Invited paper

EuroRegio2016
June 13-15, 2016
Porto · Portugal

SOME PARADOXES OF THE "CYLINDRICAL SAXOPHONE"

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Abstract
The story of the "cylindrical saxophone" started with Benade (1988). The basic idea is that when the length of the missing part of a truncated cone is smaller than the wavelength, and therefore smaller than the length of the truncated cone, the behavior of a conical reed instrument has similarities with that of a cylindrical pipe excited by a reed on its side, at an intermediate location. The shorter part of the cylinder has to be equal to that of the missing part of the cone. This similarity allowed to get caricatures of the pressure waveform inside the mouthpiece, in the form of a Helmholtz (2-state motion), which is well known for bowed string instruments. However some paradoxes remain with this analogy. If the waveform is a Helmholtz motion, the negative pressure episode has a duration corresponding to the resonance frequency of the short length, i.e., a frequency which does not fulfill the condition of the analogy. Furthermore using the simplest approximation deduced from the analogy, the waveform of the radiated sound by the conical instrument should be a Dirac comb. This means that all the Fourier coefficients have the same magnitude, without missing harmonics. After a short review of the literature, these paradoxes will be discussed.

Keywords: musical acoustics, conical instruments, saxophone

PACS no. 43.75.ef, 43.75.Pq

1 Introduction

The idea of the analogy of a saxophone with a cylindrical tube excited at a certain distance of one extremity appeared for the first time in a paper by Benade [2], with the name “cylindrical saxophone”. Irons [1] previously found that an excellent approximation for the eigenfrequencies of a reed conical instrument is given by the eigenfrequencies of an “open-open” cylinder of the same length L, and this was used in particular by Benade [3] or Nederveen [4]. The condition of this analogy is the following: the length of the missing cone, x_1 (see Figure 1) needs to be much smaller than the wavelength, i.e.,

kx_1 <<1, or x_1 <<L/π , where k is the wavenumber and $L=x_1 + \ell$ is the total length of the cone. However it can be remarked that the condition can be extended if the mouthpiece has a volume equal to that of the missing cone [5], because this reduces the inharmonicity between the two first eigenfrequencies. Obviously for a cylinder x_1 is intinife and the analogy fails.
A consequence of this analogy is the analogy of the mouthpiece pressure signal with a rectangular signal (see Figure 2), which was first explained by Gokhstein [6], and developed by Dalmont et al [7,8], together with the analogy with a stepped cone. This rectangular signal is named Helmholtz motion in the literature on bowed strings. Gokhstein showed both experimentally and theoretically that the duration of closure of the reed is independent of the played note, i.e., of the equivalent length of the resonator. This duration is related to the round trip of a wave over a length equal to that of the missing part of the cone.
In this presentation, we discuss several paradoxes of this analogy. For the sake of simplicity, we call a truncated cone with reed and mouthpiece “saxophone”.

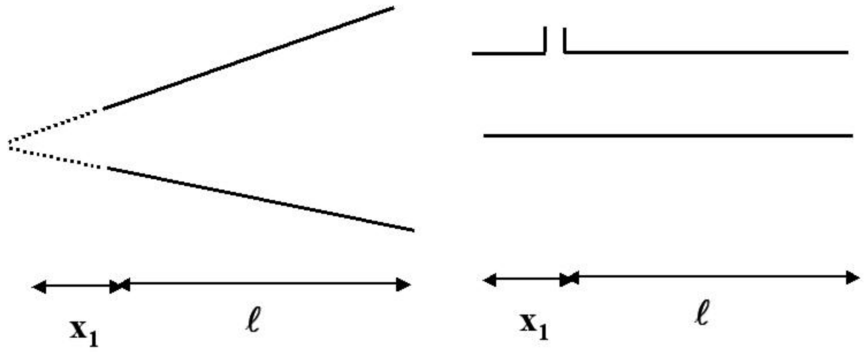

Figure 1. A truncated cone (on the left) and the equivalent “cylindrical saxophone"” (on the right). For the latter, the mouthpiece is placed on the side of the cylinder.

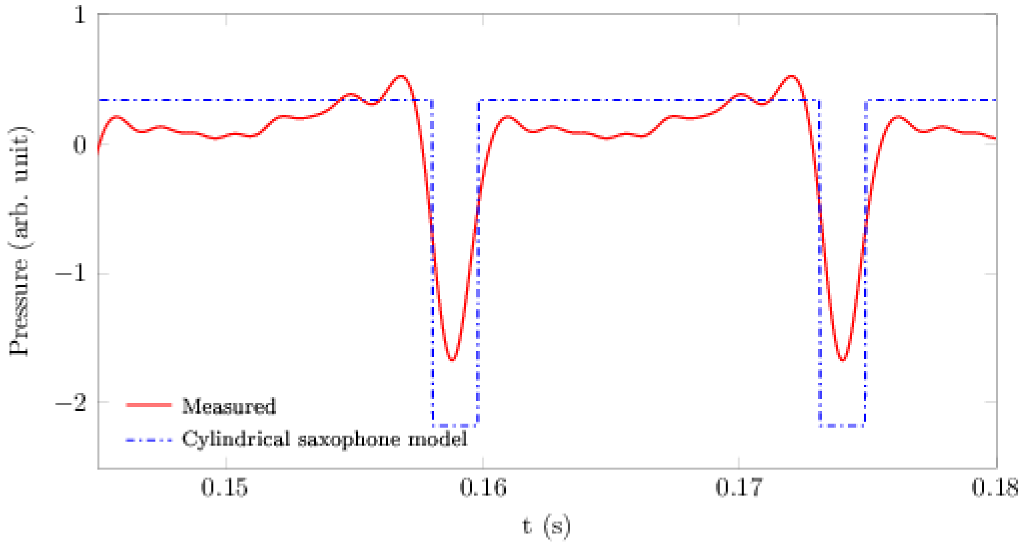

Figure 2. Periodic signal of the mouthpiece pressure of a barytone saxophone, and the approximation by a rectangle signal (pure Helmholtz motion). The duration of the of negative pressure state, which corresponds to the reed beating, is common to the different notes with different values of the equivalent length ℓ of the truncated cone.

2 Three paradoxes

i) The inharmonicity of a truncated cone is positive, because at the limit of a cylinder, the natural frequency series is transformed from 1,2,3... into 1,3,5... The mouthpiece of the saxophone reduces the inharmonicity of the truncated cone. For a cylindrical saxophone, the mouthpiece adds an important inharmonicity, which can prevent the sound production. In this case inharmonicity is negative: at the resonance frequencies the mouthpiece is equivalent to a closed tonehole, which dicreases the resonance frequencies, and the shift increases with frequency. The patent by Yamaha [9] seems to propose several solutions to the problem, but no results are published. Recently a solution based on a coaxial resonator was presented [10].

ii) For a cylindrical saxophone, the reed closure episode (negative mouthpiece pressure) has the same duration whatever the note, i.e., whatever the length of the cone. Therefore inside the mouthpiece, there are anti-ormants (missing harmonics when the ratio x_1/L is rational), corresponding to the natural frequencies of the length x_1. Measurements in a saxophone mouthpiece confirms the existence of anti-formants at frequencies close to the natural frequencies of the length x_1. This is parodoxical because the length x_1 has by definition the same order of magnitude as the corresponding natural frequencies, at which the analogy cannot be valid. This is explained by the consideration of the values of the impedance of the saxophone at frequencies that are multiple of the fundamental one, because the sound is periodic [11, 12].

iii) A cylindrical saxophone radiates by two sources at its extremities. Both radiated pressures have a flat spectrum (a Dirac comb). However missing harmonics are expected, similarly to the even harmonics of a simplified clarinet: in [11] this was explained by considering a difference in phase between the two sources (when their external distance is ignored). For a saxophone, the analogy considers one source only, i.e., the opening the longer extremity, thus missing harmonics do not exist.

3 What are the most important parameters for a saxophone?

A consequence of the analogy with the cylindrical saxophone is somewhat paradoxical also for the saxophone. For the simplest model of a cylindrical saxophone, that leads to the Helmholtz motion, there are two caracteristic lengths only: the two lengths on the two sides of the mouthpiece. The radius does not intervene in the model, because no losses are considered. The analogy leads to the conclusion that the apex angle of a saxophone is of secondary importance on the waveform, i.e., on the low frequencies, when it is compared to the length of the missing cone, and this is not intuitive. Therefore the comparison between ancient and modern saxophones based on the apex angle [13] is disputable. The length x_1 is more important and should be studied.

Figure 3 shows an example of the mouthpiece pressure in two saxophones with the same length $L=x_1 + \ell$, i.e., with a very close playing frequency, and two different lengths x_1 . The calculation is done with the simplest model, which was shown to give realistic signals in the mouthpiece [14-16]. The positive pressure episode is close to the value of the corrresponding mouth pressure p_m (the excitation pressure), which is the exact value for the Helmholtz motion. It is almost independent of the value of x_1 . The negative episode of the Helmholtz motion is given by the following formula [17]:

$$p = -\, p_m\ (1-\beta)/\beta \quad \text{with } \beta = x_1\ /L \qquad (1)$$

For the values of the cases shown in Figure 3, x_1 = 0.2m and 0.3m, respectively, the ratio of the two minima is found to be 1.65 for the saxophone model and 1.7 for the cylindrical saxophone model. This confroms the importance of the length x_1 for reed conical instruments.

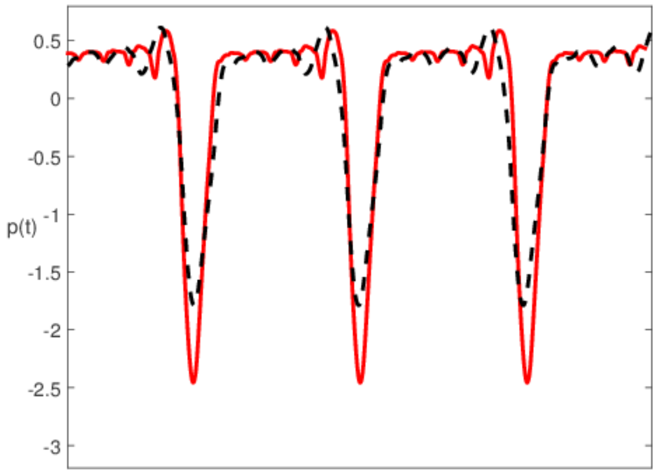

Figure 3: Two waveforms for the mouthpiece pressure calculated with the minimum model. The total length is the same for the two cases: L=1.3m. Solid line: x_1 = 0.2m . Dotted line: x_1 = 0.3m. The pressure p(t) is reduced by the static closure pressure.

4 Conclusions

When comparing instruments of the saxophone family, or when comparing a soprano saxophone and an oboe (which have the same lowest note), it is better to consider first the length of the missing cone. Obviously the values of the apex angle and of the radii are important for the higher frequencies, which are sensitive to frequency-dependent losses and to the reed dynamics. Furthermore the study of the nonlinear characteristic shows that there is a significant difference between the maximum values of the flow entering the instrument for an oboe and a saxophone [6,18].

Acknowledgements

This work has been supported by the French ANR research projects CAGIMA. We acknowledge useful discussions with Sami Karkar, Jean-Pierre Dalmont and Bruno Gazengel.

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