Scattering control by using correlated disorder

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Abstract
In this work, we show the possibility to control the scattering of acoustic waves by using 1D or 2D disordered distributions of rigid scatterers embedded in air. The scattering produced by a distribution of scatterers is related to its spatial Fourier transform, which is known as the structure factor in condensed matter physics. This structure factor only depends on the point distribution and can be used to design different types of materials with different scattering properties: stealth materials that suppress the back scattering over some set of frequency; hyperuniform materials that suppress the back scattering in the long wavelength limit and introduce isotropic scattering at higher frequencies; and equiluminous materials that produce an omnidirectional constant scattering over a set of frequencies. Several examples of these systems are shown and discussed in this work from the theoretical and experimental point of views.

Keywords: Hyperuniform materials, Equiluminous materials, Stealth materials, sound diffusion.

1 Introduction

The ability to manipulate waves has long been one of the main goals in various areas of physics and engineering. Many-body scattering systems [1] and metamaterials [2, 3] offer promising prospects to deal with this challenge due to their ability to be tuned and reconfigured. Properly designed highly disordered many-body systems have recently attracted attention as a tool for scattering manipulation. The introduction of local correlations between the positions of the scatterers constituting the disordered system allows to control the scattering of an incident radiation [4, 5, 6]. In particular, stealth materials consist of multiple scatterers distributed in such a way as to completely suppress the scattering of the sample over a broadband frequency range [7].

In this work, we develop a route to engineer 2D acoustic materials consisting of multiple rigid cylinders, which possess the desired scattering properties under the incidence of a plane wave [8]. We characterize the scattering pattern of a set of scatterers under the approach of weak scattering by its structure factor. We validate this hypothesis calculating the scattered far-field amplitude using the multiple scattering theory that considers all scattering orders. We develop an optimization technique, which optimizes the positions of scatterers that lead to a chosen value of the structure factor over a given frequency range.

2 Material designed based on the structure factor formalism

Let us consider a plane acoustic wave impinging N rigid cylinders of radius R_0 located at positions $\vec{r}_j$ ($j = 1, .., N$) inside a square $L \times L$ domain. The response of these N scatterers can be conveniently characterized by the structure factor $S(\vec{G})$, where $\vec{G}$ is a vector in the reciprocal space, which is proportional to the scattered intensity in the Born approximation

$$S(\vec{G}) = \frac{1}{N}\left|\sum_{j=1}^{N} e^{i\vec{G}\vec{r}_j}\right|^2 \propto I_{sc}. \tag{1}$$

Imposing the constraint $S(\vec{G}) = S_0$ when $|\vec{G}| \in [|\vec{K}_1|, |\vec{K}_2|]$, the following types of materials can be distinguished: hyperuniform ($S_0 = 0$, $|\vec{K}_1| = 0$), stealth ($S_0 = 0$, $|\vec{K}_1| \geq 0$) and equi-luminous ($S_0 > 0$, $|\vec{K}_1| \geq 0$). In order to obtain a distribution of scatterers possessing the required structure factor (and thus the required scattering pattern), we use a numerical optimization algorithm, which minimizes the potential

$$\phi(\vec{r}_1, \ldots \vec{r}_N) = \sum_{|\vec{G}| \in [|\vec{K}_1|, |\vec{K}_2|]} \left(S(\vec{G}) - S_0\right), \tag{2}$$

accounting for the constraint of non-overlapping scatterers $|\vec{r}_i - \vec{r}_j| \geq 2R_0 \quad \forall i \neq j$.

Figure 1(a) shows the spacial distribution of $N = 100$ cylinders with $R_0 = L/100$ constructing a hyperuniform material with $S(\vec{G}) = 0 \in [0, 0.5 \cdot 2\pi N/L]$. The structure factor is shown on Fig. 1(b). It is suppressed, $S(\vec{G}) = 0$ everywhere in the target region except the forward scattering (corresponding to $\vec{G} = (0, 0)$). The wave vector of the incident plane wave $\vec{k}_0$ and the scattered wave $\vec{k}_s$ are introduced following the von Laue formulation. According to this, the constructive interference takes place if $\vec{k}_s - \vec{k}_0 = \vec{G}$ (see Fig. 1(c)). For elastic scattering $|\vec{k}_0| = |\vec{k}_s|$ and the possible vectors $\vec{k}_s$ form an Ewald circumference centered at the origin of the vector $\vec{k}_0$. In Fig. 1(d) the polar plot of the normalized scattered intensity is shown. For the dashed circle in Fig. 1(c) which fully lies in the $S(\vec{G}) = 0$ region the scattering is completely suppressed, while for the solid circle there is a strong backward scattering is exhibited corresponding to the part of the circle lying in the $S(\vec{G}) \neq 0$ region.

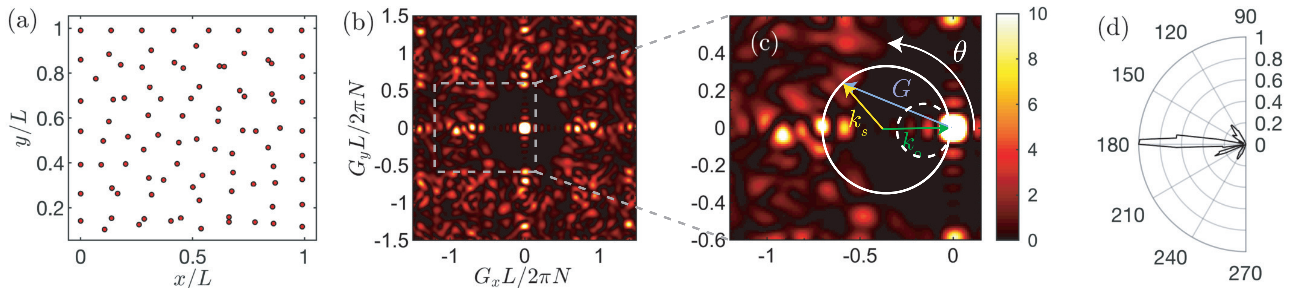

Figure 1: (a) spacial distribution of $N = 100$ scatterers with $R_0 = L/100$ composing a hyperuniform material, (b) corresponding structure factor, (c) zoomed region of the structure factor and the Ewald circumference, (d) normalized scattered intensity as a function of the scattering angle for the solid circle in (c).

3 Multiple scattering formalism

The relation between the scattered intensity and the structure factor, Eq. (1), holds only for the case of weak scattering (Born approximation). In order to validate the hypothesis of weak scattering approximation in our materials we consider the framework of the multiple scattering theory which considers the whole scattering orders [9, 10]. It is based on the idea that the field that impinges the i-th cylinder is constituted of both the incident plane wave and the scattered waves by all the other cylinders. In this approach the scattered intensity is defined by the scattered far-field amplitude, $I(\theta, \omega) \propto |P_s^f(\theta, \omega)|^2$, which is a function of the scattering angle θ and the frequency ω. In its turn, for a plane wave of the form e^{ikx} the far-field amplitude is expressed via the scatterers positions $(r_i, \theta_{\vec{r}_i})$ and the scattering coefficients A_n^i provided by the multiple scattering theory for the

i-th cylinder and scattering order n,

$$P_s^f(\theta,\omega) = \frac{2}{k}\sum_{i=1}^{N} e^{-\imath k|\vec{r}_i|\cos(\theta-\theta_{\vec{r}_i})} \sum_n (-i)^n A_n^i e^{\imath n\theta}. \tag{3}$$

4 Results

In this section we show the design of a stealth material presenting a scattering suppression area, i.e., $S(\vec{G}) = 0$, between $[|\vec{K}_1|, |\vec{K}_2|] = [0.8 \cdot 2\pi N/L, 1.2 \cdot 2\pi N/L]$. Figure 2(a) shows the distribution of scatterers for the designed stealth material with $N = 64$ scatterers obtained from the optimization method. The structure factor of the stealth material (Fig. 2(b)) exhibits an annular region of scattering suppression delimited by $|\vec{K}_1|$ and $|\vec{K}_2|$ shown by the circumferences in red continuous lines. For a given range of frequencies (a particular example is shown in Fig. 2(b)), the corresponding Ewald circumference (white continuous line) overlaps the scattering suppression area by its whitish area. This implies the total suppression of the back-scattering for all the scattering vectors $\vec{k}_s$ in the whitish area. Moreover, we can see that this scattering suppression mechanism is produced at any angle of incidence (an other Ewald circumference with dashed white lines is depicted in Fig. 2(b) for another angle of incidence). The polar plot of the scattered intensity for this Ewald circumference is shown in Fig. 2(c) and it clearly matches the expected behavior from the structure factor, showing the back scattering suppression. Figure 2(d) illustrates the spacial distribution of the scattered pressure $|P_s^f|$ for a plane wave incident from the left. It also confirms the suppression of the back scattering.

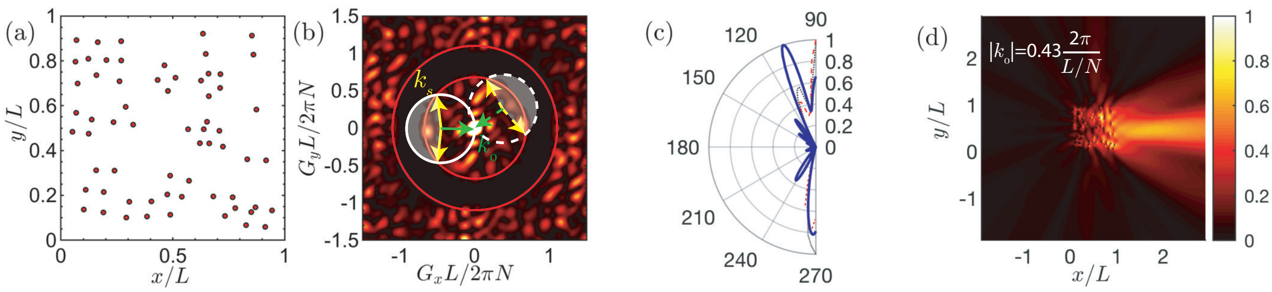

Figure 2: (a) Stealth material made of a distribution of $N = 64$ cylinders. (b) Structure factor of a stealth material. (c) Polar plot of the scattered intensity calculated from the structure factor (black line) and from multiple scattering (blue line). (d) Map of the scattered pressure, $|P_s^f(\theta,\omega)|$, for the wavevector with $|\vec{k}_s = 0.43\frac{2\pi N}{L}$ for the normal incidence.

5 Conclusions

With the methodology shown in this work we demonstrate that distributions of sub-wavelength particles are good candidates to achieve target scattering properties, in particular for stealth materials with broadband and omnidirectional back scattering suppression. We have described the scattering properties of such systems by the structure factor in the framework of a single scattering approach and by the far-field scattered amplitude within the multiple scattering theory, validating this approach. Numerically optimizing the structure factor of the sample we design the appropriate distributions of scatterers which, though counter intuitive, present the target scattering pattern.

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