Inverse design of a Helmholtz resonator-based acoustic metasurface for low-frequency sound absorption using deep neural network

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Abstract
Helmholtz resonators are widely accepted as narrowband low-frequency sound absorbers. In this study, with the help of deep neural network (DNN), an optimized design of acoustic metasurface comprising four inhomogeneous Helmholtz resonators is proposed. The absorption characteristics of the proposed structure are studied using electro-acoustic analogy based analytical formulations. The analytical method is compared against full-field finite element simulation results obtained from COMSOL Multiphysics. Further, DNN-based inverse design strategy is deployed to optimize the design of metasurface. The obtained optimal design showed quasi- perfect sound absorption in the frequency range of 300-350 Hz with deep sub-wavelength thickness. The proposed design and optimization strategy are very promising for future needs in noise control engineering.

Keywords: Helmholtz resonator, Deep neural network, Metasurface, Inverse design

1 Introduction

Low frequency noise is considered as one of the pervasive environmental pollutant. The attenuation of low frequency noise is essential, though it is very difficult due to its larger wavelength. Naturally occurring objects such as vegetations [1] and passive absorbers such as porous materials [2] are not efficient to mitigate low frequency noise. Even the resonant absorbers such as quarter wavelength resonators [3] or Helmholtz resonators [4] can't absorb low-frequency noise over a broad range of frequency. In this context, it is necessary to propose a better low frequency sound absorber with broadband absorption characteristics.

The still emerging field of acoustic metamaterials shed light into the research of effective low frequency noise abatement. Space-coiling type [5], membrane type [6] and resonator-based [7] structures are the common implementations of acoustic metamaterials. Recently, Wu et al. [5] designed and constructed a metamaterial absorber with microperforated panel and coiled up Fabry- Perot channels. Wang et al. [6] demonstrated effective low frequency noise elimination using membrane constrained acoustic metamaterial. Similarly, a compact acoustic metasurface comprising inhomogeneous Helmholtz resonators is also proposed for low frequency sound absorption [7]. However, the quest for a broadband low frequency absorber with reduced thickness still remains as a challenging task.

In recent years, data-driven methods such as machine learning shows rapid developments in all fields of engineering and physics. Especially, deep learning techniques are widely used in inverse designing problems of nano photonics [8], electromagnetics [9] and vibration [10]. Recently, deep learning-based designing methods are also started to use in acoustic problems. Among them, the inverse design methodology using deep neural network proposed in the previous work [11] needs special attention owing to its superior performance and excellent prediction capability. In this work also, the same inverse design strategy [11] is used to optimize the structure of the proposed acoustic absorber.

In this study, the analytical and numerical examinations of a novel acoustic metasurface comprising inhomogeneous Helmholtz resonators is considered. Initially, the analytical scheme is established using equivalent medium theory and electro-acoustic analogy. Then the validity of the scheme is assured using full-field finite element simulations. Later, using a deep neural network based inverse design scheme, an optimal design of metasurface absorber is accomplished.

This paper is structured as follows: The geometric description of the metasurface is detailed in Section 2. The analytical methodology used for the determination of absorption coefficient of the metasurface is described in Section 3. The details of the numerical methodology are dealt in Section 4. Then the deep neural network based inverse design of the absorber is described in Section 5. Finally, the concluding remarks are detailed in Section 6.

2 Geometric considerations of the metasurface

In this study, the acoustic characteristics of a metasurface comprising four Helmholtz resonators with inserted neck (HRIN) is considered. The physical model of the proposed metasurface is given in Figure 1(a). HRIN is an altered configuration of traditional Helmholtz resonator, where the neck is totally inserted into the cavity. As shown in Figure 1(b) four such HRINs are parallelly arranged to build the metasurface. The schematic diagram of a single HRIN is shown in Figure 1(c). In order to achieve broadband noise mitigation in the low frequency regime, the geometric parameters of the proposed metasurface are inversely speculated using Deep Neural Network (DNN). The required dataset for the training purpose of DNN is generated using analytical model. The details of the analytical scheme are given in the following section.

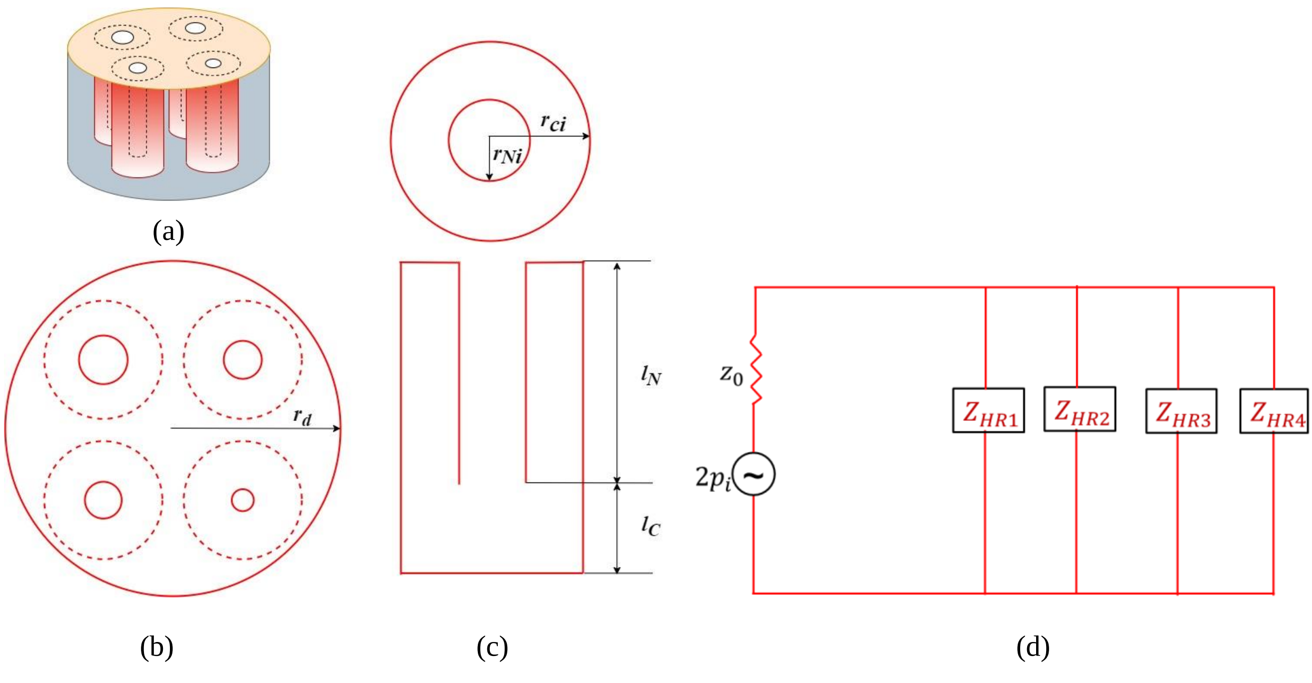


Figure 1 – (a) Physical model of the metasurface. (b) Top view of the metasurface and (c) schematic diagram of the HRIN, where l_N is the neck length, l_C is the cavity beyond neck length, $r_N = d_N/2$ is the neck radius r_C is the cavity radius and r_d is the radius of the metasurface structure. (d) Equivalent electro-acoustic circuit of metasurface, where Z_{HRi} is the acoustic impedance of the i^{th}HRIN.

3 Analytical methodology

The absorption characteristics of the proposed metasurface is analytically evaluated using the electro-acoustic analogy. The acoustic impedance of a single HRIN depends on normalized specific acoustic resistance (R_{HR}) and normalized specific acoustic reactance (X_{HR}). Note that, the normalized specific acoustic resistance [12],

$$R_{HR} = \frac{\sqrt{8\eta\rho\omega}(\frac{l_N}{d_N}+1)}{\varepsilon_T \rho c}, \tag{1}$$

where l_N is neck length, d_N is diameter of neck, η is the dynamic viscosity of air, ρ is the density of air, c is the velocity of sound in air, ε_T is the perforation ratio and ω is angular frequency. Here, $\varepsilon_T = S_n/A_0$ where, S_n is the cross-sectional area of neck and A_0 is the cross-sectional area of cavity.
Also, the normalized specific acoustic reactance is [12],

$$X_{HR} = \frac{kl_e}{\varepsilon_T} - \frac{1}{kl_{RC}}, \tag{2}$$

where $l_e = (l_N + \delta)$, is the effective neck length, in which δ=0.85d_N, is the end correction of neck and l_{RC}=$l_N + l_C$-$\varepsilon_T l_e$ is the modified cavity length.
Thus, the acoustic impedance of a HRIN (Z_{HR}) is,

$$Z_{HR} = Z_0(R_{HR} + jX_{HR}), \tag{3}$$

where, $Z_0 = \rho c$ is the acoustic impedance of air.
The metasurface proposed in this study comprises of four Helmholtz resonators which are parallelly connected. According to the electro-acoustic analogy represented in Figure 1(d), the total acoustic impedance of the metasurface (Z_t) can be formulated as

$$Z_t = \frac{1}{\sum_{i=1}^{4} \frac{S_i}{Z_{HRi}}}, \tag{4}$$

where, $S_i = \frac{\pi r_{Ci}^2}{\pi r_d^2}$ is the area ratio of HRIN. From Z_t , the absorption coefficient of metasurface (α), is obtained as

$$\alpha = 1 - \left|\frac{Z_t - Z_0}{Z_t + Z_0}\right|^2, \tag{5}$$

4 Numerical model

In order to validate the analytical model, full-field finite element simulations are carried out in COMSOL Multiphysics software. The frequency domain interface with pressure acoustics module is used to analyse the sound propagation through the metasurface. The sound pressure distribution inside the model is governed by Helmholtz equation of the form,

$$\nabla . \left(\frac{1}{\rho}(\nabla p)\right) + \frac{k^2 p}{\rho} = 0, \tag{6}$$

where p is the acoustic pressure and k is the wave number. The thermo-viscous losses inside the neck have to be modelled separately. For this, narrow region feature under pressure acoustic module is chosen. The walls of the metasurface are modelled as perfectly rigid and the inside fluid medium is chosen as air. The domains are discretized using tetrahedral element and the maximum element size is chosen as $\lambda/20$, where λ is the wavelength corresponding to highest frequency of interest. The narrow regions such as necks are modelled using very fine mesh of high resolution. Then by using two-microphone impedance tube method [4, 13] the absorption characteristics of the metasurface is determined over a frequency regime of 100-500 Hz.

Initially the absorption characteristics of a metasurface absorber comprising four inhomogeneous HRINs are analyzed using numerical and analytical schemes. The dimensions chosen for modelling the absorber are given in Table 1. The comparison of the analytical and numerical results is given in Figure 2, which shows good agreement. The absorption characteristics of the metasurface shows four absorption peaks corresponding to the resonance frequencies of the individual HRINs. Indeed, each HRIN has almost perfect absorption and they spread within the frequency regime of 250-400 Hz.

The present study mainly focused to propose the design of a metasurface, which has quasi-perfect sound absorption in the frequency regime of 300-350 Hz. For this, deep neural network based inverse design strategy is used. The details of the inverse design using deep neural network (IDDN) are detailed in the following section.

Table 1 – Geometric parameters of the metasurface absorber with 4 different HRINs. For all HRINs the parameters kept constant are l_{Ni}=35 mm, l_{Ci}=5 mm and r_{Ci}=18 mm, where subscript 'i' corresponds to the number of HRIN. The radius of metasurface r_d=50 mm.

Number of HRIN	d_{Ni} (mm)
1	7
2	8
3	9
4	10

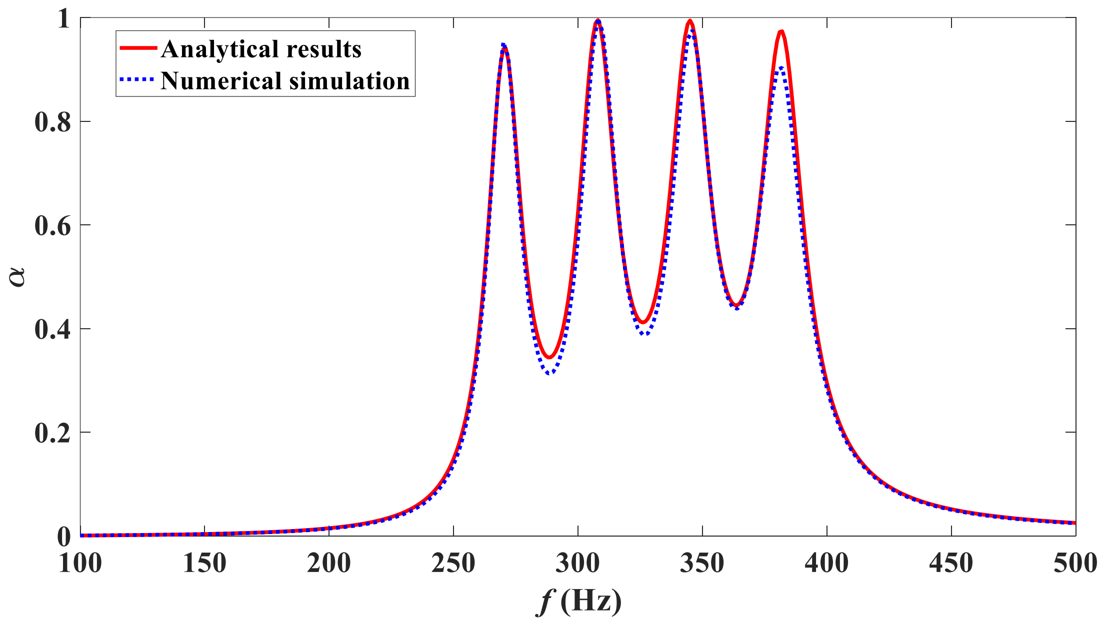


Figure 2 – Sound absorption characteristics of metasurface from analytical and numerical simulations. The dimensions of the metasurface are given in Table1.

5 Inverse design of metasurface

To accomplish the optimum dimensions of the metasurface, IDDN scheme is deployed. IDDN scheme is a data driven prediction technique. For this large amount of data is required, which is generated using analytical scheme. Using the generated data, the IDDN maps the relation between outputs and inputs. It has three main stages – training, validation and testing. Once the suitable deep neural network is created, it will be trained and validated using available dataset. After the successful completion of these steps the performance of the network on a mutually exclusive test data is conducted. If the performance of the model is satisfactory on test data, it is deployed for required application. The detailed explanation of the IDDN scheme is available in the previous work [11].

Dataset generation

Here, IDDN is used to map the relation between absorption characteristics and corresponding geometric parameters. When a desired absorption spectrum is given to the trained neural network it will predict the corresponding geometric parameters. In this study, the desired absorption characteristics of the metasurface will give as the inputs and the neural network will predict the geometric parameters as the outputs. The dataset required for the IDDN scheme is generated using analytical methodology. Using Eqn. (10) the absorption coefficients corresponding to an arbitrary set of nine geometrical parameters are computed over a frequency range of 0-500 Hz. Neck radii and neck lengths of HRINs as well as the radius of the metasurface are randomly varied for the data generation. During data generation all other physical and material parameters are kept constant. The dimensions chosen for the data generation are given in Table 2. To maintain a compact size, the total thickness of the metasurface is limited to 40 mm. Hence the maximum neck length chosen for training is fixed as 35 mm and cavity beyond neck length is chosen as 5 mm, while all other geometric parameters are chosen according to the frequency of interest. Using custom python scripts, a total of 262144 data samples are generated. Among them 80% of data is assigned for training and remaining data is used for validation. Also, a mutually exclusive test set of 2000 samples are generated for testing.

Table 2 – Dimensions chosen for the dataset generation for the prediction of geometrical parameters of metasurface.

Parameters	Limit of random values (mm)
r_{Ni}	2-5
l_{Ni}	20-35
r_d	45-60

5.1 Deep neural network architecture

The architecture of the custom DNN model used for the inverse prediction is detailed in this section. As shown in Figure 3, the DNN architecture takes absorption coefficients corresponding to 0-500 Hz as the inputs and it predicts the geometric parameters as outputs. For better performance and improved prediction accuracy the value of r_C is given as an additional input along with 500 absorption coefficients. The input data is a one-dimensional array. To process such a data 1D convolutional layers are appropriate. Hence initial layers are chosen as 1D convolutional layers and dense layers are used for complex feature extraction. In between 1D convolution layers max pooling layers are used for reducing the dimensions of feature map. To prevent the model from overfitting, Dropout layers are used in between dense layers. The complete details of the architecture are given in Table 3.

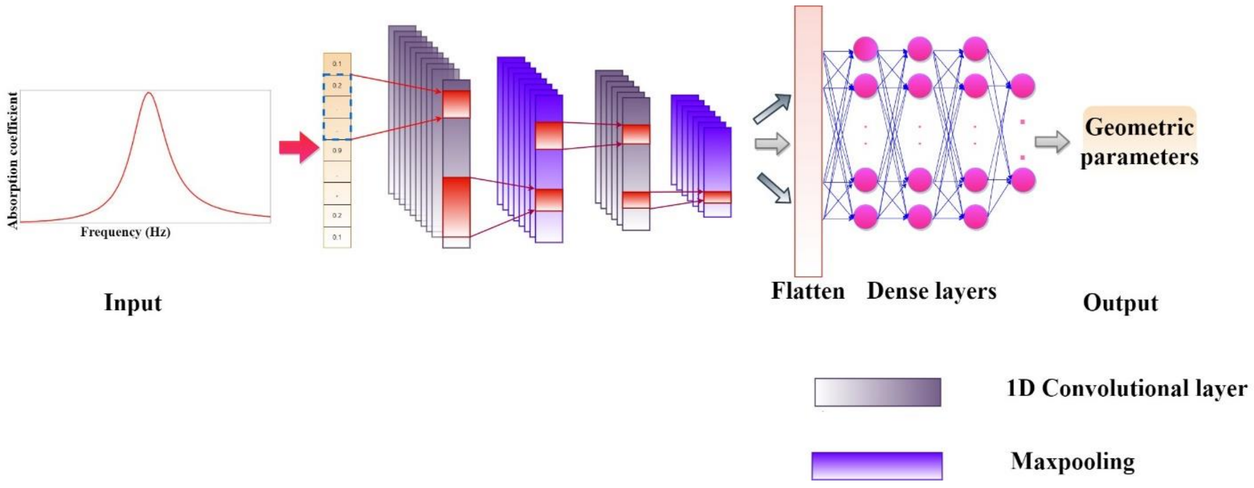


Figure 3 – Schematic representation of DNN architecture.

Table 3 – Details of layer parameters of the proposed DNN architecture.

Layer name	Layer parameters	Output shape
Conv 1D	64 × 3, Strides = 1, Input shape = (501, 1), Activation = ReLU	499x64
Max pooling 1D	Pool size=3, Strides=1	166x64
Conv 1D	32 × 3, Strides = 1, Activation = ReLU	164x32
Max pooling 1D	Pool size=3, Strides=1	54x32
Flatten	-----	1728
Dense	1024, Activation = ReLU, Dropout rate =0.2	1024
Dense	1024, Activation = ReLU, Dropout rate =0.1	1024
Dense	512, Activation = ReLU	512
Dense	256, Activation = ReLU	256
Dense	128, Activation = ReLU	128
Dense	64, Activation = ReLU	64
Dense	32, Activation = ReLU	32
Dense	8, Activation = ReLU	8

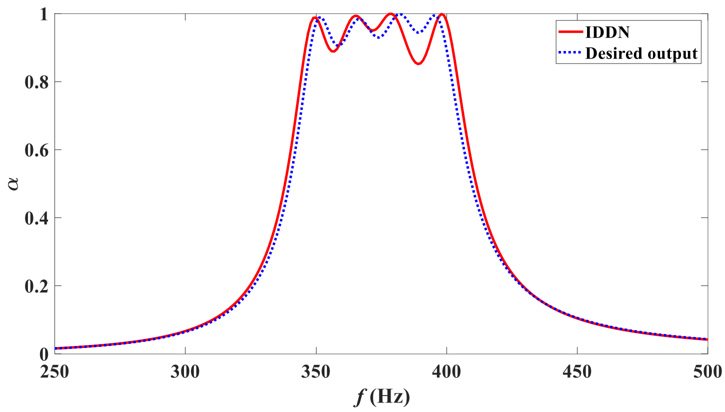
The model is trained using Adam optimizer for 1000 epochs with an initial learning rate of 0.001. The learning rate is decreased in steps up to 10^{-6}. The batch size is chosen as 1000 and the mean absolute error (MAE) is chosen as the loss function. MAE is defined as,

$$\mathrm{MAE} = \frac{1}{n}\sum_{i=1}^{n}\left|y_j - \widehat{y_j}\right|, \tag{7}$$

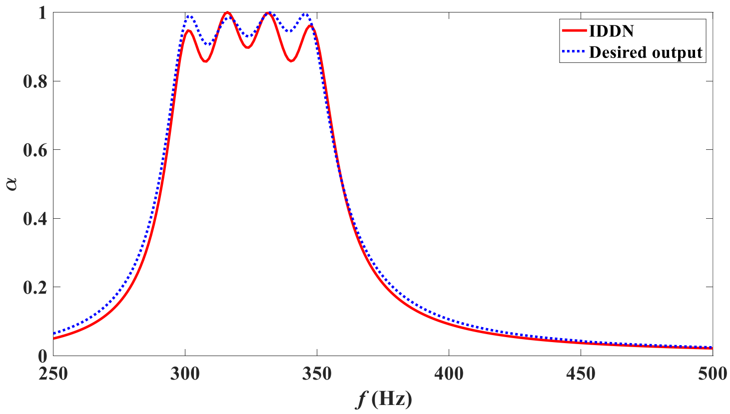
where y_j is the actual value, $\widehat{y_j}$ is the predicted value, and 'n' is the number of data samples. When MAE between the predicted geometric parameters and the actual geometric parameters is reduced to 0.2 in the test set, the training process of IDDN is stopped and the model is saved. Using the saved model further predictions are done.

5.2 Prediction results

After successful training and testing the DNN model is used to predict the geometric parameters of metasurface having desired absorption characteristics. For instance, two such desired absorption spectrums (blue dashed lines in Figure 4) are randomly generated (for S1 & S2) and passed through the neural network. In addition to the absorption spectra the value of r_d is also given as the input and it is set as 50 mm. Hence the IDDN will predict the geometric parameters of the metasurface having 50 mm radius and desired absorption spectra. The prediction results of IDDN scheme are given in Table 4. Using the predicted parameters, the absorption spectra is recreated using the analytical methodology and it is compared against desired spectrum. The obtained frequency response shows good agreement with the desired spectrum.



(a)



(b)

Figure 4 – Comparison between the desired absorption characteristics and the absorption characteristics generated using the geometric parameters predicted by IDDN method for metasurface models (a) S1 and (b) S2.

Table 4 – Predicted dimensions of metasurface models S1 and S2. For both models, l_{Ci} = 5 mm, r_{C_i} = 18 mm and r_d =50 mm

Metasurface model	d_{Ni} (mm)	l_{Ni} (mm)
S1 (90% absorption in 350-400 Hz.)	7.6, 8.46, 8.4, 9.2	25.63, 30.6, 32.1, 30
S2 (90% absorption in 300-350 Hz.)	7.1, 8.1, 7.1, 8.3	26.2, 34.5, 31.5, 33.3

6 Conclusions

In this study, with the help of deep neural network, optimized design of acoustic metasurface consisting of four inhomogeneous Helmholtz resonators is proposed. For that a novel design of acoustic metasurface is proposed and its absorption characteristics are evaluated using analytical scheme. The analytical scheme is validated using full- field finite element simulations. Then, using IDDN scheme the geometric parameters of the metasurface are predicted and optimal designs are proposed. The conclusions arrived from this study are listed below.

- The proposed metasurface absorber consisting of four inhomogeneous Helmholtz resonators exhibits excellent low frequency sound absorption.
- Using the IDDN method, two metasurface models with quasi-perfect sound absorption (more than 85%) is proposed. The proposed models, S1 and S2, exhibit broadband sound absorption in the frequency range of 300-350 Hz and 350-400 Hz respectively.
- The proposed metasurfaces are compact, and their overall thickness is 4 cm only, which is deep sub-wavelength in scale (λ/25, for *f*=350 Hz).

The proposed metasurface models as well the artificial neural network based inverse design strategy has potential applications in noise control engineering.

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