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A NUMERICAL TECHNIQUE FOR OBTAINING SEA COUPLING LOSS FACTORS IN DOUBLE WALLS

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Abstract
Double walls usually consist of two leaves of material connected by steel studs. Aside from improving the structural performance, studs create a vibration transmission path which connects the two leaves. There is interest in reliable models of the acoustic performance of these structures, for the frequency range required in regulations. Statistical energy analysis allows reaching high frequencies with a low computational cost. However, the best SEA approach for modelling double walls is not clear in the literature. The cavity may be considered as a subsystem or treated as a connecting device between the two leaves. The effect of the cavity is also often neglected compared to the coupling provided by the studs. In this work, numerical techniques are used to evaluate these approaches and to define a combined deterministic–statistical approach that accounts for all the transmission phenomena.

Keywords: double walls, building acoustics, statistical energy analysis, computational vibroacoustics.

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1 Introduction

Modelling double walls with statistical energy analysis (SEA) is not a straightforward task. There is not an expression for the coupling loss factor between the wall leaves in the literature that accounts for all the relevant transmission phenomena [1,2]. Depending on the author, the cavity may be considered as a subsystem or treated as a connecting device between the two leaves. The effect of the cavity is also often neglected compared to the coupling provided by the studs. In this work numerical techniques are used to evaluate these approaches and to define a combined deterministic–statistical approach that accounts for all the transmission phenomena.

Double walls are a popular configuration of lightweight structures, consisting of two leaves of material (generally plasterboard) connected by steel studs. Aside from improving the structural performance, studs create a vibration transmission path which connects the two leaves. There is interest in reliable models of the acoustic performance of this type of structures for the frequency range required in regulations (50–5000 Hz).

Models of the sound transmission through double walls couple the structural vibration of the leaves and studs with the sound propagation through the cavity. To do so, there are different approaches. On the one hand, deterministic models can be used. They are based on solving the structural dynamics equation for the solid domain and the Helmholtz equation for the fluid domain. These equations are expressed in the frequency domain and can be solved either numerically [3,4] or analytically with the help of assumptions and simplifications [5,6]. On the other hand, energy-based formulations such as the statistical energy analysis [7] can be used.

Deterministic computations have a large computational cost when dealing with large domains as those used in building design at the higher frequencies required by regulations. SEA seems to be the best alternative for this kind of problems. However, some parameters required by this technique, such as the coupling loss factor (CLF), are not straightforward to obtain for certain configurations. Either experiments or simulations have to be performed for fitting their values.

Some authors [8, 9, 10] have used finite element methods (FEM) to obtain the energy fluxes in dynamic problems and then estimate SEA parameters from them. Others like Maxit and Guyader [11] estimate the coupling loss factors from modal parameters of the SEA subsystems. From a different point of view, Thite and Mace [12] deal with the idea of obtaining robust estimators of these parameters from the deterministic results.

In this work, a combined approach that allows solving vibroacoustic problems with a reasonable computational cost is proposed. It is based on estimating the SEA coupling loss factors from deterministic problems and applying them to solve large vibroacoustic problems with the statistical energy analysis.

2 Methodology

In this work, the double wall is approached as an SEA system consisting of two subsystems: the leaves. Anything located between them is treated as a connection and the CLF associated to it is computed numerically. The SEA formulation for a system consisting of two subsystems is

$$\begin{Bmatrix} P_1^{\text{in}} \\ P_2^{\text{in}} \end{Bmatrix} = \omega \begin{bmatrix} \eta_{11}+\eta_{12} & -\eta_{21} \\ -\eta_{12} & \eta_{21}+\eta_{22} \end{bmatrix} \begin{Bmatrix} \langle E_1 \rangle \\ \langle E_2 \rangle \end{Bmatrix}, \tag{1}$$

where η_{ii} and $\langle E_i \rangle$ are the internal loss factor and averaged energy of subsystem i respectively, P_i^{in} is the input power in subsystem i and η_{ij} is the coupling loss factor between subsystems i and j. This coupling loss factor satisfies the consistency relationship

$$\eta_{ij} n_i = \eta_{ji} n_j, \tag{2}$$

where n_i is the modal density (number of modes per Hz) of subsystem i.

The general procedure is to use SEA to compute the averaged energies of the subsystems. The input powers are usually known for a given excitation and, for most of the subsystems used in building acoustics, the internal loss factor can be computed with analytical expressions, available in the literature [2]. However, the analytical expression for the coupling loss factor is only available for simple connections.

In this work, the SEA formulation for a system consisting of two subsystems is used to estimate the coupling loss factor. The averaged energies of the subsystems are obtained from the numerical simulation of the same vibroacoustic problem and Eq. (1) is used to compute η_{12}. Since the energy values are frequency dependent, the CLF obtained will also depend on the frequency, and therefore the result of the computation will not be a single value but a CLF law in terms of the frequency.

For the double wall, the effect of the air cavity and the studs have been considered separately. On the one hand the CLF associated to the air cavity is obtained. To do so, the vibroacoustic problem of the sound transmission through a double wall without studs is solved numerically. On the other hand the CLF associated to the studs is computed from the results of the deterministic analysis performed by Poblet-Puig et al. in [13].

2.1 Estimation of the coupling loss factors

The CLF calculations are based on the SEA formulation for a system consisting of two subsystems (1) when the excitation is only applied to subsystem 1 ($P_2^{in} = 0$). If the internal loss factors and modal densities of the subsystems are known, the numerical computation of the averaged energies allows to isolate the coupling loss factor from the second equation of system (1), leading to the following expression

$$\eta_{12} = \frac{\eta_{22} \langle E_2 \rangle}{\langle E_1 \rangle - \frac{n_1}{n_2} \langle E_2 \rangle} . \tag{3}$$

This expression is preferred to the one that would be derived from the first equation in system (1) because of its error propagation properties [14].

2.2 Effect of the air cavity

The coupling loss factor associated to the air cavity is obtained from the energies of the leaves with Eq. (3). They are computed from the numerical solution of a vibroacoustic problem. It consists of a pressure wave impinging on a double wall without studs.

The wall leaves are modelled with the thin plate equation in the frequency domain

$$D \nabla^4 u(x,y) - \omega^2 \rho_s u(x,y) = q(x,y), \tag{4}$$

expressing the vibration field in the plates in terms of the eigenfunctions of simply supported plates, see Díaz-Cereceda et al. [15]. In Eq.(4) $D = E h^3 / 12(1-\nu^2)$ is the bending stiffness of the plate (where E, ν and h are Young's modulus, Poisson's ratio and the thickness of the plate), ρ_s is its mass per unit surface, $q(x,y)$ the applied load per unit surface, $\omega = 2\pi f$ (with f the frequency of vibration) and $u(x,y)$ the plate deflection.

The air cavity is modelled with the Helmholtz equation

$$\nabla^2 p(\mathbf{x}) + k^2 p(\mathbf{x}) = 0, \tag{5}$$

where $p(\mathbf{x})$ is the pressure field, $k = \omega / c$ is the wave number in the air and c is the sound speed also in the air. Modal analysis [16] has been used for solving the acoustic part of the problem. This technique exploits the simple geometry of the problem to achieve an accurate result with less computational cost than a finite element discretisation. However, for a more complex geometry the problem might be solved with other discretisation techniques, such as the finite layer method [16] or the finite element method [17].

The equilibrium and continuity between the cavity and the walls are enforced weakly and, once the displacement field $u(x,y)$ in a plate is known, its velocity is obtained as $v\,(x,y) = \mathrm{i}\omega u(x,y)$, where $\mathrm{i} = \sqrt{-1}$. Then, the averaged energy of the plate is computed as [1]

$$\langle E\rangle = M\langle v_{RMS}^2\rangle, \tag{6}$$

where M is the mass of the plate and $\langle v_{RMS}^2\rangle$ is the spatial mean square value of its velocity.

2.3 Effect of the studs

The effect of studs may be taken into account in different ways. One option would be to compute numerically the vibration transmitted between two leaves connected through the studs, when only one of them is excited. Then, the coupling loss factor would be computed from the energies of the leaves, in a similar approach to that of Section 2.2. In this case, the numerical technique chosen would be the finite strip method [18]. This technique allows the discretisation of the cross section of the double wall with finite elements, necessary because of the complicated shape of the studs, but allows the use of trigonometric functions along the extrusion direction, reducing the computational cost.

In this work, however, the coupling loss factor associated to the studs is computed differently. It is obtained with the analysis performed by Poblet-Puig et al. in [13]. They studied the effect of different types of metallic studs on the sound propagation through double walls. To do so, they analysed the mechanical properties of the studs within the leaf-stud-leaf system and concluded that they could be modelled as translational springs with a frequency-dependent value of the stiffness. They also computed the value of the stiffness law for different studs.

Given the equivalent stiffness of a stud, the associated coupling loss factor can be computed analytically, considering the studs as line springs and using the equivalent circuit theory [1]. Therefore, the CLF can be derived as

$$\eta_{12} = \frac{n_L \Re\{Y_2/L_y\}}{\omega M_1 |Y_1^L/L_y + Y_2^L/L_y + Y_s^L/L_y|^2} \tag{7}$$

where n_L is the number of studs. The line mobilities of the leaves, modelled as thin plates, are obtained as [19]

$$Y_i^L = \frac{1}{2(1+\mathrm{i})\rho_s c_B (f/f_c)^{1/2}}, \tag{8}$$

and the line mobility of the spring is $Y_s = \mathrm{i}\omega / K_L$.

3 Application examples

3.1 Effect of the cavity: comparison with analytical expressions.

The coupling loss factor between the two leaves of a double wall without studs is computed as described in Section 2, and compared with some analytical approaches available in the literature. In all the examples of this work, the absorption coefficient at the cavity contour is assumed to be null. This is reasonable because the cavity is not filled with absorbing material (this topic is addressed in [20]) and the smaller surface of the contour compared to the surfaces of the leaves leads to negligible losses caused by the material that covers the cavity contour.

The most common analytical approach is to consider the cavity as an SEA subsystem itself [2] (see Figure 1), and obtain its own modal density

$$n_i = \frac{4\pi f^2 V_{\text{cav}}}{c^3} + \frac{2\pi f S_{\text{cav}}}{4c^2} + \frac{L_{\text{cav}}}{8c} \tag{9}$$

and internal loss factor

$$\eta_{ii} = \frac{c\,\alpha\, S_{\text{cav}}}{8\pi f V_{\text{cav}}}, \tag{10}$$

where V_{cav} is the cavity volume, S_{cav} is the surface of the cavity boundary, L_{cav} is the sum of the length of all the cavity edges and α is the absorbing factor at the cavity boundary, which in this case is equal to zero.

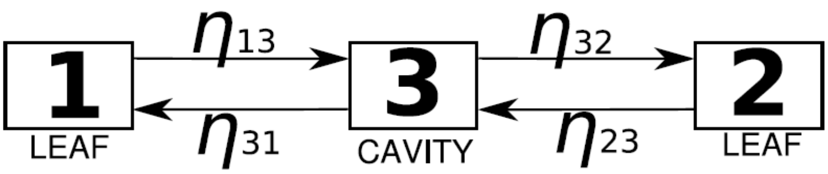

Figure 1: Sketch of an SEA model where the cavity is considered as a subsystem.

Then, the coupling loss factors between the cavity and the leaves are obtained as

$$\eta_{ij} = \frac{\rho_{\text{air}} c\, \sigma_{\text{rad}} f_c}{4\pi f^2 \rho_s}, \tag{11}$$

where f_c is the coincidence frequency between the leaf and the air and σ_{rad} is the radiation efficiency of the leaf. This efficiency is computed with the expressions defined in [21] with a small modification: the critical frequency treatment is applied for all the frequencies in a range of $f_c \pm 5$ Hz.

For comparing this approach with the others, an equivalent coupling loss factor between the leaves is obtained

$$\eta_{12}^{\text{equi}} = \frac{\eta_{13}}{2}. \tag{12}$$

This can be done because the absorption at the cavity has been assumed to be null. More details on this are provided in [14].

The other analytical approach considers the air cavity as a connection between the subsystems (leaves) as shown in Figure 2, in particular as a spring with stiffness

$$K_{\text{air}} = \rho_{\text{air}} c^2 S / H, \tag{13}$$

where H is the thickness of the cavity and S the surface of the leaves. The coupling loss factor η_{ij} between leaf i and leaf j is obtained with the electrical circuit analogy used by Hopkins [1]

$$\eta_{ij} = \frac{\Re\{Y_j\}}{\omega M_i |Y_i + Y_j + Y_s|^2} \tag{14}$$

where $Y_i = 1/8\sqrt{D_i\, \rho_{si}}$ and M_i are the point mobility and the mass of leaf i respectively and $Y_s = \mathrm{i}\omega / K$ is the mobility of the spring.

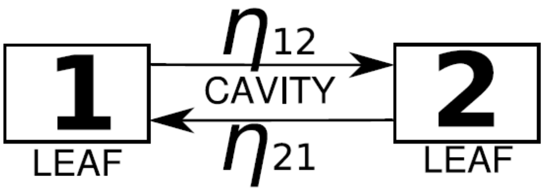

Figure 2: Sketch of an SEA model where the cavity is considered as a connection.

The basic properties of the double wall used for the comparison are summarised in Table 1. The thickness of the cavity is H = 70 mm.

Table 1: Properties of the leaves.

Variable	Symbol	Value
Leaf length in x	L_x	2.4 m
Leaf length in y	L_y	2.4 m
Thickness	h	0.013 m
Young's modulus	E	2.5×10^9 N m^{-2}
Density	ρ	692.3 kg m^{-3}
Poisson's ratio	ν	0.3
Loss factor	η	3%

The comparison between the numerical estimation of the coupling loss factor and the two analytical expressions is depicted in Figure 3. Leaving the low-frequency discrepancies aside, the estimated CLF law shows two main features: on the one hand, the importance of the equivalent stiffness of the air, specially at mid frequencies; on the other hand, the coincidence phenomenon that takes place at 2500 Hz. This phenomenon is only considered by SEA when the cavity is treated as a subsystem. In fact, SEA overestimates a little the transmission at that frequency. This may be due to the simplifications involved in the expressions used for computing the radiation efficiency.

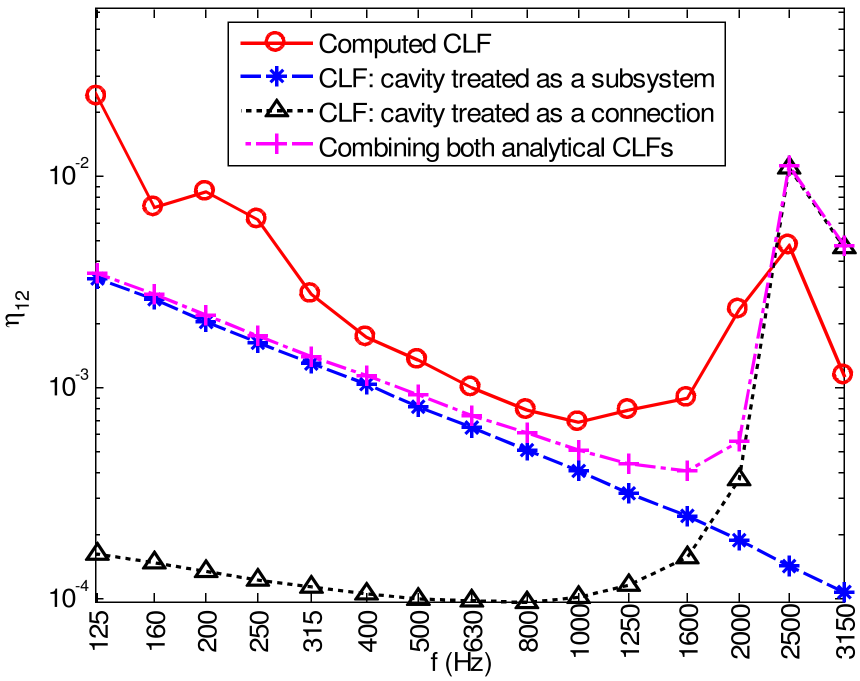

Figure 3: Comparison of the CLF estimations for a double wall with analytical expressions.

The two analytical expressions miss some physical information if used separately. The fourth curve of Figure 3 shows a more complete SEA model, which considers the cavity both as a connecting device and as a subsystem (see Figure 4). The SEA system with three subsystems is solved, adding the coupling loss factor between the two leaves defined in Eq. (14) and the equivalent coupling loss factors described in Eq. (12). The need of considering both behaviours together along the whole frequency range is evident.

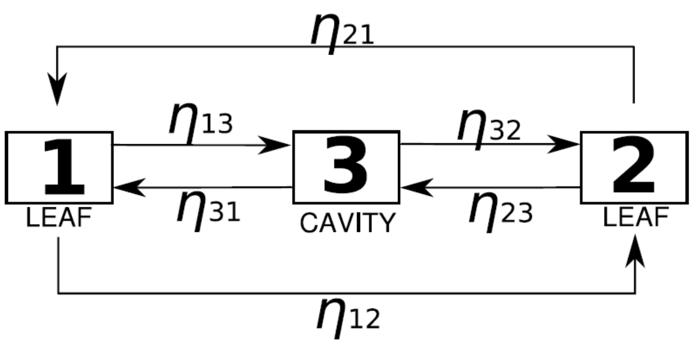

Figure 4: SEA sketch for the combination of the two techniques.

3.2 Sound reduction index of double walls

The sound reduction index between two rooms divided by a double wall is computed with SEA. The wall leaves have the same mechanical properties described in Table 1 and their dimensions are

$2\,\text{m} \times 3\,\text{m}$. The two rooms are identical, with dimensions $2\,\text{m} \times 3\,\text{m} \times 5\,\text{m}$, and the thickness of the air cavity is 70 mm.

The SEA system consists of four subsystems: sending room, leaf 1, leaf 2 and receiving room. The internal loss factors of the leaves are equal to their loss factors (Table 1), and the internal loss factors of the rooms are computed with Eq.(10), assuming an absorption of 10% in the room boundaries.

The coupling loss factor between the two leaves caused by the air cavity is computed with the technique described in Section 2. The CLF between each leaf and its adjacent room is computed with numerical simulations because the expressions provided in [21] for the radiation efficiency are only suitable for mechanical excitations, and underestimate the sound transmission when the excitation consists of a diffuse pressure field, as shown by Vigran on Figure 6.16 of [22], or by Villot on [23].

The leaf-room CLF is obtained with Eq. (3), computing the averaged energies from the numerical simulation of a system consisting of a room in contact with a leaf (see Figure 5). Since the first SEA subsystem is the sending room and the last one is the receiving room, the influence of applying the correct excitation when estimating the coupling loss factor between the room and the leaf is studied.

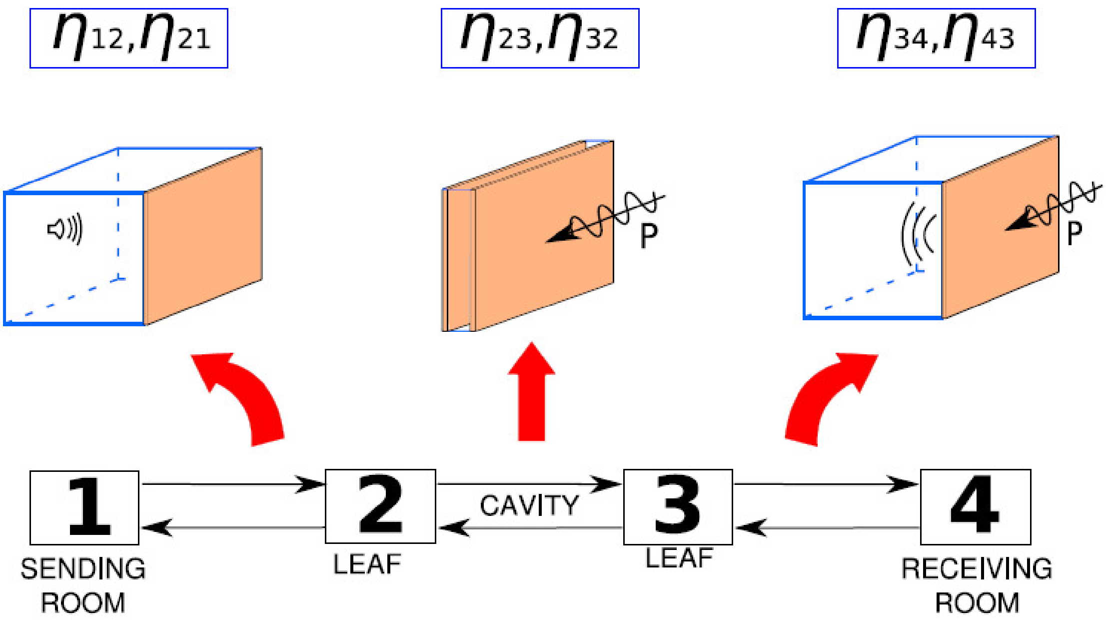

Figure 5: Sketch of the 2-subsystem vibroacoustic problems solved to obtain the coupling loss factors.

Two vibroacoustic problems have been solved, with the only difference of the excitation applied to the system. In **problem A**, the excitation is a sound source in the room (left part of Figure 5). In **problem B**, the excitation is a pressure wave impinging on the leaf (right part of Figure 5).

The most natural situation would be to estimate η_{12} and η_{21} from problem A and η_{34} and η_{43} from problem B. However, the influence of this choice is analysed here.

In Figure 6 the sound reduction index between the two rooms is shown from three different approaches. Two of them correspond to solving the problem with SEA. On the first one, η_{12} and η_{21} are estimated from problem A, and η_{34} and η_{43} from problem B. On the second one, the four coupling loss factors between the rooms and leaves are obtained from problem B. The third one is the result of a numerical calculation of the sound reduction index with a full deterministic approach, based on modelling the rooms with Helmholtz equation and the double wall as described in Section 2.2.

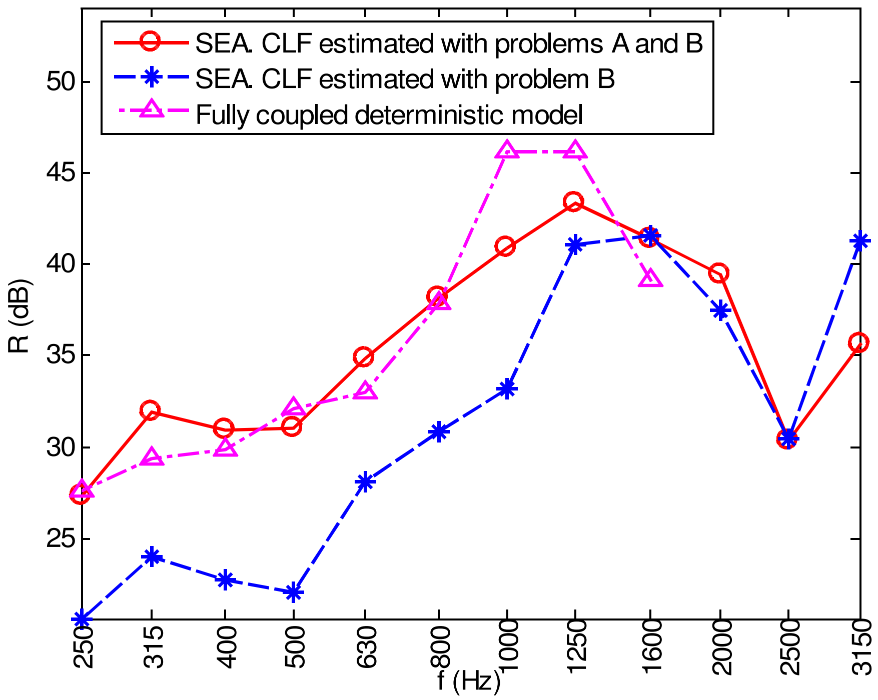

Figure 6: Sound reduction index of the double wall.

The lowest frequencies are not plotted because SEA hypotheses are not fulfilled there and the results are not reliable. The deterministic calculation is not shown for frequencies higher than 1600 Hz because the computational cost is unaffordable at these frequencies.

The trend of the results computed only with the CLFs estimated from problem B is much different than the other two. This illustrates the importance of estimating the coupling loss factors from problems with the same excitation at which the subsystems will be subjected afterwards.

3.3 Comparison of the effect of the studs and the cavity

The acoustic effect of steel studs is added here to the SEA model of Section 3.2. To do so, the coupling loss factor associated to the stud connections is computed with Eq. (7) and added to that of the air cavity, in order to reproduce the global behaviour of the double wall.

Two types of studs are considered: on the one hand a conventional S-section stud and on the other hand an acoustic stud (LR), with the shapes shown in Figure 7. The dimensions of the studs are given by *d1*=70 mm, *d2* = 40 mm, *d3* = 10 mm, *d4* = 14 mm, *d5* = 14 mm and *d6* = 28 mm.

In Figure 8 the sound reduction index between the two rooms is shown, both for the simple leaf-cavity-leaf system and for the same wall with four studs inside. The coupling loss factors between the leaves and their adjacent rooms are computed with the corresponding excitation (problems A and B respectively).

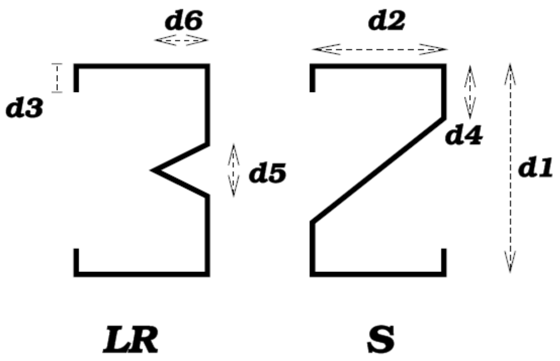

Figure 7: Section of the studs.

For the highest frequencies, the increment in the sound transmission due to the extra path added by the studs is negligible. This happens because the transmission through the air cavity for these frequencies is already large compared to the one added by the studs. For the mid-frequency range, however, the presence of the studs reduces the sound insulation of the double wall.

The improvement of the performance with the acoustic studs in front of the conventional ones happens, as expected, for the mid and high frequencies. As explained in [13], the increment of the flexibility due to the stud shape is more relevant around the eigenfrequencies where the central part of the stud acts as a spring. These eigenfrequencies do not happen in the low-frequency range.

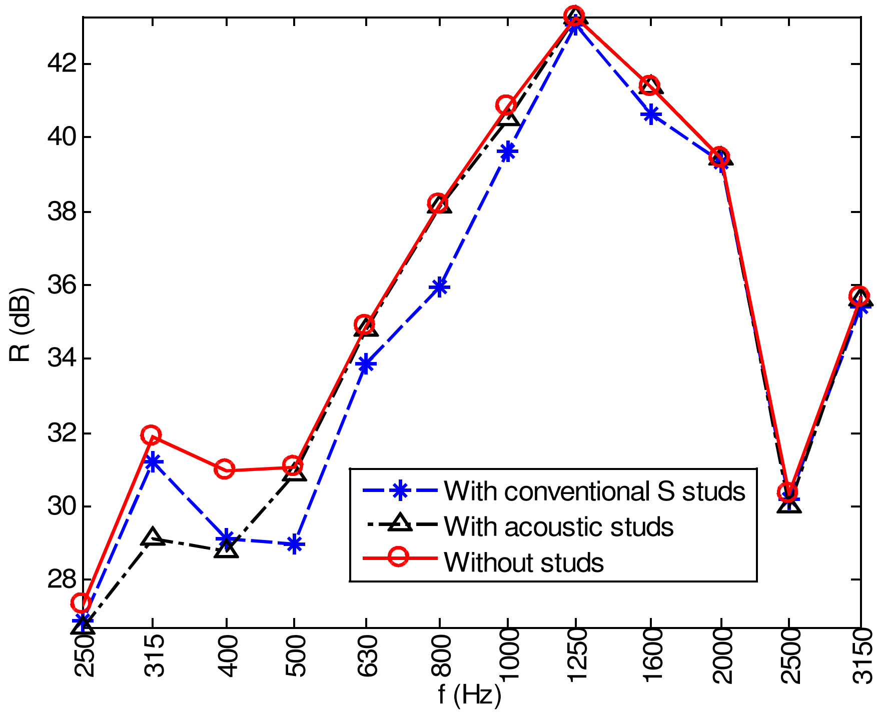

Figure 8: Effect of the studs in the sound reduction index through the double wall.

4 Conclusions

- The combination of numerical and statistical methods is useful to solve realistic vibroacoustic problems. It allows reaching the whole frequency range required by regulations with a reasonable computational cost for large domains based on the repetitions of smaller elements.
- Standard techniques used in SEA for estimating the coupling loss factor associated to double walls can not reproduce all the transmission phenomena. A numerical estimation of the coupling loss factor in these situations allows to take them into account and to detect that the addition of the two analytical expressions provides an acceptable approximation of the real behaviour.
- The vibroacoustic problem solved for the coupling loss factor estimation has to be performed with the same type of excitation as the one that will be applied in the SEA simulation where the CLF is going to be used.
- The effect of studs can be easily considered with SEA if they are treated as line springs with frequency-dependent stiffness. The coupling loss factor associated to the studs does not always involve a significant increment of the sound transmission. The effect depends on the frequency, the type of stud and the cavity properties. The use of acoustic studs improves the insulation of the double wall in front of the conventional ones at mid and high frequencies.

Acknowledgements

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