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TUNING A PORTUGUESE GUITAR’S FINITE ELEMENT MODEL TO A GIVEN SET OF EXPERIMENTAL DATA

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Abstract

It is proposed an approach to the tuning the twelve strings of a Portuguese Guitar finite element model. This is motivated by the interest in predicting, in the design phase, what are the geometric characteristics including dimensions that allows to offer a certain set of modal characteristics, in this case focused on the first three frequencies associated with modes (0,0), (0,1) and (0,2) on the guitar's top. In addition to the top, it includes a part of the arm and twelve-string with the tuning methodology for tones B, A, E, B, A, D for the Portuguese guitar of Lisbon. As far as the author knowledge, there is not, in the current literature, a study with these characteristics, that is, a study linking the geometric modeling in Computer Aided Design (CAD) with modal analysis and pretension on the strings. The six pairs of strings are tuned to two decimal places, and the frequencies of the modes (0,0), (0,1) and (0,2) are fitted to the literature values. Based on the experimental work, carried out by other authors, this work has the objective to reproduce the behavior of tested guitars, through models created in CAD and tested by the Finite Element Method (FEM).

Keywords: Portuguese guitar, CAD-FEM, String, Prestressed Modal Analysis, Tones.

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1 Introduction

The main challenge of this work is the development of prediction methodologies in CAD-FEM for the structural and acoustic response of a Portuguese guitar, through the analysis of the un-dampened modal vibrational response, including the arm and the prestressed twelve strings.

Essentially, the objective is to set a way to predict at the project stage the geometrical (and dimensions) characteristics need to achieve specific modal characteristics using a CAD-FEM model. If accomplished, it may furthering the contemporary knowledge of these instruments, so that the results produced may be used by the industries that build these instruments.

Guitar-makers like Ervin Somogyi [1], whose guitars, produced in California, can reach 31000 dollars, and Gerald Sheppard [2], compare hand-made guitars to industrially produced guitars in their articles, where they stress the quality and uniqueness of hand built guitars, while recognizing the quality of some industrially produced guitars as well as the unique characteristics of some of these guitars.

With the introduction of new materials and the further investigation of previously existing ones, it should be possible, in our opinion, to produce high quality (or very near) industriously produced guitars. In [3], the authors mention in Spanish that “El sonido en una guitarra de concierto será limpio en todas as cordas y en todos los trastes”, i.e. the sound of a concert guitar will be clean in every string and frets. They add also that the goal of state-of-the-art technology nowadays is to replace the subjective quality assessments usually associated with the concert guitar, with simulations and experimental science, in order to increase quality and lower costs. In part, it should be possible nowadays.

From it follows that the introduction of models with the ability to produce an effective simulation, within the possibilities of computer simulation using the FEM and Experimental Modal Analysis (EMA), for instance, should be helpful for the industry of music instruments. It would also be an excellent contribution to applied science, which could also learn much from the Guitar-makers.

In this work, a modal characterization of the structure without strings and with prestressed strings is reviewed, and the mode tuning is explored in the CAD-FEM environment. As far as the authors knowledge, there is no publication available on FEM in this field of the Portuguese guitar that includes the strings, the bridge and guitar modes. Previous work from one of the authors [4], was on the strings and guitar resonance frequencies, but the concern on getting simultaneously the correct resonance frequencies with the corresponding correct modes is only achieved in this manuscript. Besides the progress to a detail not seen in previous works, the authors understand that more difficult challenges still open. Examples of it are the simplifications that need to be addressed for a better model as e.g.: 1) the modelling of the chords in order to have simultaneously the correct chords frequencies and chords pretensions transmitted to the guitar, and 2) the modelling details of the bridge connection to the chords. The usefulness of the fluid-structure interaction (FSI) is also an open question within this field, e.g. to confirm which frequencies are not significant for the sound radiation.

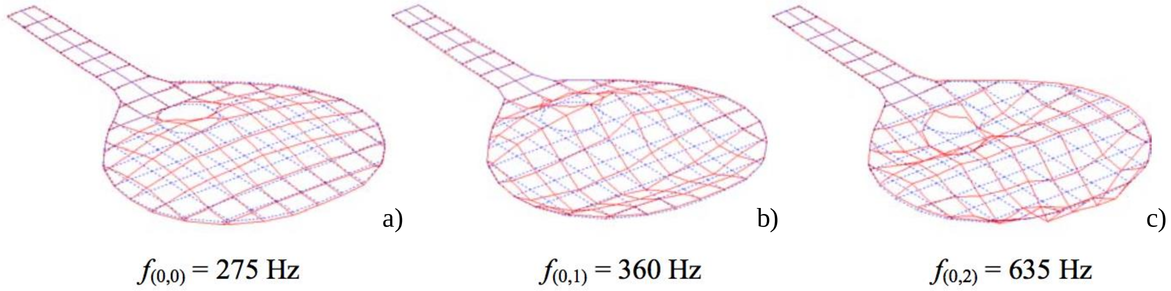
2 Brief revision of the literature

One of the first known studies of the modal response of a Portuguese guitar was made by Inácio et al. [5]. These authors obtained the modal characterization of the guitar soundboard from fully mounted Lisbon and Coimbra guitars. They compared and described the frequency response curves for several specimens, as well as some significant vibrational modes.

In the experimental setup, the guitar was suspended by rubber bands on a rigid structure. The authors used EMA to make a detailed modal identification in one of the guitars. The study mention a grid of 114 available impact points, in the soundboard as well as the arm, so that they could identify possible coupled movements.

These authors presented in [5] the following conclusions for the Coimbra guitars: 1) frequencies bellow 200 Hz are not significant for the sound radiation, 2) frequencies between 121 Hz and 160 Hz are due to cavity resonances (Helmholtz), and 3) between 250 and 450 Hz, there is, at least one resonance or resonance group that is responsible for a significant part of the radiated spectrum (see figure 1). The most important monopole mode is the (0,0) shown in Fig. 1a), which is the mode that radiates the sound more efficiently, as opposed to the longitudinal dipole (0,1) shown in Fig. 1b), where the adjacent antinodes move in anti-phase, “eliminating” the air movement inside the box. The tripole longitudinal mode (0,2) shown in Fig. 1c) shows up at 635 Hz. Others appear above these, but are not mentioned here.

In [6], the author present her study on modelling the Portuguese guitar using FEM, for the first time and as far as our knowledge. The modal analysis of the guitar produces results that were compared with the experimental results in [5]. The dimensions of the guitar and of the harmonic braces were supplied beforehand. From these dimensions the author performed the modal analysis and obtained the results, which are the first 20 frequencies and modes of vibration of the guitar, amongst which she compared the frequencies closest to the ones obtained by [5].



$f_{(0,0)} = 275$ Hz $f_{(0,1)} = 360$ Hz $f_{(0,2)} = 635$ Hz

Figure 1 – Modal forms of three resonances from one of the guitars in study (source: [5]): a) $f_{(0,0)} = 275\ Hz$; b) $f_{(0,1)} = 360\ Hz$; c) $f_{(0,2)} = 6355\ Hz$.

In [4], the authors use a Portuguese guitar model FE based on the reference geometry [6] to study the influence of adding the guitar strings and its respective modal response. The twelve steel strings are stretched in a way that their natural resonances correspond to the frequencies of the Portuguese guitar notes. For that model, these authors used pre-tensions related to temperature variations in the strings and by that way obtained frequencies corresponding to the right tones of the Portuguese guitar.

Next, it is presented the proposed methodology to perform a modal characterization of the structure without strings and with prestressed strings, and the mode tuning considering the presence of strings. Here, the concern is on getting simultaneously the correct resonance frequencies with the corresponding correct modes.

2.1 Methodology

The strategy adopted in this study uses a CAD program to model the guitar’s geometry according to the dimensions, which were taken from [7], from an Álvaro Merceano da Silveira’s Portuguese guitar, including (see Fig.s 2 and 3) harmonic braces, gluing belt (linings) and the tail block, that constitute the grid, arm with thickness variation, hand (support for fan), and the volute.

The simplifications assumed in this study are essentially the followings: neither the fan (string holding mechanism), nor the part of strings beyond nut are considered in the adopted geometry, as these elements were ignored to the modal behavior of the guitar and at this step the box is modeled with shell elements. Due to the difficulties in finding the right properties of some woods, the arm is modeled with mahogany instead of Brazilian cedar, and the parts that should be modeled with spruce are modeled with Sitka-Spruce. The strings are all considered bulk (when actually some are spiral shaped) and modeled by beam elements. Instead of a continuum string, two lengths are considered in the model, one from nut (opposite side of de arm) to bridge and another from the bridge to the nut (near head of guitar).

In the Portuguese guitar, the soundboard, the back and the sides are glued through a gluing belt that acts as reinforcement and setting the braces. In the model, a gluing grid is used which unites the braces (brace 1 at wright; brace 2 at middle and brace 3 at left) and the tail block (Fig. 3).

Afterwards, the geometry is exported from CAD to the finite elements program (FE) and the modal analysis is performed with the goal of obtaining the natural frequencies and the vibrational modes of the guitar. The obtained frequencies and modal forms for the three first modes, namely modes (0,0), (0,1), (0,2) and respective natural frequencies are compared with the frequencies and the three first vibration modes of the guitar tested experimentally by [5], see Fig. 1. This process is reproduced interactively, varying, in each iteration, at least one of the following dimensions: gluing belt, harmonic braces of the back or harmonic braces of the soundboard or all at once (three pairs of braces) or the distances between them and the distance to the exact point (position of the bridge) while checking the effect on the modal forms and frequencies. When finally the deviations relatively to the experimental values of [5] are considered acceptable, as far as this work is concerned, the bridge is positioned (protuberance of 17 mm of height by 3 mm of length, in mahogany) on the soundboard, followed by the installation of just one string, and finally the installation of all strings.

Figure 2 – a) Image of the Álvaro Merceano da Silveira's guitar (source: [7]); b) image of the CAD model developed; c) front view of the model; and d) side view of the model.

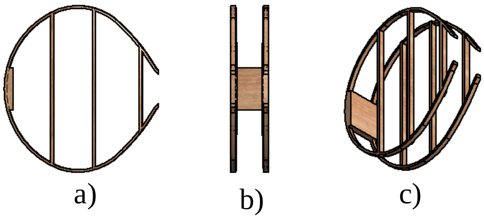


Figure 3 – Gluing grid with the braces: a) Front view, b) side view c) perspective.

In the case of the strings the method used is based on the one proposed in [4]: insertion of a pre-tension in the strings by introducing a reference temperature and the variation of the temperature of each string, depending on the desired frequency, followed by a static analysis with pre-tensions and the respective modal analysis of the complete model i.e. with strings. Thereby it is possible to create a pretension in each string, so that its natural vibration frequency in the model's complete structure corresponds to the string's natural vibration frequency in the tuned guitar.

An iterative process is also used in the case of the addition of the strings, where the results obtained with the model are compared with the experimental results of [5]. The purpose is to check the potentialities of the model adjusting. The addition of the bridge and the strings produces a downward dislocation of the frequencies and change the vibration modes. This iterative process thus determines the final guitar geometry/dimensions and was one of the focus of this work.

Special attention was given to work [6] performed without strings, which is the source of the material data, see Tab. 1, with the exception of the material used in the strings. In this work, the author begins with pre-determined dimensions that keeps constant, and performs the modal analysis, comparing the frequencies obtained with the frequencies in [5], not considering whether these modal frequencies correspond, except in the case of the first one. In this case it becomes relatively easy to find all the coincident frequencies with some modal form.

Table 1- Characteristics of the materials (source [6])

Wood	**Density (Kg/m^3)**	**E_1 (10^8 N/m^2)**	**E_2 (10^8 N/m^2)**	**E_3 (10^8 N/m^2)**	**G_{23} (10^8 N/m^2)**	**G_{13} (10^8 N/m^2)**	**G_{12} (10^8 N/m^2)**	**v_{12}**	**v_{13}**	**v_{23}**
Sitka-Spruce	390	116	9	5	0.39	7.2	7.5	0.37	0.47	0.25
Indian Rosewood	775	160	22	7.2	3	8.4	11	0.36	0.03	0.26
Ebony	1100	190	21.1	9.5	4	11.2	16.7	0.3	0.03	0.26
Mahogany	450	106.7	5.3	11.8	6.3	2.2	9.4	0.3	0.03	0.26

In [4], the author placed the strings in the model created and analysed by [6], with the previously defined dimensions and the same vibration modes and frequencies found by [6], which he kept fixed, having afterwards adjusted the strings to the nearest hundredth. It may be observed that in this case the two last modes do not correspond to the modes obtained by [5] as intended. Also, nothing is mentioned regarding the effect of the string placement on the guitar body.

The material here used to model the strings is a stainless steel AISI INOX 302 already employed by [4], whose mechanical properties are presented in Tab. 2.

Table 2 – Stainless steel (AISI INOX 302) properties [4].

Density [Kg/m^3]	7900
Young's modulus [GPa]	193
Thermal Expansion [1/oC]	0.0000172

3 Modal Analysis

According to the theory of linearized elastodynamics, the dynamic response of a linear elastic solid, in small deformations, is [8] obtained from the stress-equations of motion (Cauchy's first law of motion):

$$\sigma_{ij,j}+f_i=\rho_s\ddot{u}_i \tag{1}$$

where σ_{ij} is the stress tensor, f_i is volume force acting on the respective point of the solid, ρ_s is the density of the solid, u_i is the displacement vector, and $i, j = x, y, z$.

The weak formulation of the problem (1) can be obtained by the residual method followed by the integration by parts and the divergence theorem. The problem discretization is obtained by a choice of the finite element type and their shape functions, in this case based on the Galerkin method. In this way, the approximate solution by finite methods in terms of nodal displacements can be written as [8]:

$$\boldsymbol{M}\ddot{\boldsymbol{u}}+\boldsymbol{K}\boldsymbol{u}=\boldsymbol{f} \tag{2}$$

where $\boldsymbol{u}$ is the nodal displacement vector, **M** is the global mass matrix, **K** is the global stiffness matrix and $\boldsymbol{f}$ is the nodal force vector. These matrices and force vector can be obtained by FE assembling of the following element matrices:

$$\boldsymbol{m}_e=\int\rho\boldsymbol{N}^T\boldsymbol{N}dV_e, \tag{3}$$

$$\boldsymbol{k}_e = \int \boldsymbol{B}^T \boldsymbol{C}\boldsymbol{B} dV_e, \tag{4}$$

$$\boldsymbol{f}_e = \int \boldsymbol{N}^T \boldsymbol{F}_v dV_e + \int \boldsymbol{N}^T \boldsymbol{F}_s dS_e, \tag{5}$$

were $\boldsymbol{N}$ is the matrix of the element's shape function, $\boldsymbol{B}$ is the matrix of the extensions – nodal displacements, $\boldsymbol{C}$ is the constitutive law matrix, $\boldsymbol{F}_v$ is the vector of the volume forces and $\boldsymbol{F}_s$ is the vector of the surface nodal forces. Equation (2) restrained to static analysis gives

$$\boldsymbol{K}\boldsymbol{u} = \boldsymbol{f}\,. \tag{6}$$

Considering the thermal response due to temperature change and despised (assumed being too small) the variation of the elastic constants with the temperature can apply the principle of superposition to the Hooke's law:

$$\sigma_{ij} = C_{\,ijkl}[\varepsilon_{kl} - \alpha_{kl}\Delta T]. \tag{7}$$

Rewriting (2) in the frequency domain, for the case of free vibration with prestress we get the following eigenvalue problem:

$$\left(\left(\boldsymbol{K} + \boldsymbol{k}_{geom}\right) + \boldsymbol{\lambda}\boldsymbol{M}\right)\boldsymbol{u} = \boldsymbol{0}, \tag{8}$$

where λ is a diagonal matrix of the frequencies squared ($\lambda = \omega^2$) and $\mathbf{u}$ is the nodal displacements matrix (one eigenvector per column) of the corresponding vibration modes and $\boldsymbol{k}_{geom}$ is the geometric matrix associated with the pretension at strings.

4 Results for the model without strings

The problem analysed in this section corresponds to the determination of the first frequencies and vibration modes of the guitar without the strings, with the boundary conditions of guitar free in space, as was considered in the experimental model of [5]. Table 3 presents the final dimensions (the ones which better approximate the modes and frequencies obtained by [5]), attained after several FE analyses by trial-and-error.

Table 3 – Cross section dimensions and brace distances of the analysed guitar model.

(lxh mm)	**Top**	**Back**	**Distance to the exact point**	**Distance between braces**	**Gluing Belt**
Álvaro Merceano da Silveira's guitar					
Brace 1	12x15	12x15			-
Brace 2	8x18	8x18		48 a 52	
Brace 3	8x18	8x18	13 a 15		
CAD model final dimensions					
Brace 1	8x15	8x15			6x12
Brace 2	8x14	8x14		90,86	
Brace 3	8x14	8x14	48,5		

4.1 Finite element convergence studies

Table 4 presents the results obtained with four FE meshes, corresponding to different finite element size parameter. Fig. 4 presents the last mesh of Tab. 4.

Table 4 – Frequencies obtained with different meshes.

Modes	Mesh/ number of elements			
	1st Mesh 18172	2nd Mesh 38879	3rd Mesh 111988	4th Mesh 133624
	Frequency (Hz)			
1	292,61	277,46	276,05	276,81
2	392,1	374,74	369,82	364,03
3	585,06	540,73	519,72	513,72

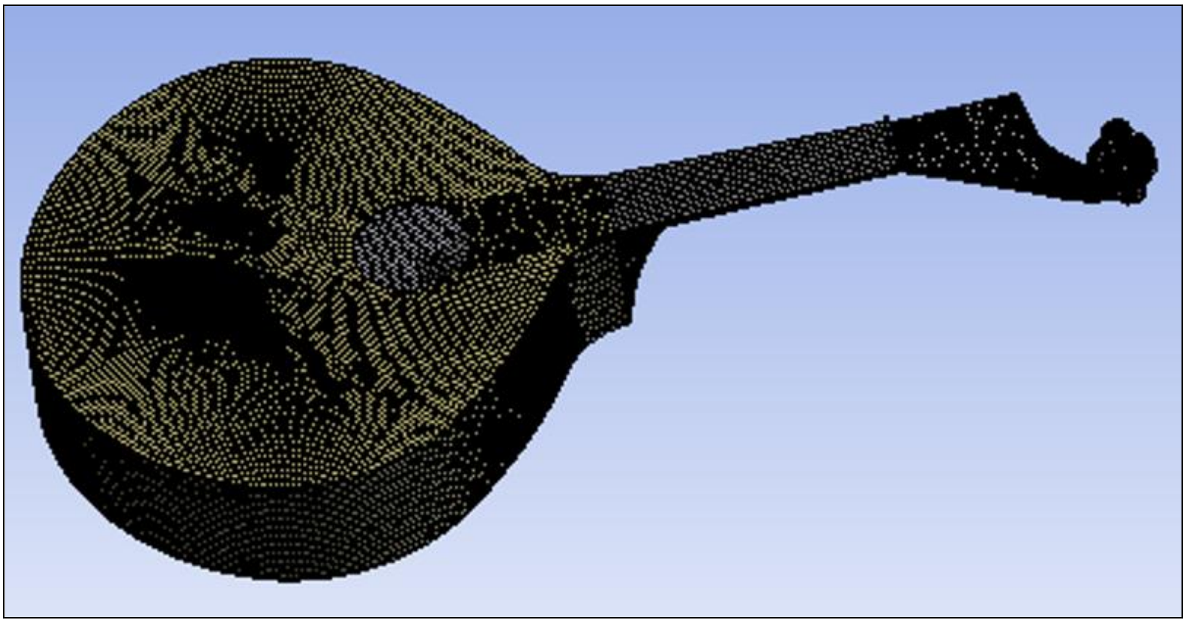


Figure 4 - 4th mesh.

In Tab. 4, we see that the indicated frequency variations between the third and the fourth meshes is not significant anymore, and, therefore, it can be considered as a fairly accurate approximation for the objectives of this work. It is true that the number of elements increases from mesh to mesh, which also corresponds to an increase in computation time; however, since the computation time is not too high, mesh 4 was chosen for the following analysis.

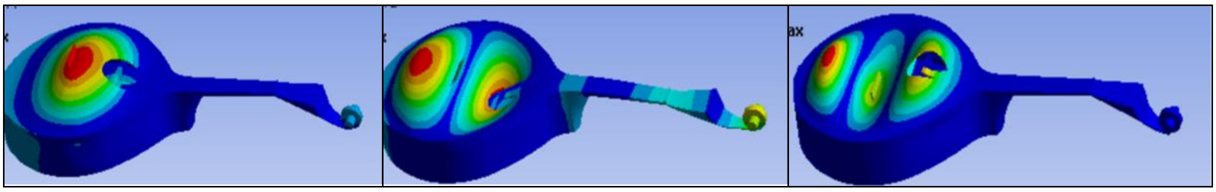
4.2 Modal analysis without strings

Next, we present at Fig. 5 the three vibration modes of the guitar (modes of interest, namely modes (0,0), (0,1), (0,2)), with bridge and without strings, obtained through FE analysis. These are the ones which better approximate the modes and frequencies obtained by [4] and presented in Fig. 1.

Table 5 presents the results and the deviations of the frequencies obtained in the FE analysis, in comparison with the results obtained by [5].

Table 5 – Frequencies of modes 1, 2 and 3, experimental and obtained through FE.

Mode Shapes	Frequency		
	Experimental (Hz)	FE Analysis without Strings (Hz)	Deviation (%)
1	275	276,81	0,66
2	360	364,03	1,11
3	635	513,72	19,1



a) b) c)

Figure 5 - Modes: a) (0,0) with 276.81 Hz; b) (0,1) with 364,03 Hz; and c) (0,2) with 513,72 Hz

4.3 Guitar with prestressed strings

At this point, the objective is to place the strings in the FE model. As mentioned before, the guitar is free in space as in the experimental model of [5]. FE analyses are performed using the mesh with the characteristics indicated in section 4.1. Some details of the strings and the bridge are represented in Fig. 6. Successive FE analyses of the model with strings were taken and the results presented in Tab. 6.

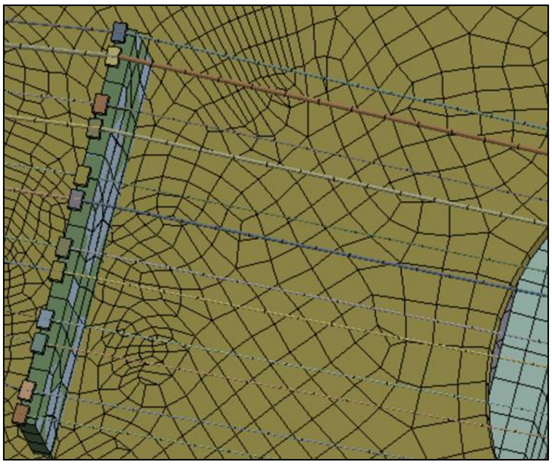


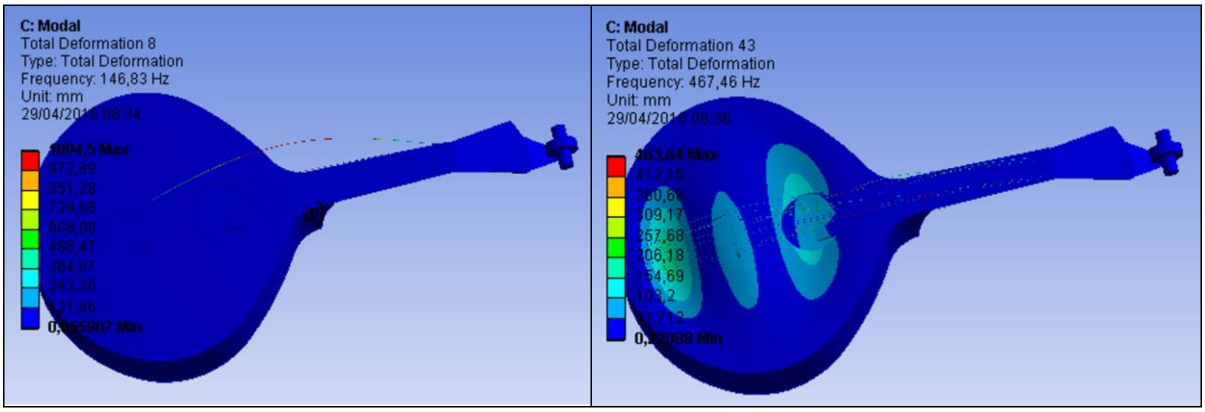
Figure 6 – Detail of the FE model of the bridge and of the whole set of strings.

Table 6 – Frequencies of modes 1,2 and 3, experimental and obtained through FE analysis.

Mode Shapes	**Frequency (Hz)**		
	Experimental	**FE Analysis with Strings**	**Deviation (%)**
1	275	275,06	0,022
2	360	364,10	1,13
3	635	467,46	26,38

Fig.7a) shows the string 11 in the fundamental mode shape and the respective tuning frequency rounded to the nearest hundredth. Fig. 7b) shows the 3rd mode of interest, i.e. the mode (0,2) ,with all the strings attached.

The fundamental frequencies of the remaining strings and the two remaining modes of vibration were found in a similar way, see Tab. 7. In it, the letter is for the pitch name of the note, and the number indicates the octave for the pitch (according to the scientific pitch notation).



a) b)

Figure 7 – Modal plots: a) mode shape of the eleventh string (146.83 Hz); b) 3th mode shape with the twelve strings (467,46 Hz).

Table 7 – String diameters for the Portuguese guitar and experimental frequencies

String	Diameter[mm] from[9]	Frequency values [Hz] – FEM	Frequency values [Hz] - from [9]
B (b4)	0,24	493,880	493,880
A (a4)	0,25	440,000	440,000
E (e4)	0,32	329,630	329,630
“B bordão” (b3)	0,50	246,940	246,940
“A bordão” (a3)	0,64	220,000	220,000
D (d4)	0,44	293,660	293,660
“D bordão” (d3)	0,79	146,830	146,830

As mentioned before, the usefulness of the FSI is also of interest within this field, e.g. to confirm which frequencies are not significant for the sound radiation and how is it related with the structure. Some works on numerical FSI vibroacoustic were published recently as e.g. [10, 11].

5 Conclusions

For the initial guitar model created without the strings, the modes (0,0), (0,1) and (0,2) were obtained in the correct form. The frequencies estimated through FE analysis had deviations of 0,66%; 1,11% and 19,1%, respectively, when compared with the experimental values of [5].

After, the response was adjusted to the experimental values of [5] by changes in the dimensions of the braces, the relative distances between them, between the braces and the exact point (where the bridge is placed, see Fig. 2 c).

The increase (or decrease) of the braces and gluing strap increases (or decrease) all frequencies due to an increase (or decrease) in stiffness and mass. This makes possible to find the first frequency, while the distance from the third brace to the second separates the frequencies from each other.

As indicated in Tab. 3, we can see substantial differences from initial model (Álvaro Merceano da Silveira’s guitar) and final CAD model.

Adding the strings to the guitar, the correct modes were also obtained and the deviations in the frequencies were of 0,022%, 1,13% and 26,38%, respectively, when compared to the experimental values of [5]. The strings were tuned to the nearest hundredth through the application of a pre-tension to each string. The effect of the bridge placement on the two previous cases was also verified.

The addition of the bridge and the strings produces a downward dislocation of the frequencies and change the vibration modes. This iterative process thus determines the final guitar geometry/dimensions and was one of the focus of this work to successful verify its potential and limitations. These results support the hypothesis that the behaviour of the guitar can be reasonably well predicted during the design stages, demonstrating that it is possible to quantify the effects caused by changes in the guitar structure, bridge and string placement and string tuning. A more detailed description of this work is published in [12].

Several questions remain for future work, as e.g.: 1. validation of the prestress in strings; 2. validation of the FE model-based dimensions changes against the response of the Portuguese guitar; 3. modelling aspects of the guitar sound quality conditioned by the guitar structure and materials.

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