Examples of constraint-based specification of room acoustic parameters

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Abstract
The paper provides an overview of the concept of room acoustics specifications, where a set of mathematical, physical or other constraints results in ranges and relationships of room acoustic parameters. In its simplest forms, this approach is known and used widely. Using new types of constraints is useful to reveal priority and relationships among room acoustic parameters and also to avoid unfeasible sets of requirements. From geometrical and statistical considerations to simple modelling approach are presented and compared.

Keywords: room acoustics, specification, room size, natural frequency, diffusion.

1 Introduction

In engineering practice, quality is always measured by some kind of technical parameters and there are usually limits assigned to those parameters, in order to help decisions or just to distinguish "bad", "hazardous", "risky" of "preferred" situations. In room acoustics the most basic relation is known since the studies by Sabine: in order to shorten the decay apparent after stopping the sound, one must add more absorption to the room, because $T \sim V/A$, where V is the volume of the room, A is the absorbing power and T is the length of the decay with respect to the $1{:}10^6$ change in power. While this relation might seem historical, it is used directly or indirectly as the basis of the most basic considerations or standards.

Since I learned into engineering room acoustics, I found different forms of rules to follow during the design, but rarely found well-founded explanations to them. For example, when I was involved to create the new Hungarian standard of room acoustic specifications, I was asked to create some guidelines that would help the reader to keep the design in favour of better acoustic. Such guidelines are based on some kind of constraints. There I chose simply to set constraints based on the timing of the first lateral reflections and did end up in a reasonable guideline for classroom sizing.

This paper revisits some of the known constraint-based guidelines and explores new ways of finding reasonable specifications.

2 Classroom Sizing

One of the most important items in the discussion of the aforementioned national room acoustic standard was the case of classrooms. Besides the debate on reverberation time requirements, a set of other suggestions were also included to support sizing of classrooms.

An important observation was, that teachers are not sticked to a position and aiming, but they walk around and turn around, and therefore their directivity and position shall not be considered. In any case it is important to direct early energies to the students. If not directly, then by reflections. Other studies have agreed, that the first 50 ms is generally acceptable as an early-late limit in energy ratios the express the effect of room acoustics on speech intelligibility and clarity [1]. In addition, the 35 ms early-late limit is suggested for cases where clarity is even more important (e.g. foreign language study, children with hearing difficulties).

If we set a constraint, that from any position there shall be reflections within the first 50 ms or 35 ms, sizes of the classroom can be given a guideline. Reverberation time control is usually started at the ceiling and hearing is more sensitive to lateral reflection, so reflections from the ceiling are not considered. Also, reflections from the floor or desks can be excluded, because they are highly position dependent and also depend on occupancy and other mobile objects.

Figure 1 shows the result of the constraint to keep the number of lateral early reflection high (taking 8 reflections to be the "best") and was included in the standard. Interestingly the results agree well with some usual other conventions (maximum floor area, maximum length, usual number of desks/children etc.). An important message of this figure is also, that to keep listeners engaged, get them closer, i.e. their number low.

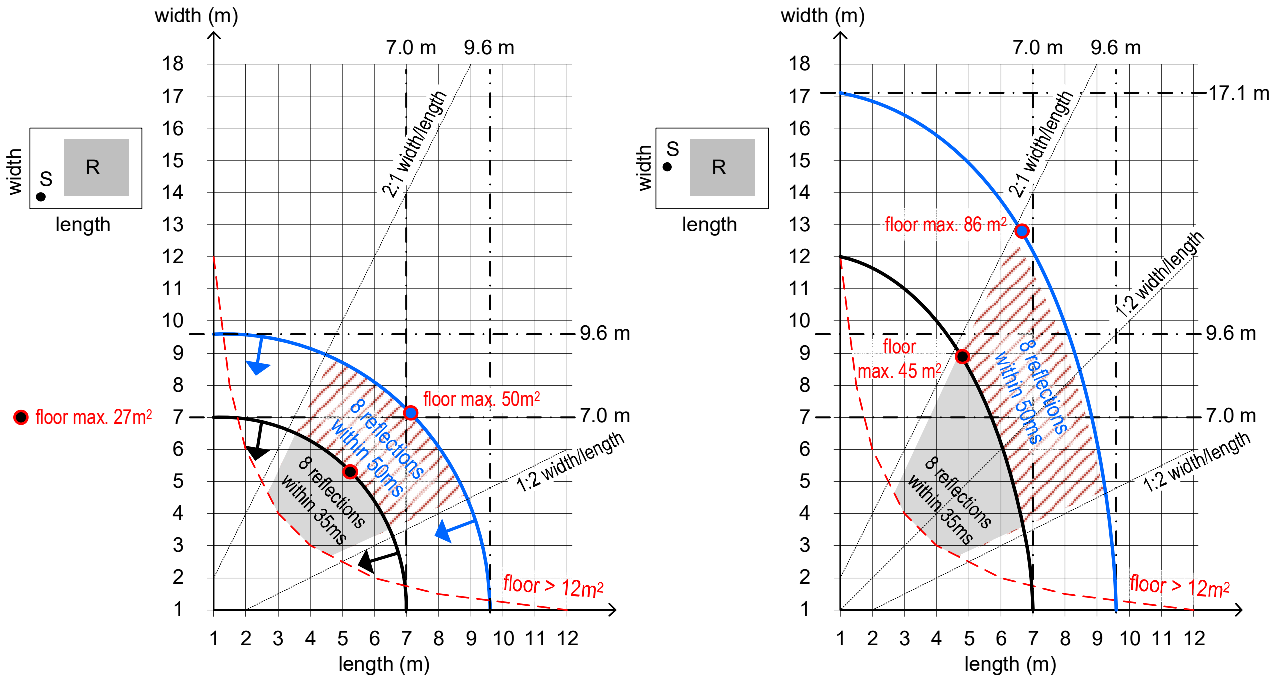

Figure 1 – Classroom sizing chart, based on minimum required number of early lateral reflections.
(left: source S is in the corner, right: source S is in front center, both: receivers R within 1m of walls)

3 Concert Hall Sizing

Setting criteria for concert hall dimensions is supported by previous works. A ratio given to support shoebox-shaped concert hall design is for example:

$$\frac{H}{W} > 0.7 \text{ and } \frac{L}{W} < 2.0 \qquad (1)$$

where H is the height, W is the width and L is the length of the hall.

To see this working, let us assume a set of constraints (e.g. [2]):

- the width of the room shall be within: $15\ m \leq W \leq 25\ m$
- number of listener seats: $800 \leq N \leq 2200$
- volume of hall: $10\ m^3 \leq V/N \leq 14\ m^3$
- maximum distance to stage (we assume it to be the center of the stage): $r_{max} \leq 40\ m$

and some usual architectural constraints:

- required floor area of audience (assuming a 0.6×0.9 m: $S_a/N = 0.54\ m^2/seat$
- stage area: $S_s : S_a = 1:2$ and $S_s \geq 50\ m^2$.
- width of corridor around audience area: 2.0 m
- maximum number of seats in a row: $N_{row} \leq 15\ seats$

From basic geometric calculations the number of seats, then volume and height can be calculated. Assuming mean absorptions (NRC) of walls 0.10 and unoccupied audience area 0.50, statistical reverberation times (Eyring) can be calculated and other room acoustic parameters (G, LF_{80}, C_{80}) can be approximated. Figure 2 shows the generalized floor plan scheme and the result for reverberation time, G using and LF_{80} using approximations found in [3] and [4] respectively.

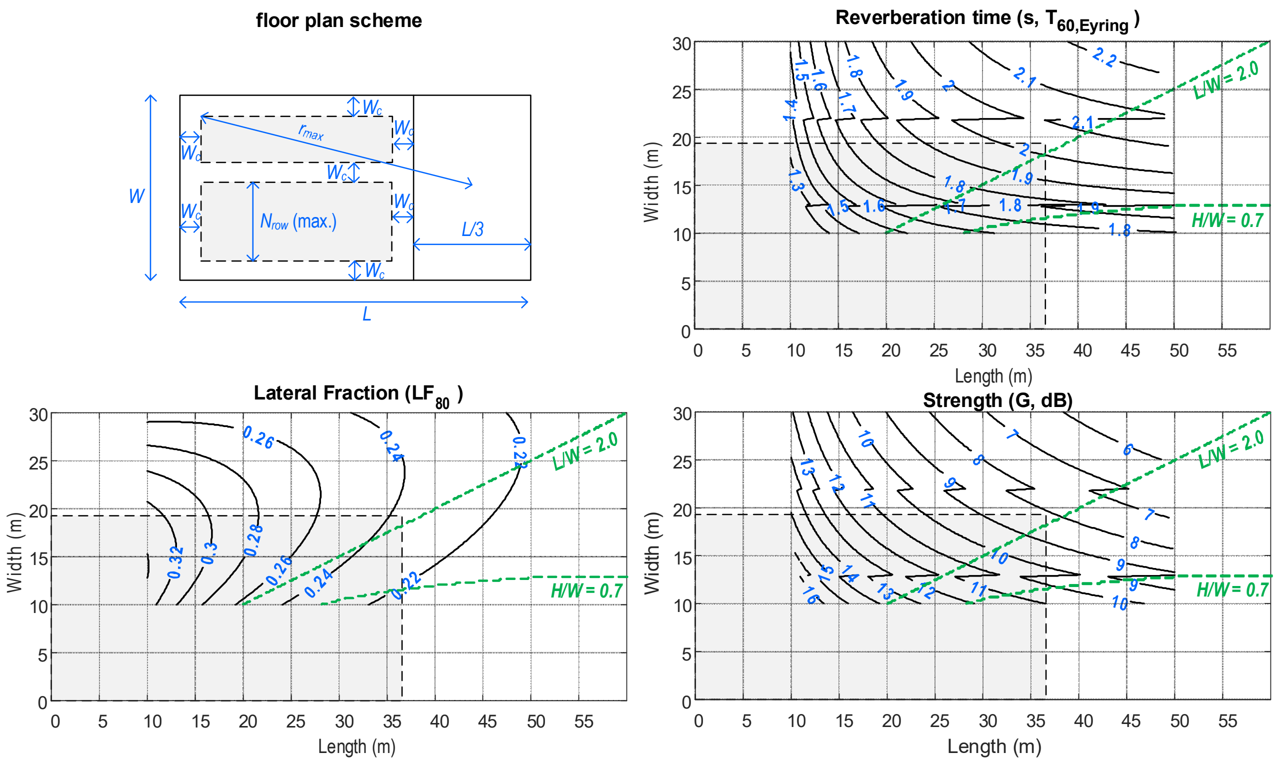

Figure 2 – Concert Hall sizing chart, based on constraints including floor plan scheme and statistical room acoustic approximations. Shadowed area denotes a 36×19 m floor area as an example.

If we set the volume constant V = 16000 m^3 as a constraint and draw reverberation time as a function of length and height, omit the maximum number of seats/row, the purpose of (1) seems to reveal (see Figure 3) as to keep the reverberation time at a maximum.

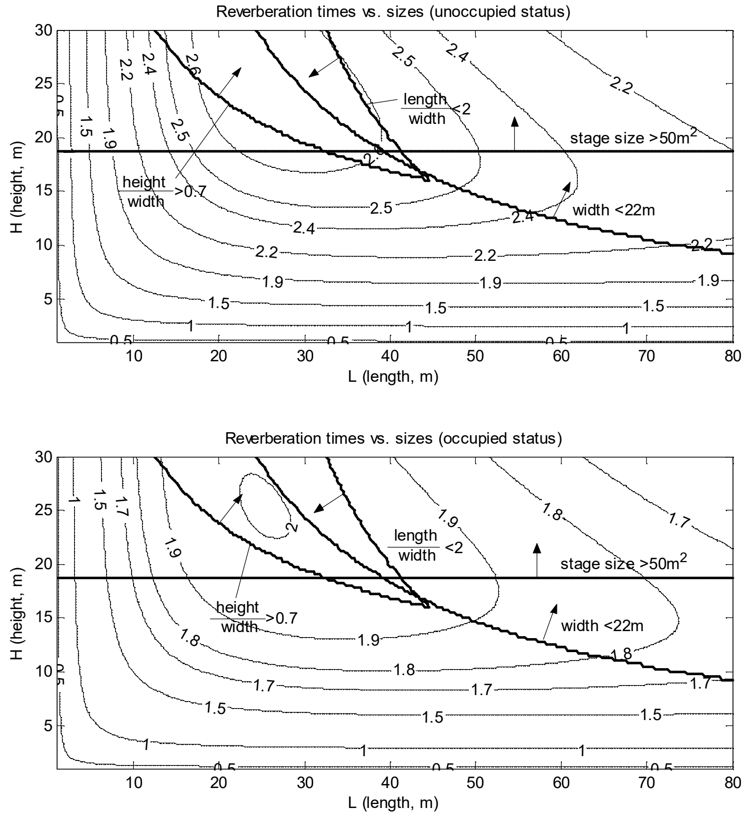

Figure 3 – Concert Hall sizing chart for reverberation time, using constraints: V = 16000 m^3, Nrow not limited, NRC of walls is 0.15, NRC of seats is 0.50 (unoccupied) or 0.80 (occupied), 0.50 m^2/seat.

4 Room Sizing for Low Frequency Transmission in a Rectangular Room

Low frequency behaviour of a room is assumed to be well controlled, if room modes are distributed evenly in the musically important low frequency region of 20-200 Hz. Suggestions to preferred ratios of length, width and height are therefore based on evaluation of distribution of natural frequencies.

Natural frequencies of rectangular spaces can be listed using equation

$$f_{n_x,n_y,n_z} = \frac{c}{2}\sqrt{\left(\frac{n_x}{L}\right)^2 + \left(\frac{n_y}{W}\right)^2 + \left(\frac{n_z}{H}\right)^2} \quad (2)$$

where c is the speed of sound, (n_x, n_y, n_z) are any combination of natural numbers if $n_x + n_y + n_z \geq 1$.

If the quality of the room is expressed as the mean square of distance of adjacent natural frequencies (based on [5]), one may get a contour, where also well-known preferred or risky ratios outline. Figure 3 also denotes preferred area of ratios suggested by [5]:

$$\frac{1.1 \cdot W}{H} \leq \frac{L}{W} \leq \frac{4.5 \cdot W}{H} - 4 \quad (3a)$$

$$L < 3 \cdot H, W < 3 \cdot H. \quad (3b)$$

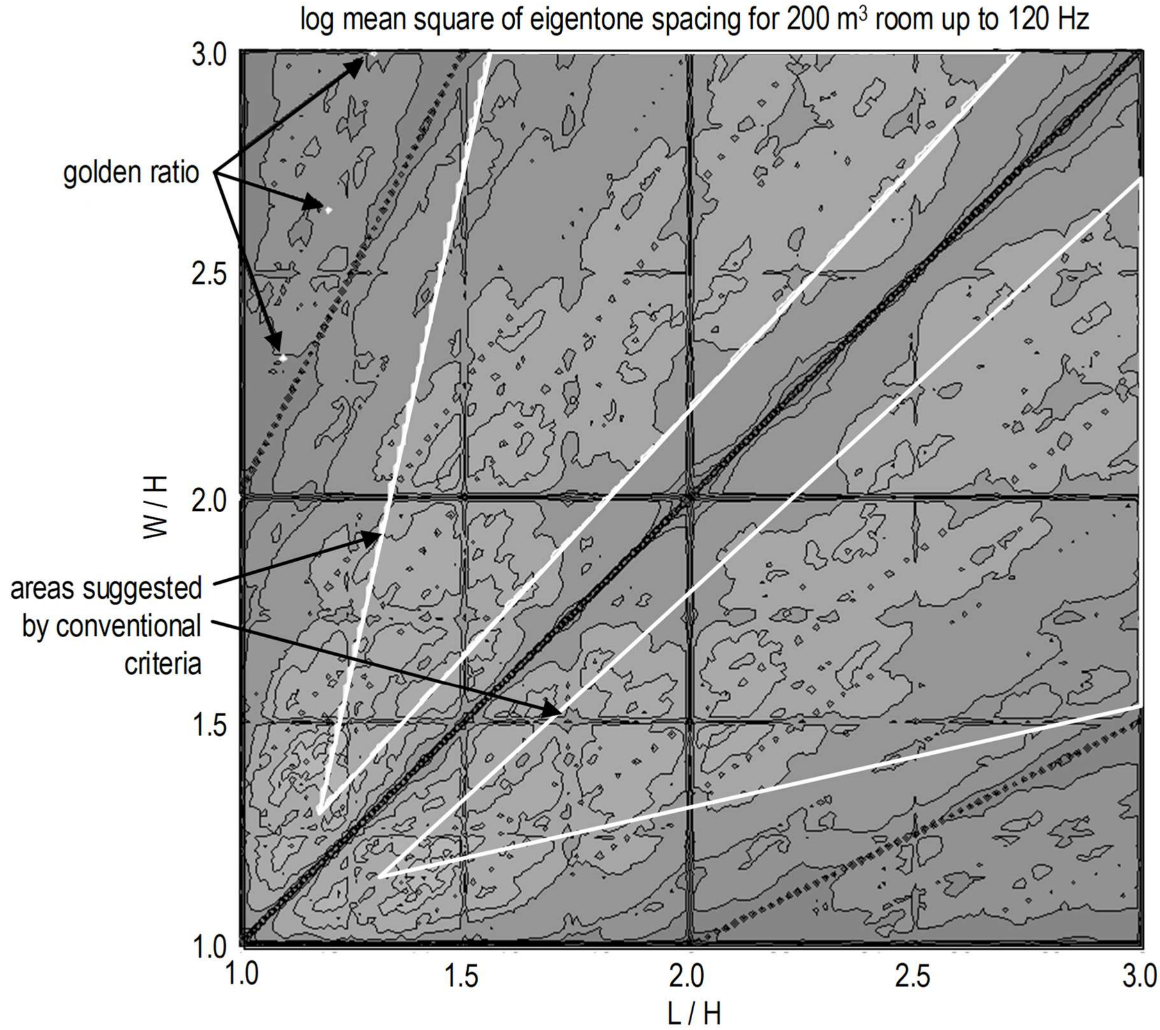

Figure 4 – Contour plot of quality of distribution of natural frequencies. White dot denote golden ratios, white line denotes are suggested by inequalities (3a) and (3b).

Despite the benefit to avoid some mistakes, Figure 4 shows that these inequations do not guarantee a flawless distribution of modes.

The topic is well summarized in [6] and introduces another type of quality descriptor called frequency space index (FSI), which is the normalized relative variance of the distance between adjacent low frequency modes. An interesting conclusion was, that the quality does depend more on W/L than W/H.

The question is therefore: do these conclusions change if instead of the distribution of eigenmodes, actual responses within a limited region the room are qualified and compared? The change of view is practical, because both instruments (sources) and listeners are using only the $1m \leq x \leq L-1$, $1m \leq y \leq W-1$ and $1m \leq z \leq 2m$ region, not the whole volume.

Responses in a rectangular enclosure can be calculated using the mirror image source method. A systematic series of calculations were run with the following settings in order to see any conclusions:

- geometry is definite (no uncertainties), reflections are purely specular,
- absorption is 0.10 on every surface,
- volume is 200 m^3,

- number of possible sources is 10, number of receivers is 25 (overall 250 responses for each dimensional variation, repeated once to have a total of 500 different random source-receiver position for each L, W and H),
- the source is a 0 $dB_{SPL@1\,m}$ omnidirectional source,
- mirror sources up to at least 20 times the diagonal of the room were collected.

Responses were calculated upon the complex summation of coherent image sources at frequencies stepped by $1/12^{th}$ octave resolution from 20 Hz to 200 Hz.

Quality of each response can be expressed based on mean absolute deviation from the mean response (Q1), the standard deviation of the response (Q2) or the difference of maximum and minimum response (Q3). Lower values denote preferred situations for all quality indicators. A single response and a set of responses for a single source position is shown in Figure 5.

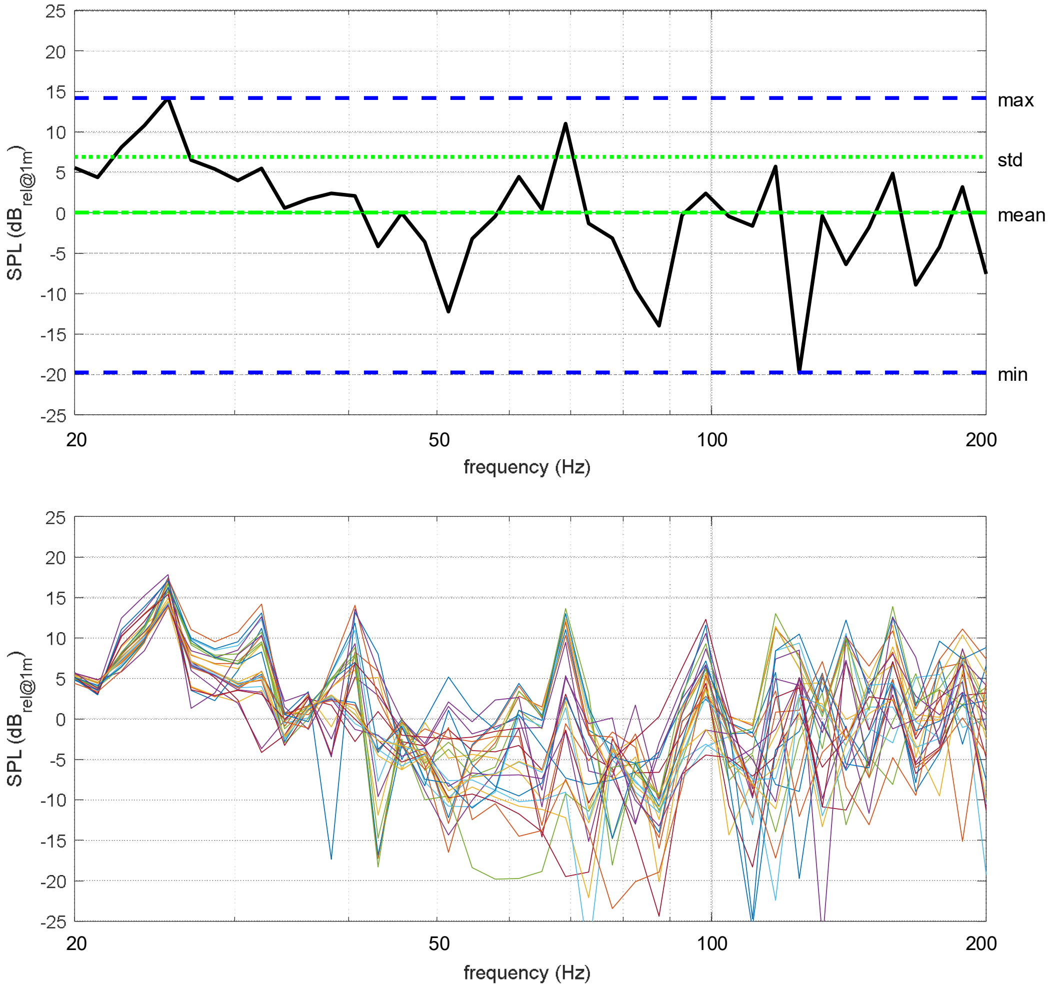

Figure 5 – Calculated responses of a shoebox MISM model, top: single source/receiver and descriptors of response ‘quality’, bottom: responses from 1 source to 25 receivers, monochromatic at 1/12th octave frequencies between 20 Hz and 200 Hz.

Using the assumptions, results of quality descriptors Q1, Q2 and Q3 are shown in Figure 6. It seems, that characteristic contours of Q1, Q2 and Q3 are similar, but Q3 shows differences more clearly. A combined qualifier Q* is the average of each normalized qualifier Q1*, Q2* and Q3*, where normalization means to scale results between 0 and 1, so that Q3* has also values between 0 and 1.

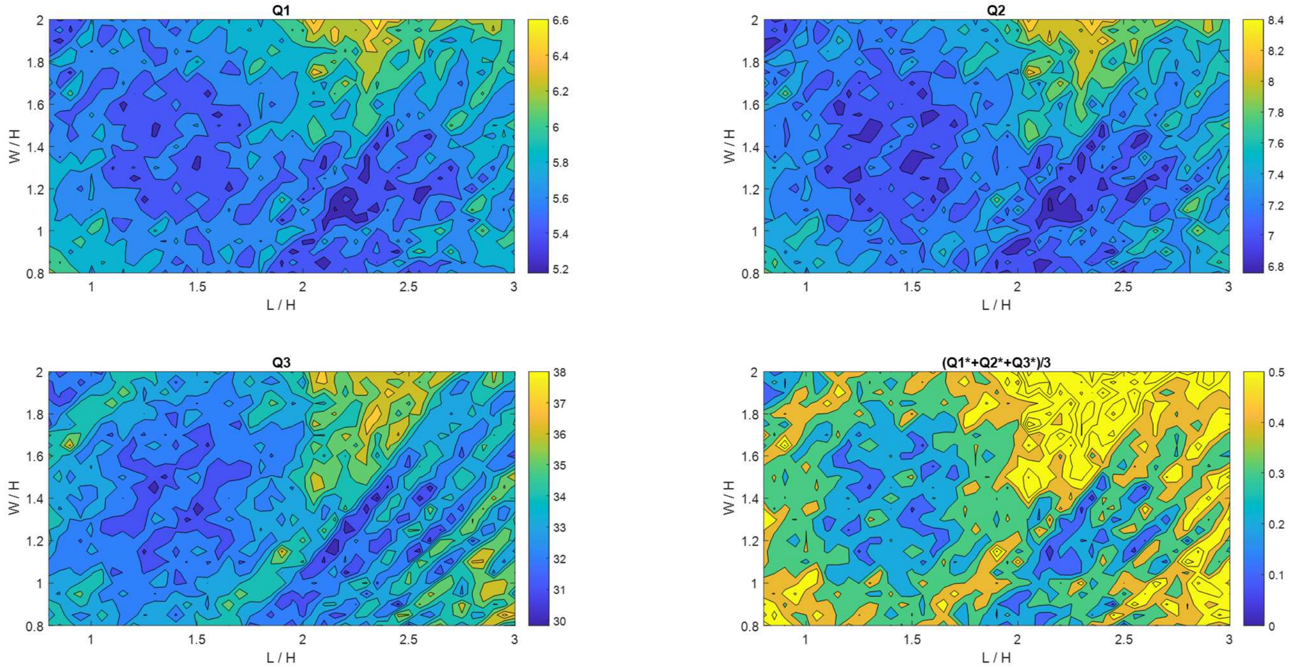

Figure 6 – Contours of low frequency response qualifiers Q1, Q2 and Q3 for different proportions of a V = 200 m3 reverberant (α = 0.1) room. Color axis is in dB (except for right bottom). Results from 1125 different ratios and 500 random source receiver responses each. Bottom right: average of normalized qualifiers, color axis limited to 0.5 to reveal a better view on minimums.

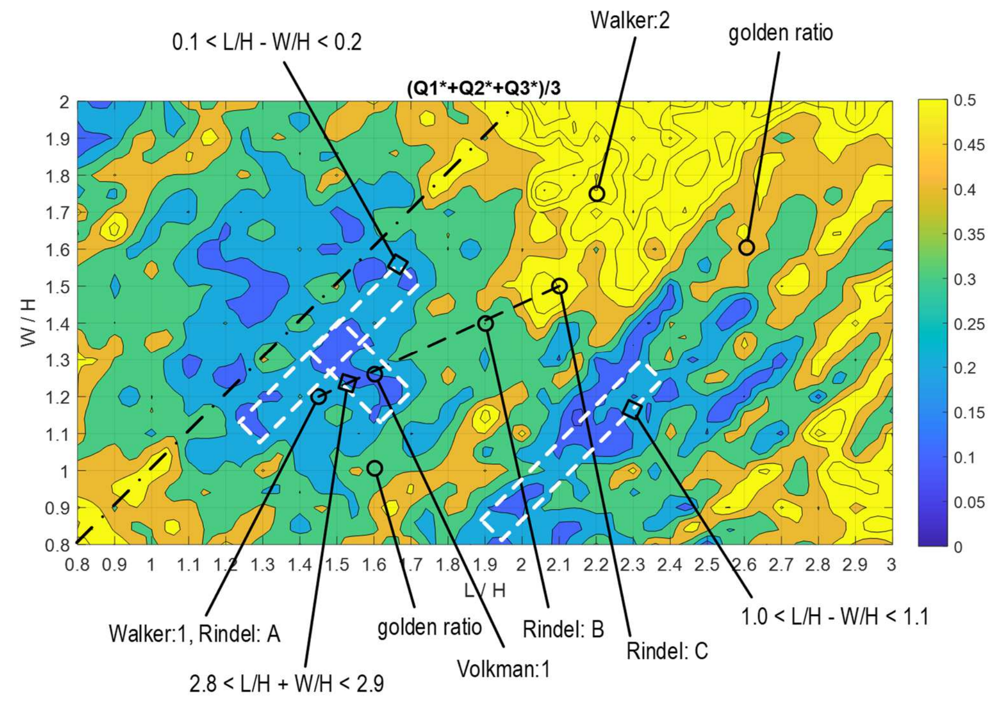

Figure 7 – Enhanced view from Figure 6, showing some preferred ratios from past works (see [6]). White dashed outlined areas denote preferred ratios for this qualifier.

Due to discrepancies from other works, effects of changes of constraints shall be tested in order to make general conclusions on preferred ratios of room sizes.

5 Minimum Required Diffusion

It would be highly important, but to the best of the author's knowledge, there is no objective measure available that could describe overall diffusity of a room, which could be used similarly to absorption power.

In theory, however it is quite simple to derive a requirement.

If we accept the fact, that the energy decay can be approximated by the function (Eyring)

$$E_{total,t} = E_0 \cdot (1-\bar{\alpha})^{\frac{t \cdot c}{\bar{l}}} \tag{4}$$

where t is time, c is the speed of sound, $\bar{\alpha}$ is the average absorption coefficient and $\bar{l} = 4V/S$ is the mean free path between reflections. The purely specular part is then

$$E_{spec,t} = E_0 \cdot [(1-s)(1-\bar{\alpha})]^{\frac{t \cdot c}{\bar{l}}} \tag{5}$$

where s is the average scattering coefficient of the room.

We may assume, that once a part of the specular incident energy is scattered, it will stay scattered, and that the total energy is the sum of the purely specular and the non-specular (or scattered) energies at any moment:

$$E_{spec,t} + E_{scat,t} = E_{total,t} \tag{6}$$

The diffuse and specular parts are equal when

$$E_{spec,t} = E_{total,t}/2 \tag{7}$$

which yields

$$t_{SD} = \frac{4ln2}{c} \cdot \frac{V}{-S \cdot ln(1-s)} \tag{8}$$

where t_{SD} is the time, where purely specular and non-specular energies are in balance.

This expression is very similar to the Eyring formula:

$$T_{Eyring} = \frac{24ln10}{c} \frac{V}{-S \cdot ln(1-\bar{\alpha})} \tag{9}$$

so (8) and (9) can be combined:
The time limit of specular-to-diffuse balance is then

$$t_{SD} = \frac{T \cdot ln(1-\bar{\alpha})}{24ln10} \frac{4ln2}{ln(1-s)} \approx \frac{T}{20} \frac{ln(1-\bar{\alpha})}{ln(1-s)} \tag{10}$$

or

$$\frac{t_{SD}}{T} \approx \frac{1}{20} \frac{ln(1-\bar{\alpha})}{ln(1-s)} \tag{11}$$

Using (11) one may set a constraint to measure a stable reverberation time, meaning that from the -5 dB point on (approx. 32% energy left in the decay) the energy decay curve the non-specular part shall be dominant. This constraint means, that $t_{SD} < T/12$, or from (11):

$$\frac{t_{SD}}{T} < \frac{1}{12} \quad \rightarrow \quad \frac{1}{12} > \frac{1}{20} \frac{ln(1-\bar{\alpha})}{ln(1-s)} \quad \rightarrow \quad s_{-5dB} \geq 1-(1-\bar{\alpha})^{0.6} \tag{12a}$$

Similarly, if the requirement is to have more scattered energy from the -10 dB point on (approx. 10% energy left in the decay), the necessary average scattering coefficient is:

$$s_{-10dB} \geq 1-(1-\bar{\alpha})^{0.3} \tag{12b}$$

These results (see Figure 8) suggest, that if the diffuse ratio is at least 0.10, then formulas based on diffuse field theory shall be surely valid up to 0.15 mean absorption coefficient, and most probably valid up to 0.15 mean absorption coefficient.

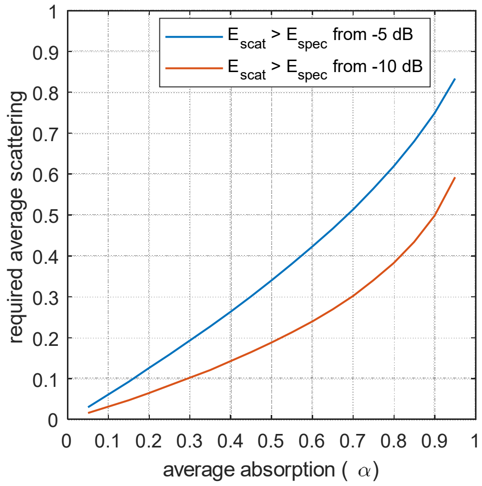

Figure 8 – Required minimum average scattering coefficient to ensure more scattered energy than specular energy from the -5 dB or -10 dB of the decay as the function of the average absorption coefficient.

6 Conclusions

Room acoustic specification is partially based on experience, but to make knowledge-based decisions, it is important to rely on simple guidelines too, that are consequences of physical, geometrical or other types of constraints.

The paper gave an overview of some of the aspects that are concluded from such constraints, mainly aiming to support sizing of rooms for clarity (classroom) or higher reverberation times (concert halls) or for a smoother low frequency response.

A simple assumption could also lead to explain, why more scattering is required along with higher absorption, if statistical formulas are expected to be valid.

Hopefully similarly practical guidelines can be constructed from simple constraints in order to make the room acoustic design an engineering art.

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