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VISCOELASTIC MATERIAL CHARACTERIZATION ON THE CONSTRAINED FORM THROUGH EQUIVALENT PROPERTIES

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ABSTRACT

This paper discusses a methodology developed for the characterization of viscoelastic materials (VEM) on its constrained form through equivalent properties for the ensemble (base material, VEM and constraining layer). At first, the experimental procedure is presented, followed by the equivalent properties characterization through curve fitting and optimization algorithms considering measured and numerical frequency response functions (FRF). Finally, specific properties for the VEM are obtained through spectral methods. Validation for the specific properties is presented.

RESUMO

Este artigo propõe uma metodologia para a caracterização do material viscoelástico na forma constrita a partir de propriedades equivalentes do sistema formado pelo composto viscoelástico na forma constrita e material base. Primeiramente, é descrito o procedimento experimental utilizado, seguido pela caracterização da estrutura com o uso de propriedades equivalentes através do ajuste de curvas e algoritmos de otimização. Por fim, propriedades dinâmicas do material viscoelástico são obtidas a partir do uso de modelos espectrais. É apresentada uma validação para a caracterização específica do material viscoelástico.

1. INTRODUCTION

Cabin noise has been an increasingly tackled issue in aircraft design in the last decades. Being intense enough, it can cause discomfort, interfere in communications and lead the crew to fatigue. Furthermore, in the long run, hearing losses may be linked to frequent exposure to the mentioned conditions. In this scenario, passive vibration control methods are widely used, where the application of viscoelastic materials (VEM) is one of the most cost-effective solutions considering the properties of this class of materials.

Application of VEM in a constrained form to the airplane fuselage can enable high levels of damping, resulting in a considerable noise reduction with relatively low mass addition. Nevertheless, a proper characterization for the properties of these materials is fundamental when considering the development of numerical models able to compute the behavior of complex structures with partial application of such a treatment.

The established methods, however, present limitations when performing the characterization of this kind of material. The ASTM E756-05 [1] standard makes use, for example, of the half-power bandwidth method to estimate the loss factor for the material, an ineffective procedure for cases of high damping. Another limitation for the standard

regards the measurement layout, where the constraining layer is removed, making the characterization for a lot of VEM solutions unfeasible.

To overcome these limitations, this paper proposes a methodology for the characterization of structures with VEM attached in a constrained form, based on the computation of equivalent properties.

2. EQUIVALENT PROPERTIES CHARACTERIZATION

The first step, inside the proposed methodology, is to obtain equivalent dynamic properties for the VEM in the constrained form[1]. For this purpose, beam specimens are tested to get the response function. Numerical and experimental FRF's are fitted peak-to-peak by optimizations algorithms. Young's modulus and loss factor are the variables used in these algorithms where the goal is to match the peak shape.

2.1 Experimental Procedure

The test specimens are beams fully covered with VEM in the constrained form as shown in Figure 1. It's recommended to use a base beam thick enough to generate FRF's with distinguishable peaks. The test specimen is fixed to an impedance head, which is connected to a shaker. This layout (Figure 2) was chosen in order to add as little mass as possible. The beam is excited by a random noise signal set in the spectrum analyzer and the responses are measured in terms of force and acceleration. With this data, the point accelerance is calculated.
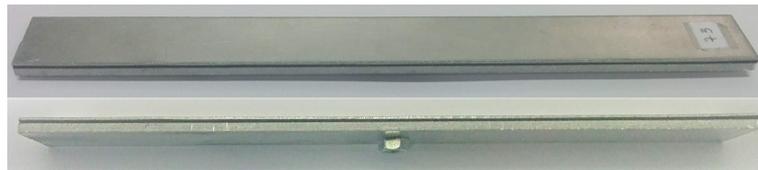
Figure 1: Test Specimen
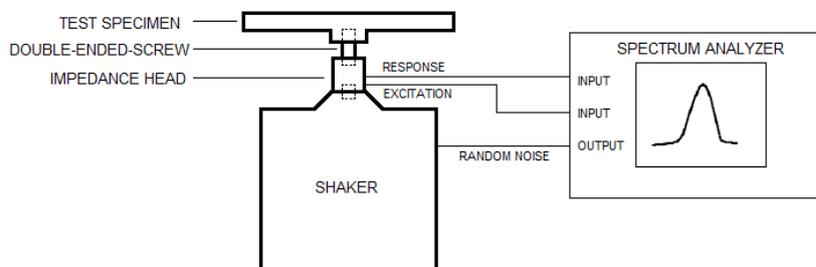
Figure 2: Test Layout

2.2 Numerical FEM Model

The numerical model was developed using the finite element method and 1D beam bending elements with the assumptions discussed in [3]. The model represents the testing specimen as a homogeneous beam with thickness and mass given as the sum of each layer. Mass corrections were also applied to represent the measurement system mass. The structural damping was applied to the system with the use of complex modulus approach.

2.3 Curve Fitting - Optimization Procedure

The equivalent dynamic properties are obtained by the curve fitting between experimental and numerical point accelerances. At first, the Young's modulus is estimated by matching the natural frequencies, which are the peak's frequencies of the experimental FRF and the square root of the numerical autovalues. This step cosiders an undamped system and solves it through a modal analysis, therefore it has a low computational cost leading to a good initial Young's modulus to the next step.

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[1]A complete description of the approach developed is discussed in [2]

Structural damping is then applied to the system, which is solved now by the direct method. The curve fitting is done by optimization alghoritms where the main function is the difference between experimental and numerical curve shapes and the variables are the Young's modulus and loss factor. This curve fitting is done to each mode separately, then one Young's modulus and one loss factor for each natural frequency is estimated.

3. SPECIFIC PROPERTIES CHARACTERIZATION

This section presents two different methods used in this work for VEM specific properties characterization.

3.1 RKU Method

One of the first models developed and still used nowadays for a passive constrained layer beam is the one developed by Ross, Kerwin and Ungar [4]. The model, commonly referred to by its acronym RKU, develops an approach to obtain equivalent complex stiffness properties for a given configuration, and is used as a reference in this paper. Some of its assumptions are:

- Energy dissipation occurs mainly through shear on the viscoelastic layer;

- The stiffness of the viscoelastic material is considerably lower than the elastic layers' materials;

- The viscoelastic layer is assumed to hold both of the elastic layers a fixed distance apart[2];

- The beam is simply supported at its edges (or long enough so that edges' effects are negligible) and vibrates in a natural frequency.

Consider the three layered composite illustrated in Figure 3.
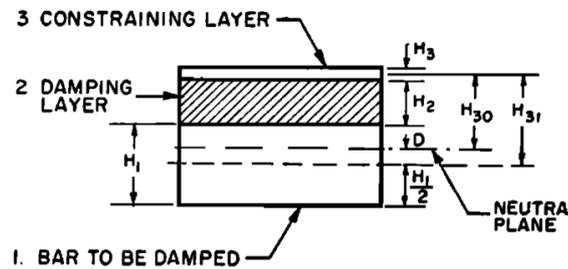
Figure 3: PCLD Beam [4]

One may then express the total bending moment as:

$$M = B\frac{\partial \phi}{\partial x} = \sum_{i=3}^{3}(M_{ii} + F_i H_{i0}).$$

(1)

where B is the equivalent complex stiffness ($EH^3/12$) and ϕ is the flexural angle of the base beam. M_{ii} is the moment exerted by the forces on the ith layer about its own neutral plane, F_i is the net extensional force on the ith layer and H_{i0} is the distance from the center of the ith layer to the composite beam neutral plane.

Through such a procedure one can find the equivalent complex stiffness (observing that H_{i1} is the distance from the center of the ith layer to the original neutral plane) as:

$$B = E_1 H_1 D^2 + E_2^* H_2 (H_{21} - D)^2 + E_3 H_3 (H_{31} - D)^2 -$$

$$\left[\frac{E_2^* H_2}{2}(H_{21} - D) + E_3 H_3 (H_{31} - D)\right]\left(\frac{H_{31} - D}{1 + g^*}\right) - \frac{E_2^* H_2^2}{12}\left(\frac{H_{31} - D}{1 + g^*}\right) + \sum_{i=1}^{3}\frac{E_i H_i^3}{12}$$

(2)

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[2]This is a critical assumption and is further discussed in this work for the cases in which it's not valid anymore.

where D defines the neutral plane offset from its original position in the base beam, given by:

$$D = \frac{E_2^* H_2(H_{21} - H_{31}/2) + g^*(E_2^* H_2 H_{21} + E_3 H_3 H_{31})}{E_1 H_1 + E_2^* H_2/2 + g^*(E_1 H_1 + E_2^* H_2 E_3 H_3)} \tag{3}$$

and the shear parameter g^* given by:

$$g^* = \frac{G_2^*}{E_3 H_3 H_2 k_n^2} \tag{4}$$

where k_n is the wavenumber ($2\pi/\lambda$, with λ being the wavelength) for the nth mode. For different boundary conditions the first mode can be affected so corrections have to be made in the equations [5]. For the free-free case, focus of this work, corrections are not required.

Observe that, through Equation 2 one can find equivalent properties for the given configuration and therefore find natural frequencies and equivalent loss factors for each mode when properties are known for each layer. Alternatively, the same equation can be used to find specific properties for the viscoelastic layer, considering that equivalent properties were already obtained through the procedure discussed above (and properties for the elastic layers are known).

As observed, the major restriction for the theory is the assumption considering that all layers partake the same motion, that is, the flexural wavelength is the same in the three layers. For cases where the shear parameter (Equation 4) assumes small values relative to the unity, waves can propagate in the middle layer itself, so that the elastic layers become uncoupled [4].

3.2 1D Spectral FEM

A different approach, discussed in [6], models wave propagation in a given laminate considering only a 1D model for its cross section (A-B) as illustrated below, Figure 4, and is discussed in this paper as an alternative to the limitations observed for the RKU model.

For the 1D mesh, one can consider a 3D displacement field given by:
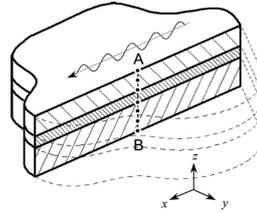
Figure 4: Laminate Finite Element 1D Mesh: A-B [6]

$$\mathbf{d}(x,y,z,t) = \mathbf{N}(z)\mathbf{q_0} \, exp(i\omega t - ikx) \tag{5}$$

where $\mathbf{N}(z)$ is the shape function matrix and $\mathbf{q_0}$ the nodal DOFs vector. With this, the time average system's kinetic energy can be written as:

$$T = \frac{\omega^2}{4}\mathbf{q_0}^H \mathbf{M}\mathbf{q_0}dxdy \tag{6}$$

where $\mathbf{M}$ is the mass matrix given by:

$$\mathbf{M} = \int_z \rho(z)\mathbf{N}^H(z)\mathbf{N}(z)dz \tag{7}$$

Going further, the time average strain energy can be written as:

$$U = \frac{1}{4}\mathbf{q_0}^H \mathbf{K}\mathbf{q_0}dxdy \tag{8}$$

where **K** is the stiffness matrix given by:

$$\mathbf{K} = \int_z \mathbf{B}^H(k,z)\mathbf{D}(z)\mathbf{B}(k,z)dz \tag{9}$$

where **B** is the strain-displacement matrix and **D** is the material constitutive matrix. The complete matrices' formulation can be found in [6].

Equations of motion for the cross section can be formulated by inserting the kinetic and potential energies into the time-average form of Lagrange's equations. In general, the nonconservative form of Lagrange's equations should be used to account for dissipation through the cross section. Nevertheless, the following analysis adopts a simplified approach in which the equations of motion are generated assuming that the laminate is conservative and that the resulting cross-sectional strain energy distribution is used to estimate the power dissipated by the section (see [6]), through a Modal Energy Deformation method.

The assumptions above are likely to be valid if the cross-sectional strain energy distribution of a given propagating wave is not significantly modified by increasing levels of damping. Inserting the energy equations (8 and 6) into the conservative form of Lagrange's equations one can find the characteristic equation:

$$[\mathbf{K_r}(k) - \omega^2\mathbf{M}]\mathbf{q_0} = 0 \tag{10}$$

where the subscript r indicates the real part of the materials' stiffness is used when formulating **K**. For a specified frequency ω, Equation 10 takes the form of a generalized quadratic eigenvalue problem for the wavenumber k.

Finally, one can compute loss factor values for a given wave type through its modal shape. With the eigenvectors ϕ found solving Equation 10, one can rewrite it to be:

$$\eta_k = \frac{\sum_n \eta_n \phi^H \mathbf{K}_r^{(n)} \phi}{\phi^H \mathbf{K}_r \phi} \tag{11}$$

The inverse characterization procedure with this formulation was designed in two steps. The first one aims to obtain the viscoelastic layer's stiffness value for a given mode. It starts computing a wavenumber using analytical homogeneous beam theory and the equivalent stiffness property. In the sequence an optimization algorithm is used to compute wavenumbers with Equation 10 iteratively (changing VEM stiffness values) until there's a convergence with the value calculated analytically. With the VEM stiffness known, flexural wave shape (computed with Equation 10) and equivalent loss factor in hands, one can simply use Equation 11 to compute the loss factor for the VEM layer.

4. RESULTS AND FURTHER DISCUSSIONS

This section presents the results obtained for two different numerical cases of constrained VEM. The results below are intended to validate the methods used and present the limitations for the RKU method.

4.1 Case 1

For the first case, the base beam and constraining layer are made of aluminum, with thickness of $2mm$ and $0.2mm$ respectively. The VEM layer is also $2mm$ thick and has constant properties with frequency for reasons of simplicity and model validation:

- *Aluminum* $\rightarrow E = 70GPa$, $\rho = 2700kg/m^3$, $\nu = 0.3$, $\eta = 0.01$

- *VEM* $\rightarrow E = 0.1GPa$, $\rho = 400kg/m^3$, $\nu = 0.4$, $\eta = 0.7$

A model with these properties was created and solved using Nastran®. Frequency response functions were then used to compute equivalent properties as illustrated below:
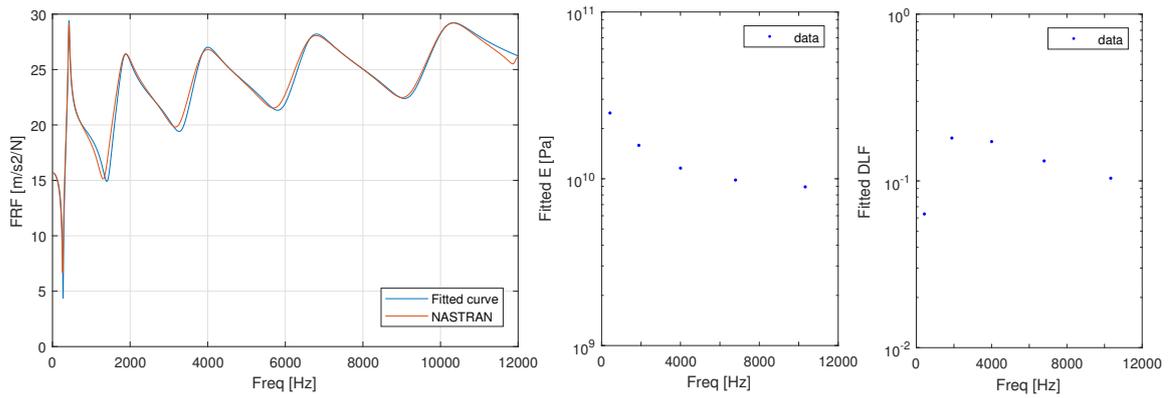
Figure 5: FRF fit and equivalent properties

Using these modal equivalent solutions and assuming the VEM properties unknown, RKU and SFEM procedures were used to obtain its specific properties. The results are illustrated below.
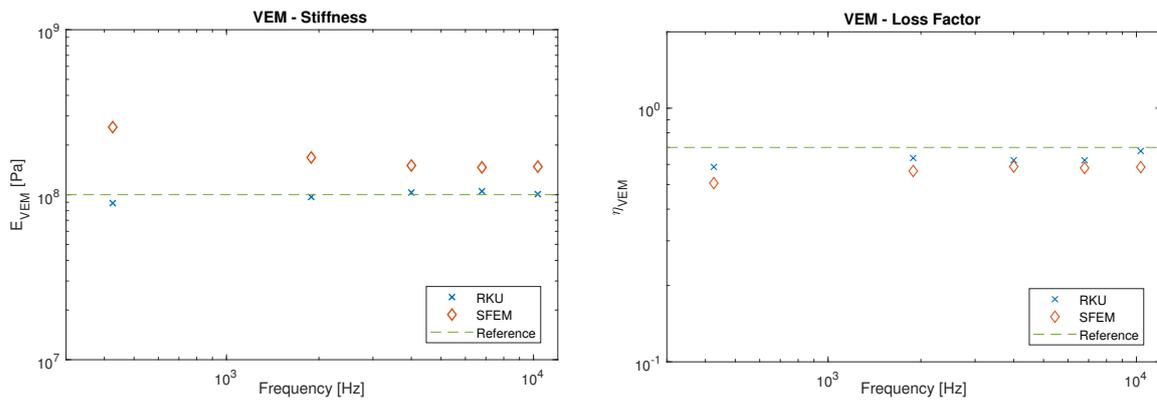
Figure 6: VEM obtained properties: Stiffness and Loss Factor

It's visible that both of the methods present good approximations to the expected values. For the SFEM model, stiffness values, however, are usually overestimated. This happens once, as discussed in the formulation, only the real part of materials' properties are being used to obtain the wavenumbers with Equation 10, so that wavenumbers (the same way as natural frequencies) are underestimated.

4.2 Case 2

The second case is similar to the Case 1 with two modifications: an increase of 1mm for the VEM layer (total thickness of $3mm$) and a reduction of its stiffness by a factor of 10 (considerably reducing the shear parameter for each mode, given in Equation 4):

- *Aluminum* $\rightarrow E = 70GPa$, $\rho = 2700kg/m^3$, $\nu = 0.3$, $\eta = 0.01$

- *VEM* $\rightarrow E = 0.01GPa$, $\rho = 400kg/m^3$, $\nu = 0.4$, $\eta = 0.7$
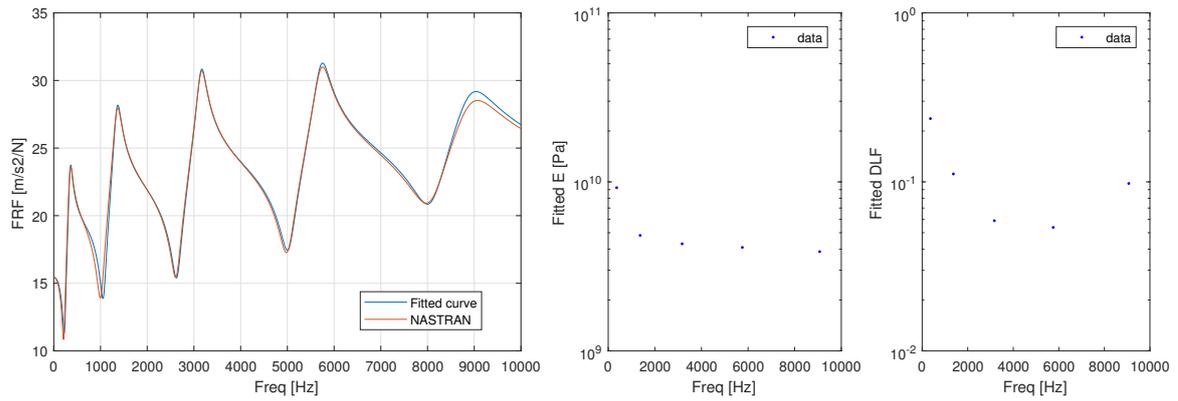
Figure 7: FRF fit and equivalent properties

Same procedure was used to compute specific properties for the VEM layer:
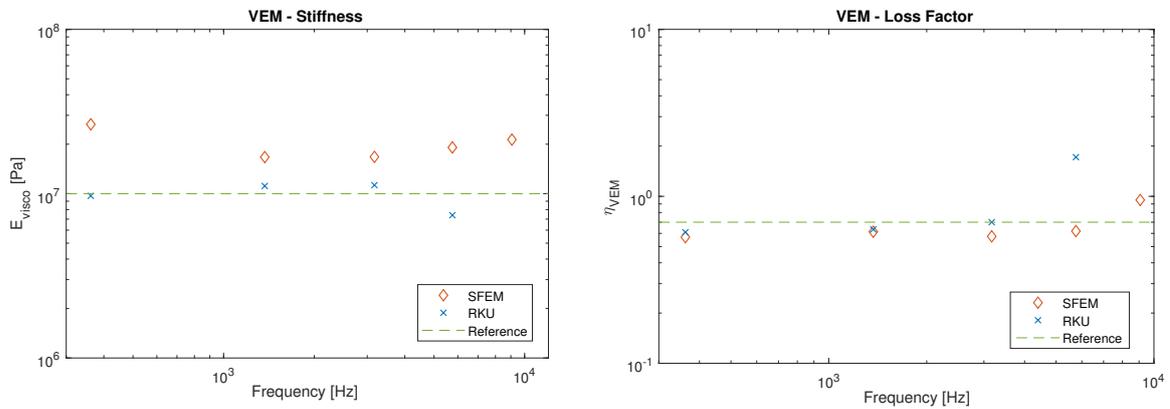
Figure 8: VEM obtained properties: Stiffness and Loss Factor

For this case, once again stiffness for the VEM layer was overestimated, but notice that RKU method was not able to compute values for the 5th mode[3], and the results for the 4th already start deviating from the reference considerably. The flexural wave solution for the 5th mode is illustrated below:
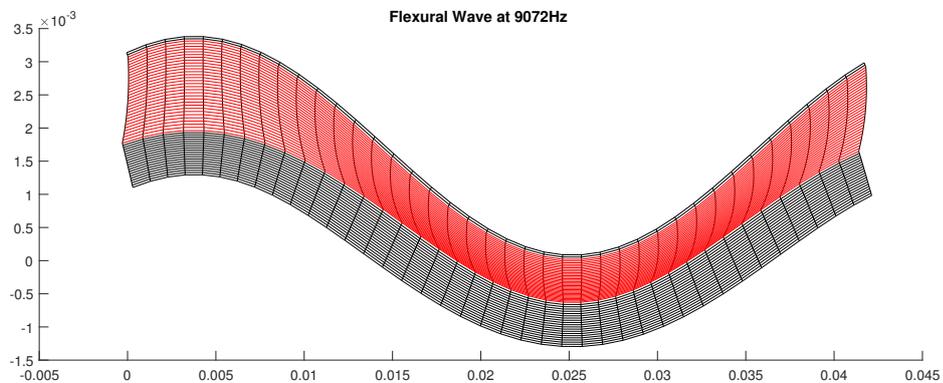
Figure 9: Waveshape, 5th mode - 9072Hz

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[3]The inverse characterization with RKU results in a 3rd order polynomial equation. For the cases where the result has a physical meaning one of the roots (stiffness) has to be positive. For the last mode in this case, all of the roots become negative.

The figure shows that two of the RKU assumptions are not valid anymore. First, the shear field in the VEM layer is no longer a linear function along the thickness axis. Second, it becomes clear that the transverse displacement is not the same for all of the layers, so that they are not hold a fixed distance apart as previously assumed.

5. CONCLUSIONS

The main purpose of this text was to present a methodology to estimate the Young's modulus and the loss factor for VEM in a constrained form, bypassing the difficulties imposed by the current ASTM E756 standard. Characterizing the material with equivalent properties and later using spectral methods proved to be feasible for this purpose. Two spectral methods were applied, RKU and SFEM. Two validation cases were presented where each of these methods was analyzed and the results converged to the input data.

Although they achieve the main objective, the application of this methodology has some limitations. The use of a homogeneous beam to obtain the beam's dynamic properties is a source of errors. Besides that, when one tries to characterize a VEM with a greater thickness and smaller Young's modulus, the RKU method limitations become clear and the proposed approach with the SFEM is an interesting alternative.

Finally, as mentioned, a more precise approach can be described in future works to take into account structural damping when computing the wavenumbers with Equation 10 and avoid overestimating the VEM stiffness. Valid to observe also that the validation cases studied here are simplifications considering constant properties for the VEM material. Future works can comprise experimental cases as well as different spectral methods [7] that don't require the intermediate step of obtaining equivalent properties.

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