumber dispersion curves and the sound transmission loss of the investigated structures, which are described in section 3. Results are presented and discussed in section 4.

2 Representative volume element (RVE) approach

In structural mechanics and vibroacoustic FE simulations, the unit-cell approach, also known as the representative volume element (RVE) approach, is widely used. The method consists in modelling only a small portion of an infinite large repetitive structure, which can be considered as a modular unit, by applying periodic boundary conditions. According to the objective of the simulation, different kinds of periodic boundary conditions, types of analysis or excitations are required.

2.1. Equivalent elastic properties

The static elastic properties of homogenised element representing a composite, or multilayer structure can be evaluated from the homogenised compliance matrix, expressing the relationship between strains and stresses. For an orthotropic elastic solid the stress and strain relation can be expressed in the compliance form as:

$$\varepsilon = \mathbf{C}\sigma \qquad \rightarrow \qquad \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{xy} \\ \varepsilon_{xz} \\ \varepsilon_{yz} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_x} & -\frac{\nu_{yx}}{E_y} & -\frac{\nu_{zx}}{E_z} & 0 & 0 & 0 \\ -\frac{\nu_{xy}}{E_x} & \frac{1}{E_y} & -\frac{\nu_{zy}}{E_z} & 0 & 0 & 0 \\ -\frac{\nu_{xz}}{E_x} & -\frac{\nu_{yz}}{E_y} & \frac{1}{E_z} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2G_{xy}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2G_{xz}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2G_{yz}} \end{bmatrix} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{xz} \\ \sigma_{yz} \end{Bmatrix} \tag{1}$$

Using Comsol Multiphysics the coefficients of the equivalent compliance matrix $\mathbf{C}$ can be easily obtained by using the cell periodicity node, which defines the six load cases which will be used in a stationary analysis, and allows the definition of the average strain periodic conditions on pair of boundary elements. The average strain matrix of the created equivalent material is populated with boolean variables. The nine independent constants characterising an orthotropic elastic solid are thus computed from the compliance matrix, given in Equation 1.

Since **C** is a symmetric matrix the following relations hold:

$$\frac{\nu_{yx}}{E_y} = -\frac{\nu_{xy}}{E_x}; \qquad \frac{\nu_{zx}}{E_z} = -\frac{\nu_{xz}}{E_x}; \qquad \frac{\nu_{zy}}{E_z} = -\frac{\nu_{yz}}{E_y} \tag{2}$$

2.2. Wavenumber dispersion curves

Assuming the CLT panel laying in the xy-plane, the wavenumber dispersion curves can be computed by using the FE-unit cell approach. Bloch-Floquet periodic boundary conditions are applied to each domain along the y and x directions, in order to simulate the laterally infinite nature of the problem. In fact, the dynamic field $u(x, y, z)$ of the system unbounded in the xy-plane can be evaluated by analysing a single unit cell through the Bloch–Floquet periodicity relations:

$$\forall M, N \in \mathbb{Z} \qquad u(x + ML_{y,cell}, y + NL_{z,cell}, z) = u(x, y, z)e^{-j\left(k_x ML_{z,cell}x + k_y NL_{y,cell} + k_z\right)} \tag{3}$$

The frequencies associated with a certain value of the wavenumber k propagating with a given heading angle ϕ can be obtained through and eigenfrequency analysis; each frequency is associated with a different propagating mode. The number of frequencies determined for each wavenumber k depends on the number of modes computed in the simulation.

2.3. Sound transmission loss

To compute the TL, the modelled structure has to be coupled on both sides with acoustic fluid domains. Bloch-Floquet periodic conditions are applied on the lateral boundaries of both the elastic solid and the fluid domains. Besides, the latter also need to be truncated in the orthogonal direction, preventing reflection from those boundaries back into the domains. Different techniques are available to approximate the Sommerfeld radiation condition, such as for example non-reflective boundary conditions, or the use of perfectly matched layer elements (PML). In this study, a non-reflective boundary condition is applied to truncate the fluid domain. An incidence plane wave is applied to the fluid domain on the source side as a background pressure $p_b = p_0 e^{-j\mathbf{kr}}$, with arbitrary amplitude p_0, incidence angle θ, and angular frequency ω. A frequency analysis needs to be performed to evaluate the sound power transmission coefficient $\tau(\omega, \theta, \phi)$; which can be computed as the square of the ratio of the transmitted $p_t(\omega, \theta, \phi)$ to the incidence sound pressure $p_i(\omega, \theta, \phi)$ obtained from the FE analysis.

$$\tau(\omega, \theta, \phi) = \left|\frac{p_t(\theta, \phi)}{p_i(\theta, \phi)}\right|^2 \tag{4}$$

The diffuse field sound power transmission coefficient τ_d can be computed by integration over the incidence and azimuthal angles, respectively θ and ϕ. The diffuse sound field TL of the CLT panel is finally computed as $TL = -10 \log \tau_d$. A more detailed description of this method can be found in [9].

3 Investigated CLT structures

Two different kinds of CLT plates were considered in this study, constituted by three or five plies, of total thickness h equal to 0.09 m and 0.15 m respectively, and density $\rho = 450\ \mathrm{kg/m^3}$. Three different layups were considered for each kind of structure, having a different thickness ratio between the even and the odd layers, as indicated in Table 1. The material properties of each layer, associated with the orthotropic elastic properties of the timber boards, were taken from Table 1 of ref. [4]. The CLT plates were modelled by implementing the RVE approach, described in the previous section, using the commercial software Comsol Multiphysics. For each analysed structure the nine independent elastic constants of the equivalent orthotropic homogeneous element were computed. The wavenumber dispersion curves associated with the different propagation modes, modelling both the layered CLT panels and the associated homogeneous orthotropic elements, were determined along the two principal and the 45° directions: k_x, k_y and k_{45}. Moreover, also the TL were computed from multilayer and equivalent homogeneous models. Results are presented and compared in the next section.

Table 1: Layup configurations of the considered CLT structures.

CLT panel	#ply	layer thickness [m]			grain orientation
		layup a)	layup b)	layup c)	ϕ[rad]
3-ply	1	0.030	0.0375	0.020	0
	2	0.030	0.0150	0.050	$\pi/2$
	3	0.030	0.0375	0.020	0
5-ply	1	0.030	0.040	0.020	0
	2	0.030	0.015	0.045	$\pi/2$
	3	0.030	0.040	0.020	0
	4	0.030	0.015	0.045	$\pi/2$
	5	0.030	0.040	0.020	0

4 Results

The RVE static analysis allowed determining the nine independent constants to described the considered CLT plates as equivalent homogeneous structures. The equivalent elastic and shear moduli as well as the three Poisson's ratios computed for the different structures are given in Figure 1. Moreover, the equivalent properties are compared with the orthotropic elastic constants of the timber used as input in the FE model. The number of layers of the panel mainly affects the equivalent elastic modulus of the homogenised orthotropic plate. In fact, the 5-ply structure exhibited a slightly higher elastic modulus compared to the 3-ply panel in all the investigated configurations. Anyway, the ratio of the layer thickness seems to have the highest influence on the equivalent elastic properties, either on the elastic and shear moduli and on the values of Poisson's ratio. Even though, negligible variations were found for the elastic modulus along the $z-$direction and the shear modulus in the $4xy-$plane. This set of equivalent elastic properties guarantee that the compliance (or stiffness) matrix which relates strains and stresses of the homogenised orthotropic element is identical to the global matrix computed for the layered element.

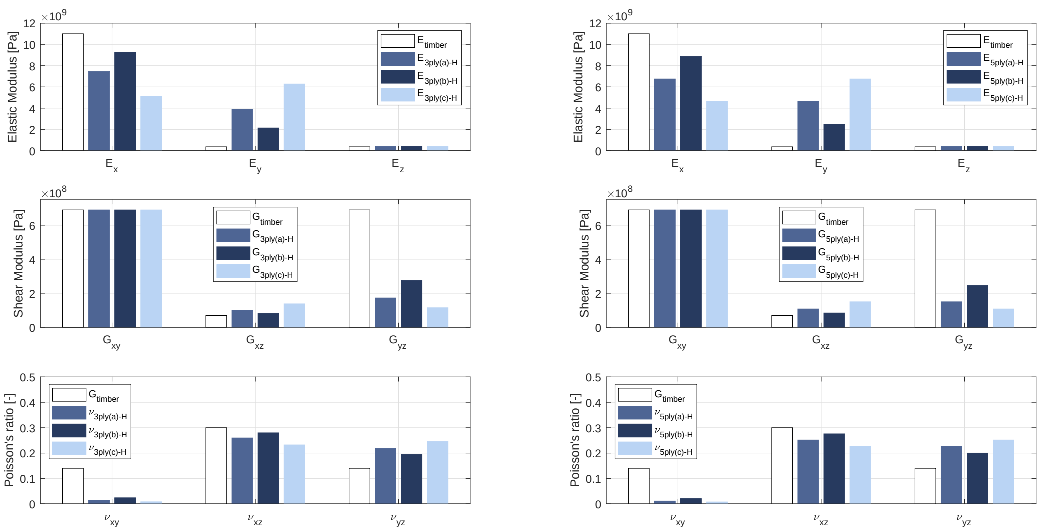

Figure 1: Homogenised elastic properties of the equivalent orthotropic CLT plates.

To understand if the equivalent static orthotropic properties are suitable to accurately describe also the CLT dynamic behaviour, the wavenumber dispersion relations of the first six propagating modes were computed. In a homogeneous isotropic or orthotropic element wave motion is described by the Rayleigh-Lamb theory of guided waves in plates [10, 11]. In Figure 2 and 3, the dispersion curves obtained from homogenised FE-models are compared with the curves computed for the 3-ply (a) and 5-ply (a) layered structures respectively. Both for the 3-ply and the 5-ply elements a significant deviation can be observed in the behaviour of layered and homogenised structures. The propagating mode m_0 is the first antisymmetric, or bending mode, which plays a key role in sound transmission. Along any investigated direction and both for the 3-ply and 5-ply elements, only at the very low frequency the homogenised dispersion curves accurately approximated the dispersion curves computed for the layered structures, while in the mid- and high-frequency range a drastic deviation can be observed. The propagating mode m_1 describe in-plane or horizontal shear motion. The homogenised approach provided a good approximation of the dispersion curves associated with this mode only within the frequency range in which there is a linear relationship between wavenumber and frequency. Similar considerations can be made by comparing the dispersion curves associated with the propagating mode m_2 which, at least in the range where there is a linear relationship between wavenumber and frequency, describe the motion due to an extensional wave. Generally, a poor agreement was found for higher modes, which describe more complicated combinations of longitudinal and shear waves.

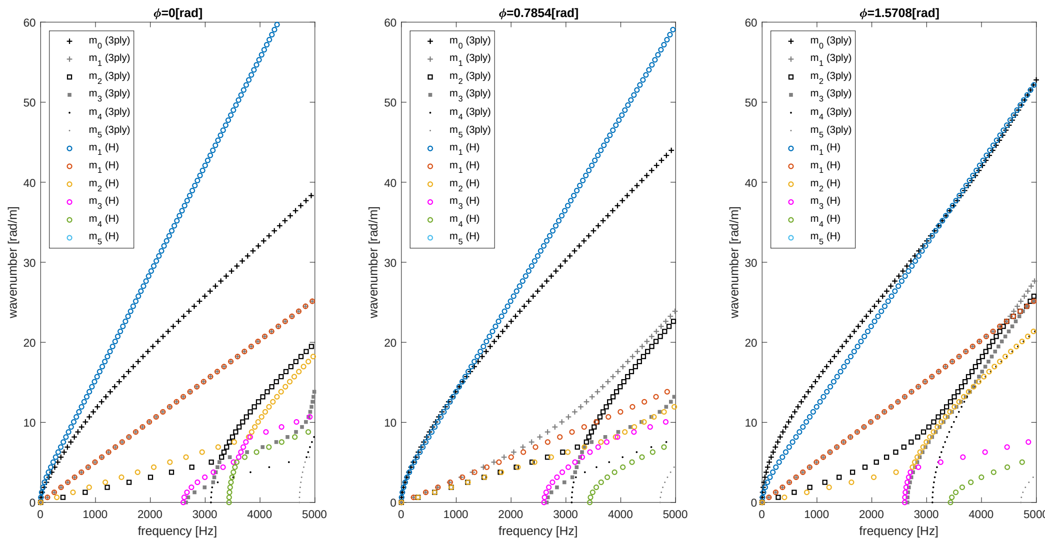

Figure 2: Wave propagation modes computed along different directions for a 3-ply CLT panel modelled as a layered element (3ply) and as an equivalent homogeneous orthotropic plate (H).

To highlight the effects of the significant deviations observed in the dispersion curves obtained from the two models on the computed acoustic performance, the sound transmission loss of the CLT panels, both in the configuration (a), modelled as layered (3ply and 5ply) or homogenised orthotropic (H) elements are compared in Figure 4. For the 3-ply structure, a surprisingly satisfying agreement was found between the TL curves, considering the discrepancies observed by comparing the dispersion curves of the same structure. The two coincidence frequency, associated with the orthotropic plate's principal directions, fall in a range of the spectrum in which a very small deviation was found between the dispersion curves associated with the bending mode. On the other hand, it was not possible to accurately approximate the TL of the 5-ply structure by using the homogenised element. A significant shift can be observed between the two TL spectra, due to a large deviation between the dispersion curve, especially associated with the bending mode, even at low frequencies.

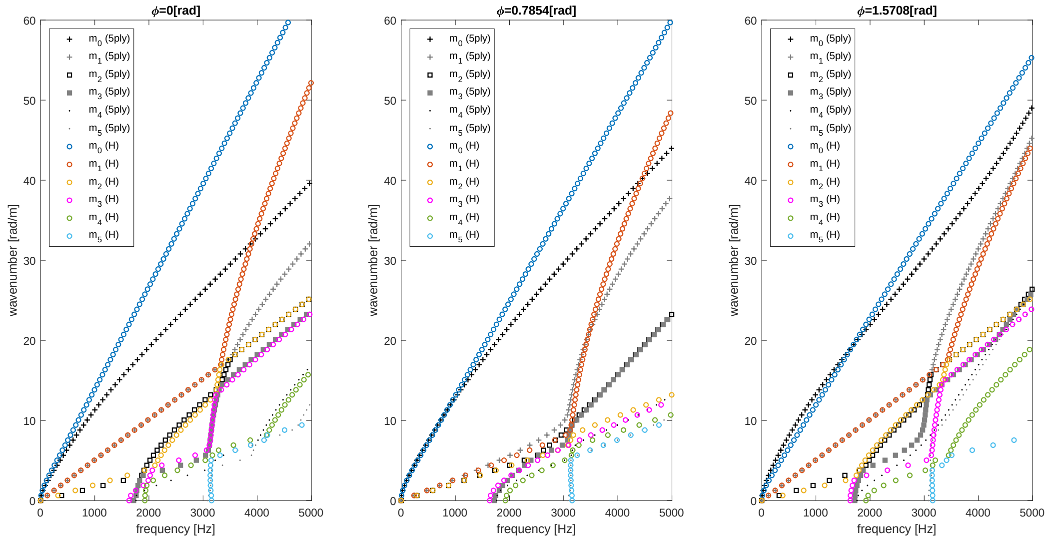

Figure 3: Wave propagation modes computed along different directions for a 5-ply CLT panel modelled as a layered element (3ply) and as an equivalent homogeneous orthotropic plate (H).

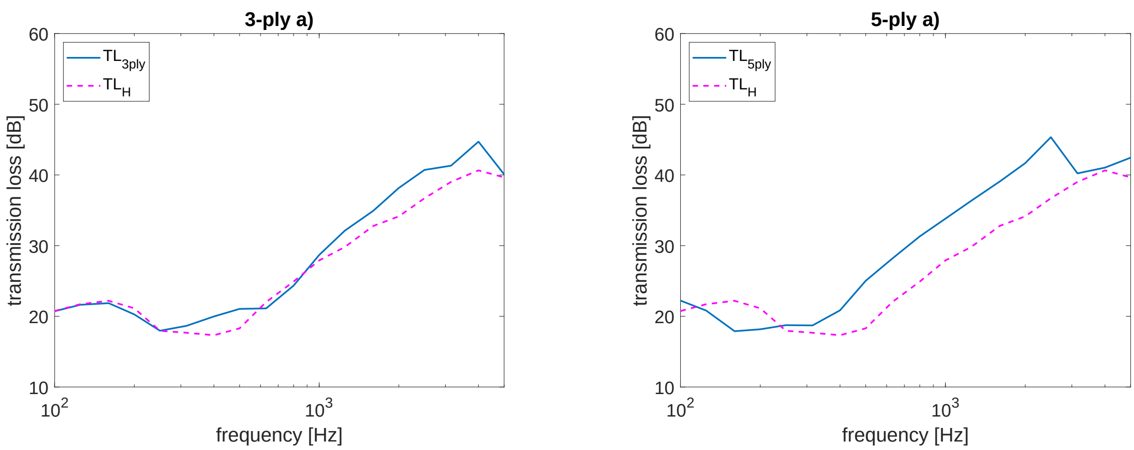

Figure 4: Numerical sound transmission loss of 3-ply and 5-ply CLT elements, modelled as layered or homogeneous orthotropic elements.

5 Conclusions

In this study, the elastic properties of equivalent homogeneous orthotropic elastic panels were computed through static finite element (FE) simulations, based on the representative volume element (RVE) approach, analysing the influence of the layup configuration of the panel and the ratio of the layers' thickness on the global elastic behaviour of the CLT panel. CLT elements with a different number of layers and a different ratio of their thickness were considered. While the computed equivalent properties, providing an identical global compliance matrix relating strains and stresses of the material, are suitable to approximate the static mechanical behaviour of CLT plates, characterised as equivalent orthotropic homogeneous solids, they cannot be used for vibroacoustic simulations. In fact, by comparing the dispersion curves computed for layered and equivalent homogeneous orthotropic elements, large discrepancies were found for all the propagating modes starting from the low-mid frequencies. Therefore, it is clear that the static equivalent orthopaedic elastic properties are not suitable to describe wave motion in CLT panels, at least not within the entire frequency range of interest in building acoustics. These findings were confirmed by the comparison of the TL spectra computed for the CLT panels modelled either as layered or homogeneous orthotropic solids. Even though for the 3-ply element in the (a) layup configuration, only small differences were observed, for the 5-ply panel substantial deviations between the TL spectra were found. The follow-up of this project will investigate the possibility to determine the orthotropic elastic properties of the equivalent homogeneous panel to approximate CLT structures in vibroacoustic analysis.

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