EMPTY LATTICE MODEL FOR PHONONIC CRYSTALS WITH ELASTIC ANSIOTROPY

Yan-Feng Wang, Vincent Laude

Institut FEMTO-ST, CNRS UMR 6174, Université de Franche-Comté, Besançon, F-25030, France
{fhyd212@163.com, vincent.laude@femto-st.fr}

Abstract
For phononic crystals, the Bragg band gaps generally, but not always, open around high symmetry points of the first Brillouin zone. The commonly accepted explanation stems from the empty lattice model: assuming a small material contrast between the constituents of the unit cell, avoided crossings in the phononic band structure appear at frequencies and wavenumbers corresponding to band intersections; for scalar waves the lowest intersections coincide with boundaries of the first Brillouin zone. In case the phononic crystal contains ansiotropic solid materials, however, its overall symmetry is not dictated solely by the lattice symmetry. We construct an empty lattice model for phononic crystals composed of elastic anisotropic materials, relying on their slowness surfaces. Even in the case of isotropic constituent materials, avoided crossings can appear at intersections between bands for elastic waves of different polarizations, i.e. shear and longitudinal, because these are coupled by periodicity in the phononic crystal. For bands with similar polarization, avoided crossings can appear at reciprocal lattice points that do not sit at boundaries of the first Brillouin zone.

Keywords: phononic crystal, anisotropy, slowness curve, empty lattice model, avoided crossing.

PACS no. 62.20.D-, 63.20.D-

1 Introduction

Phononic crystals are periodic functional composites made of two or more materials with different mass densities and elastic constants [1]. They can give rise to complete band gaps, within which the propagation of sound, acoustic waves, or elastic waves is prohibited. Moreover, phononic crystals also have intriguing properties in the passing bands, leading to their use as building blocks for acoustic metamaterials. Phononic crystals have potential applications in many fields, and the related studies have attracted a rapidly growing interest (see, e.g., the review by Hussein et al. [2]).

Due to the periodicity of the phononic crystal, the basis of Bloch waves, rather than plane waves, is well suited for the description of wave propagation [1]. The eigenstates of Bloch waves are governed by periodicity and can be labeled with respect to a Bloch wave vector expressed in the basis of reciprocal lattice vectors [3]. When the band structure (the relation between frequency and wave vector, also termed dispersion relation) is investigated, it is convenient to restrict the Bloch wave vector to a primitive unit cell of the reciprocal lattice, with the universal choice being the first Brillouin zone (BZ). Following Brillouin [4], this choice eliminates the discontinuities of the dispersion relation except at the zone boundaries. For a given direct Bravais lattice, the first BZ is canonically defined as the Wigner-Seitz cell of the reciprocal Bravais lattice. As such its symmetries are those of the symmetry group associated with the reciprocal Bravais lattice. When the periodic perturbation in the phononic crystal has a lower symmetry than the direct lattice, however, the first BZ "would correspond to the actual periodicity and symmetry of the perturbation, but not the direct lattice itself" [4]. Furthermore, even for a given first BZ, the irreducible BZ depends on the symmetry of the components [5] or of the unit cell [6]. If care is not taken to plot the band structure along a relevant path of the first BZ, erroneous conclusions regarding complete band gaps may be drawn.

For scalar wave propagation in anisotropic photonic crystals, variations of the BZ with material anisotropy were studied recently [5,7,8]. Sivarajah et al. in particular investigated band intersections and avoided crossings in the dispersion curves of an anisotropic photonic crystal [8]. They found that both band intersections and avoided crossings appear at high symmetry points of the Bragg plane BZ, which may be different from the first BZ with the introduction of refractive index anisotropy. In the case of vector (elastic)waves propagating in phononic crystalswith anisotropic solid components, the influence of the elastic anisotropy on band gaps has been the main subject of investigation [1,9,10]. Little attention has however been paid to avoided crossings appearing between modes with different polarizations [11], even in the case of isotropic phononic crystals [12,13]. Furthermore, the physical origin of the positions of those avoided crossing has not been clarified.

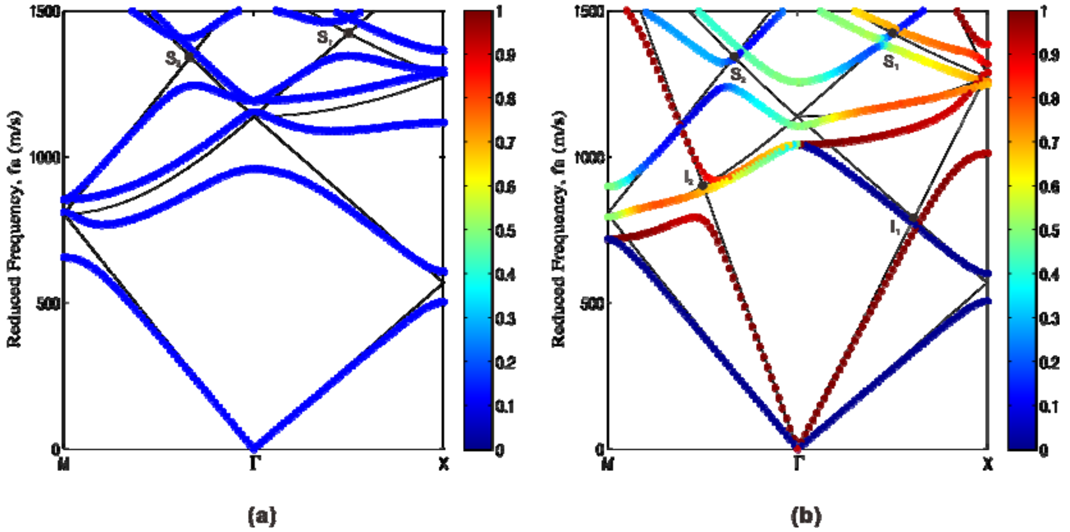

Fig.1 Band structures for (a) the out-of-plane waves and (b) the in-plane waves of epoxy with contrast ($\rho_B/\rho_A = 3$, d/a = 0.3). The color scale represents the polarization amount in the propagation direction, from 0 (shear) to 1 (longitudinal). Solid lines are for the empty lattice model.

2 Results

In this paper, we construct an empty lattice model for vector elastic waves propagating in phononic crystals with anisotropic solid components. The empty lattice model predicts the positions of band intersections in reciprocal space. Its significance appears when the periodic modulation in the phononic crystal does not vanish anymore, since intersections can become avoided crossings and band gaps open as a result of the existence of avoided crossings. For scalar and isotropic waves, the first intersections are lying exactly on the boundaries of the first BZ and subsequent intersections also occur at high symmetry points of higher order BZ. For vector waves with different polarizations, intersections can define curved closed contours different from the first BZ boundary. They occur inside the first BZ. For vector and anisotropic waves with the same polarization, intersections also in general form curved closed contours different fromthe first BZ boundary, unless the symmetry of the composing solid is the same as the symmetry of the Bravais lattice.

Here, we study avoided crossings in the phononic band structure by considering a small periodic perturbation in the two-dimensional square-lattice phononic crystal. The ratio of inclusion (B) diameter to lattice constant is d/a = 0.3. Material of the matrix (A) is either epoxy or rutile. Material parameters (mass density ρ and elastic constants c) of the inclusion B are assumed proportional to those of the matrix, that is

$$\frac{\rho_B}{\rho_A} = \frac{c_B}{c_A} \; . \tag{1}$$

This constriction guarantees that bulk velocities are the same in the two materials so that avoided crossings occur around the intersections predicted by the empty lattice model for the matrix. The material constant contrast ratio defined by Equation (1) equals 3 in all subsequent examples.

Band structures for square-lattice phononic crystalswith isotropic (epoxy) and anisotropic (Z-cut rutile, X-15° orientation) solid components are shown in Figures 1 and 2, respectively. For pure shear waves, it can be checked that avoided crossings occur at high symmetry points X and M. In addition, in Figure 1a, an avoided crossings also appear around the S_1 and S_2 points. A directional band gap could even be generated were a larger contrast considered, as we checked numerically. This phenomenon can also be observed in Figure 2a when a larger frequency range is presented. For scalar waves propagating in a one-dimensional phononic crystal, such phenomenon can not be found [14].

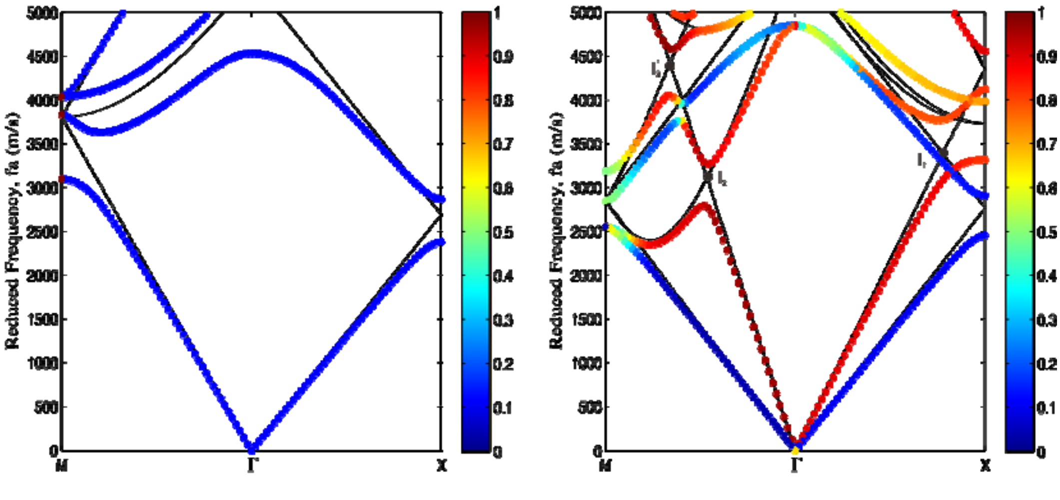

Fig.2 Band structures for (a) the out-of-plane waves and (b) the in-plane waves of Z-cut rutile for the X-15°orientation with contrast ($\rho_B/\rho_A = 3$, d/a = 0.3). The color scale represents the polarization amount in the propagation direction, from 0 (shear) to 1 (longitudinal). Solid lines are for the empty lattice model.

For in-plane waves, in Figures 1b and 2b, avoided crossings occur not only at the high symmetry points, but also inside the first BZ, for instance at the first intersections I_2 and I'_2 of the two waves with different polarizations in the Γ-M direction. All avoided crossings appear at intersections of the empty lattice band structure. Similar results were found in two-dimensional phononic crystals with isotropic solid components [13] or holes in an anisotropic matrix [11], where avoided crossing is generally accompanied by a transfer of the polarization fromone band to the other. It is also noted that the intersection point I_2 can be tuned by changing the orientation of the anisotropic solid. A relatively large band gap could thus be expected when the frequencies of points I_2 and M are the same. It can be remarked that when considering oblique incidence in a one-dimensional phononic crystal, coupling of the longitudinal and slow transverse waves can also result in a band gap occurring inside the BZ in addition to the high symmetry points [12].

3 Discussion

For general phononic crystals composed of several different solids, the ratios of mass densities and elastic constants can hardly be made equal. Then Equation (1) is no longer valid and the empty lattice model should be modified.One possible direction is to consider the effective properties (effective mass densities and effective elastic constants) of the phononic crystal considered as an acoustic metamaterial. Effective properties have been widely investigated in recent years [15–17]. The band structure of the periodic metamaterial could then be reproduced partly by using effective parameters, although maybe only in the low frequency range where the long wavelength approximation is valid. This would then make it possible to construct an effective empty lattice model with effective mass densities and elastic constants.

Acknowledgements

The authors would like to thank Anurupa Shaw for helpful discussions. Financial support by the Labex ACTION program (Contract No. ANR-11-LABX-0001-01) is gratefully acknowledged.

References

[1] Laude, V. *Phononic Crystals: Artificial Crystals for Sonic, Acoustic, and Elastic Waves*. Walter de Gruyter: Berlin, Germany, 2015.

[2] Hussein, M. I.; Leamy, M. J.; Ruzzene, M. Dynamics of phononic materials and structures: Historical origins, recent progress, and future outlook. *Appl. Mech. Rev.* **2014**, 66, 040802.

[3] Ashcroft, J.; Mermin, N. *Solid State Physics*. Harcourt Inc.: Florida, USA, 1976.

[4] Brillouin, L. *Wave Propagation in Periodic Structures: Electric Filters and Crystal Lattices*. Dover Publications: New York, USA, 1946.

[5] Khromova, I.A.; Melnikov, L.A. Anisotropic photonic crystals: Generalized plane wave method and dispersion symmetry properties. *Opt. Commun.* 281,2008, 5458–5466.

[6] Wang, Y.F.;Wang, Y.S.; Zhang, C. Bandgaps and directional propagation of elasticwaves in 2D square zigzag lattice structures. *J. Phys. D: Appl. Phys.* 47, 2014, 485102.

[7] Alagappan, G.; Sun, X.; Shum, P.; Yu, M.; den Engelsen, D. Symmetry properties of two-dimensional anisotropic photonic crystals. *J. Opt. Soc. Am. A* 23, 2006, 2002–2013.

[8] Sivarajah, P.; Maznev, A.; Ofori-Okai, B.; Nelson, K. What is the Brillouin zone of an anisotropic photonic crystal? *Phys. Rev. B* 93, 2016, 054204.

[9] Wang, Y.Z.; Li, F.M.; Huang, W.H.; Jiang, X.; Wang, Y.S.; Kishimoto, K. Wave band gaps in two-dimensional piezoelectric/piezomagnetic phononic crystals. *Int. J. Solids Struct.* 45, 2008, 4203–4210.

[10] Lin, S.C.S.; Huang, T.J. Tunable phononic crystals with anisotropic inclusions. *Phys. Rev. B* 83, 2011, 174303.

[11] Achaoui, Y.; Khelif, A.; Benchabane, S.; Laude, V. Polarization state and level repulsion in two-dimensional phononic crystals and waveguides in the presence of material anisotropy. *J. Phys. D: Appl. Phys.* 43, 2010, 185401.

[12] Tamura, S.; Hurley, D.; Wolfe, J. Acoustic-phonon propagation in superlattices. *Phys. Rev. B* 38, 1988, 1427–1449.

[13] Wu, T.T.; Huang, Z.G. Level repulsions of bulk acoustic waves in composite materials. *Phys. Rev. B* 70, 2004, 214304.

[14] Wang, G.; Yu, D.; Wen, J.; Liu, Y.; Wen, X. One-dimensional phononic crystals with locally resonant structures. *Phys. Lett. A* 327, 2004, 512–521.

[15] Lai, Y.;Wu, Y.; Sheng, P.; Zhang, Z.Q. Hybrid elastic solids. *Nat. Mater.* 10, 2011, 620–624.

[16] Zhu, R.; Liu, X.; Hu, G.; Sun, C.; Huang, G. Negative refraction of elastic waves at the deep-subwavelength scale in a single-phase metamaterial. *Nat. Commun*. 5, 2014, 5510.

[17] Wang, Y.F.; Wang, Y.S.; Zhang, C. Two-dimensional locally resonant elastic metamaterials with chiral comb-like interlayers: Bandgap and simultaneously double negative properties. *J. Acoust. Soc. Am.* 2016. (accepted)