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MODAL ANALYSIS OF A PORTUGUESE GUITAR USING A 3D FINITE ELEMENT MODEL WITH STRING TENSION

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ABSTRACT

A simplified finite element model of the Portuguese guitar is presented. The authors present here their studies on the inclusion of the strings tension as well as a discussion on their influence in the modal response of the guitar. The twelve hard steel strings are tensioned to resonate at a natural frequency corresponding to the guitar tuning frequencies. At this stage, no acoustic-fluid was considered inside and outside the guitar. The interaction of the string with the guitar body through the bridge is being tested to explore the potential and limitations of predicting the guitar behavior at early design steps.

1. INTRODUCTION

The acoustic guitar has been a subject of study for the last years. Currently, an acoustic guitar can be handmade or factory made. Each one is made differently, for different purposes and different markets, and with different intent, aim and skills. Factories need to make instruments which are good enough to sell to a mass market, whereas luthiers – craftsman specialized in the construction of musical instrument – need to make instruments which are successful tools for musicians. Guitars could be considered handmade if the tool for its construction could be used with a degree of freedom dictated by the needs of the work and the will of the operator.
Additionally in the craft building process of a string musical instrument it is required to deal with different types of wood, with well-defined characteristics, which makes impractical the type of investigation that we are going to present in this article. Other differences between handmade and factory made acoustic guitar can be consulted in the literature, as in e.g. [1].

In this investigation we use numerical models to determine natural frequencies, corresponding to the guitar tuning frequencies, and the vibration modes, which can be tuned to toward the production of the desired sound of a musical instrument. In the study of acoustical instruments, it is important to relate the behavior of a guitar structure with its prestress strings. But what is the influence of prestress strings with the modal response of the Portuguese guitar? To answer this question, it is essential to develop methods for analytical and numerical prediction of the instrument structural behavior. Therefore, by varying the structural parameters it is possible to obtain different modal parameters without need for constructing multiple prototypes [1, 2].

Motivated by the pleasure of playing guitar and the wish to understand the behavior structural of the Portuguese guitar, we are interested in developing a numerical method to study the interaction of the strings with the guitar body. The Portuguese guitar is a pear-shaped instrument with twelve metal strings (six courses). Different from most acoustic guitars, this one has a bent soundboard with a similar bridge, although smaller in size, to the bridge of a violin, a neck usually with 22 fixed metal frets and it's tuned by a fan-shaped mechanism, consisting in twelve screws, acting as pegs, mounted with small gliding pins where the strings are attached to adjust its tension. It is an instrument that requires a soundboard to resist the stresses imposed by string tension and needs to have an attractive appearance. Therefore, it becomes evident (especially for mass production) the need and advantage of using computational tools that aid the process of building musical instruments and also allow the innovation. In this paper we use numerical modal analysis, calculated by finite element method (FEM), to determine the dynamic behavior of the Portuguese guitar. Note that the twelve hard steel strings are tensioned to resonate at a natural frequency corresponding to the guitar tuning frequencies. The modal analysis technique allows to determine the natural frequencies and the corresponding mode shapes of structural systems.

One of the first studies of the Portuguese guitar in these fields was developed by Inácio et al. (2004), as far as the authors knowledge. In their paper they present the results of an experimental modal identification performed on the soundboard of fully-assembled Lisbon guitars. They compared and described the frequency response curves for several specimens as well as their significant vibratory modes. In the table 3 we can see their experimental model results about the mode shapes of three resonances of the soundboard of a guitar. According with this results, the authors concluded that the most important of the three mode shapes it is the corresponding to the (0,0) monopole mode for a Portuguese guitar. This one, is the one that radiates sound more efficiently in contrast with the (0,1) longitudinal dipole mode, where adjacent antinodes move in anti-phase and eliminate any net volume flow, and the (0,2) longitudinal tripole mode that shows up only at 635 Hz [3].

Another study of the Portuguese guitar was the study of Marques et al. (2013). In their work they developed a model for a twelve strings (six pairs) guitar, such that the strings are coupled with the instrument body through the moving bridge. This is a relevant component for energy transmission from the strings to the guitar soundboard and back. The guitar body is modeled as a simple plate and the string assume only planar vertical motions. How we can see in their article they show the string notes and the corresponding frequencies according to the standard tuning of the Lisbon Portuguese guitar that were obtained in experimental identifications. Notice that the frequencies represent the fundamental frequency corresponding to the active part of the strings, this means, the length between the nut and the bridge should vary [4]. The bridge is an important element for good guitar sound and for the guitar tuning. However, due to its complex geometry and the influence of its position on the soundboard, so that the guitar is correctly tuned, we chose not to model this component and only consider its height [5]. Once the bridge is a component that will induce a certain height in the string, it was considered 17mm for the height of this component.

At section 2 is presented a brief reference to the elastodynamic finite element theory. It is followed by a description of the geometric and finite element model of the guitar and the strings. At section 4, authors present their preliminary results in this study including a comparison with the experimental results from [3].

2. THEORETICAL FOUNDATIONS

In this section we are presenting a concise mathematical formulation of the infinite element method for the structural modal analysis. The formulation presented here is developed with detail in Bathe [6]. Based on the theory of elasticity, the dynamic behavior of an elastic solid for the linear case (small deformations) defined on the boundary □□of the domain □□can be written in index notation as (strong form or Cauchy law):

$$\sigma_{ij,j} + f_i = \rho_s \ddot{u}_i \qquad (1)$$

where σ is the stress tensor, *f* is the vector body of body force, ρ_s is the material volumetric density, *u* is the displacement in the coordinate direction i = x, y, z. Then, it can be obtained in its integral form (weak form) by weighted residual method, whose choice of the function is based on the Galerkin method. Thus, the approximate solution for finite element in terms of the nodal displacements can be written as follows:

$$\mathbf{M}\ddot{\mathbf{d}} + \mathbf{K}\mathbf{d} = \mathbf{q} \tag{2}$$

where **d** is the nodal displacement vector, **M** is the global mass matrix, **K** is the global stiffness matrix and **q** is the force vector of the element, which are obtained by assembling the elementar matrices and load vectores:

$$\mathbf{M}^{\mathrm{e}} = \int_{\Omega} \boldsymbol{\rho}_s \mathbf{N}_s^{\mathrm{T}} \mathbf{N}_s \, d\Omega \tag{3}$$

$$\mathbf{K}^{\mathrm{e}} = \int_{\Omega} \mathbf{B}_s^{\mathrm{T}} \mathbf{D} \mathbf{B}_s \, d\Omega \tag{4}$$

$$\mathbf{q}^{\mathrm{e}} = \int_{\Omega} \mathbf{N}_s^{\mathrm{T}} \mathbf{f} \, d\Omega + \int_{\Omega} \mathbf{N}_s^{\mathrm{T}} \mathbf{t} \, d\Gamma \tag{5}$$

where $\mathbf{N}_s$ is the matrix of shape functions of the solid element, $\mathbf{B}_s$ is the strain-displacement matrix nodal, **D** is the matrix of the constitutive laws, **f** is the vector forces volume and **t** is the vector of surface forces. However, the residue in the domain of waste is the sum of all the elements used in its discretization, which leads to the process of assembling the elementary matrices into a global matrix.

Therefore, the equation (1), restricting it to static analysis, can be written in terms of a global matrix as:

$$\mathbf{K}\mathbf{d} = \mathbf{q} \tag{6}$$

Stresses **σ** can be obtained from (6) by the Hooke's law. Then, rewriting equation (2) in the frequency domain for vibration free condition is obtained:

$$((\mathbf{K} + \mathbf{K}(\boldsymbol{\sigma})) + \boldsymbol{\Lambda}_s \mathbf{M})\mathbf{d} = \mathbf{0} \tag{7}$$

where $\boldsymbol{\Lambda}_s$ is a diagonal matrix of the squares of the natural frequencies on solid domain and **d** is a matrix of nodal displacement vectors (one by column) of the corresponding forms of vibration modes. This solution is also known as structural modal analysis.

3. PROCEDURE

3.1. Guitar model

The finite element geometry model of the guitar was built in the FE program according to the information from [7], for the dimensions of a Lisbon Portuguese guitar. This modeling contemplates the body (soundboard, back, side), the soundboard and the back plate brace, the neck, the frets and the neck joint (linking the body to the neck). Figure 1 shows the guitar main dimensions and the components included in the Finite Element (FE) model.

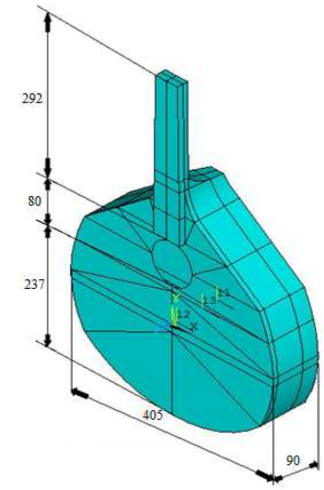


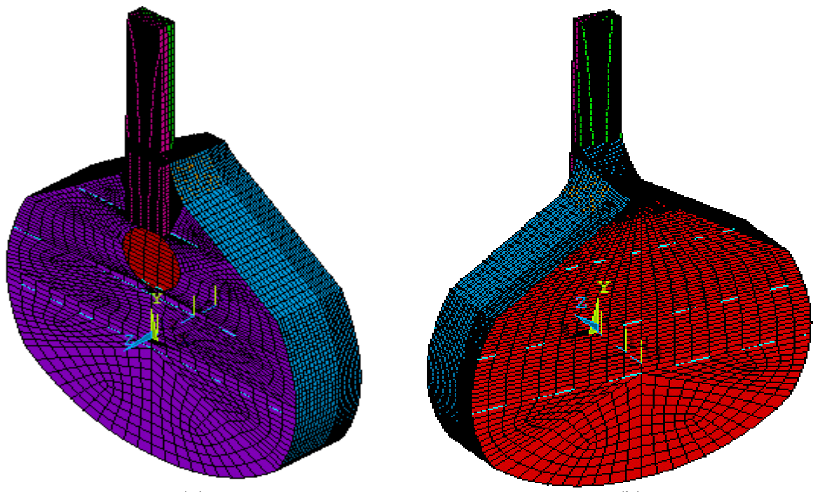
Figure 1 – Main dimensions included in FE model of a Lisbon Portuguese guitar box. Dimension in mm.

The materials used to build a Portuguese guitar are woods that are orthotropic materials. Thus, the different woods used are the Spruce for the soundboard, the soundboard brace and the back plate brace and the neck joint; Indian Rosewood for the back and the side; Ebony for the frets and Mahogany for the neck. However, it was difficult to identify the correct mechanical properties of the Spruce and it was substitute Sitka-Spruce for the simulation [8,9]. The table 1 presents the mechanical properties of the woods used in the FE model, where Ei represents the Youngs modulus in the direction "i", Gij represents the shear modulus between the directions "i" and "j", and □ij represents the Poisson ratio between the directions "i" and "j".

Table 1 - Wood mechanical properties [9].

Wood	$\rho\left[\frac{Kg}{m^3}\right]$	E_1 [GPa]	E_2 [GPa]	E_3 [GPa]	G_{23} [GPa]	G_{13} [GPa]	G_{12} [GPa]	ν_{12}	ν_{13}	ν_{23}
Sitka-Spruce	390	11.6	0.9	0.5	0.039	0.72	0.75	0.029	0.02	0.25
Indian Rosewood	775	16	2.2	0.72	0.3	0.84	1.1	0.36	0.03	0.26
Ebony	1100	19	2.11	0.95	0.4	1.12	1.67	0.3	0.03	0.26
Mahogany	450	10.67	0.53	1.18	0.63	0.22	0.94	0.3	0.03	0.26

The finite element model of the guitar, at this stage, doesn't consider acoustic-fluid inside and outside the guitar. The structural is modeled with three different elements. Thereby, for the soundboard brace and the back plate brace we used a 2 node beam finite element based on the Timoshenko beam theory (with reduced integration at shear strains) and the soundboard, the back and the side are modeled with 4 node isoparametric plate finite element based on the Mindlin plate's theory. The neck, the frets and the neck joint we model with resource at 3D solid 8 nodes elasticity finite element. Several meshes were created in order to obtain a parametric mesh with the fewest possible number of distorted elements. The numeric model also includes the orthotropic wood properties. The figure 2 shows the meshes developed.



(a) (b)

Figure 2 - FE model of a Lisbon Portuguese guitar box mesh: a) Frontal isometric view; b) Rear isometric view.

3.2. Guitar model with the twelve strings

To respond to the big question in this article we created a numerical model that includes the strings of a Portuguese guitar in the FE model described above. In a first step we developed a numerical model that allowed studying the strings one by one. This first model was created for each of the twelve strings and we considered just the string without the guitar. With this we wanted to know the natural frequency of each string corresponding to the guitar tuning frequencies. For each string was considered a length of 166*mm* since the soundboard until the bridge and a length of 453mm since the bridge until the nut. So, we made a model with three *points,* corresponding to the soundboard, the bridge and the nut, and two elements. In this paper, the material used to build the strings is stainless steel and the thickness of each string is in accordance with [10]. According with the American Iron and Steel Institute (AISI) the designation of the material is AISI INOX 302 and the mechanical properties used are in table 2.

Table 2 – String mechanical properties [11]

Density [Kg/m^3]	7900
Young's modulus [GPa]	193
Thermal Expansion [1/ºC]	0.0000172

The finite element used to model the strings is again the 2 node beam finite element based on the Timoshenko beam theory (with reduced integration at shear strains). With this finite element model we are studying the influence of prestress strings with the modal response of the guitar. Thus, we formulated a thermal-stress analysis to introduce a prestress in each string according with the characteristics of each one.

To formulate a thermal-stress analysis we started to introduce a reference temperature. Then we defined the temperature of the string using a “body force temperature” to the elements of the string. The aim to create a temperature difference, consist in getting a tension in a string that corresponds to the natural frequency to the guitar tuning frequencies. Note that the temperature across the string needs to be uniform so that the tension along the string will be uniform as well.

After input all the constraints, we did a static analysis with prestress in order to introduce a tension in the string. To finish the study and find the natural frequency of the string, we did a modal analysis with the prestress analysis on. Note that, to obtain the natural frequency corresponding to the guitar tuning frequency, it was need to vary the temperature of the string until getting the correct valor corresponding to the guitar tuning frequency of each string. The proceeding described above was repeated for each string in order to find the natural frequency corresponding of each one.

Once created and validated the first numerical model, include the twelve strings in the finite element model of the guitar. The geometric, the dimensions, the material and the finite element of each string is the same that the first model. In this new FE model the strings of each pair are separated by a distance of 4*mm* and each pair is separated by a distance of 8*mm* [4]. Figure 3 shows the guitar with the strings and the main dimensions of the same in the FE model. Note that each string is modeled accordance with its diameter.

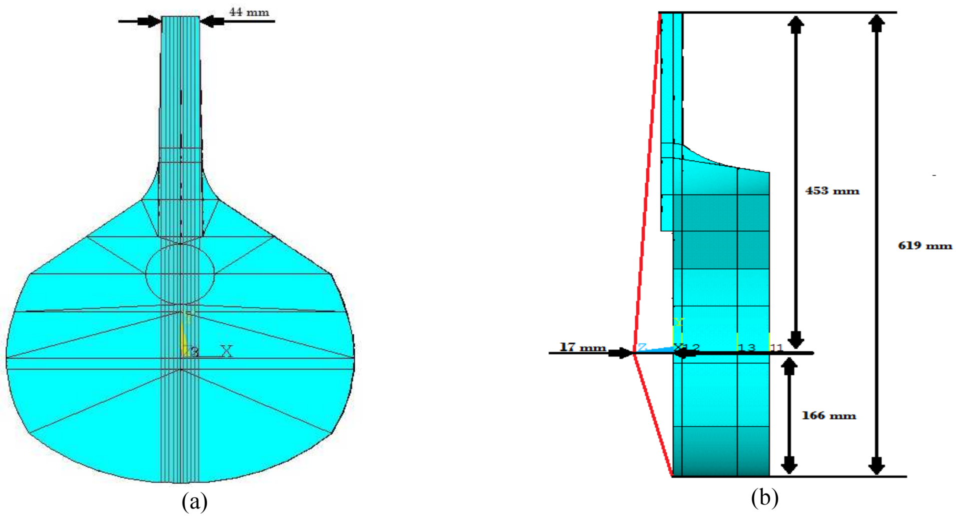


Figure 3 - Main dimensions included in FE model of the guitar with twelve strings. Dimension in mm.: a) Front view of the FE model of the guitar with twelve strings; b) Side view

This new FE model has the aim to study the influence of prestress strings in the guitar body and to study the modal response of the same. Like the first model, we formulated a thermal-stress analysis to introduce a prestress in each string and then we did a modal analysis to study their influence in the modal response of the guitar. To formulate a thermal-stress analysis we introduced the reference temperature and we defined the temperature of the string using a “body force temperature” to the elements of the string.

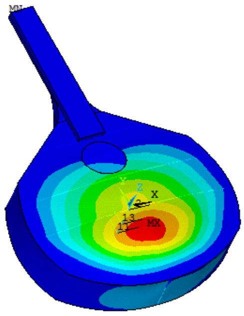
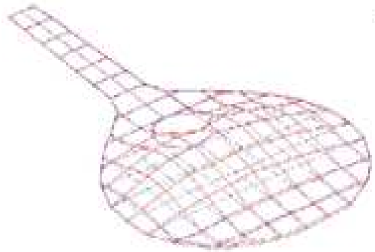
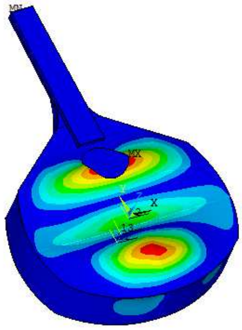
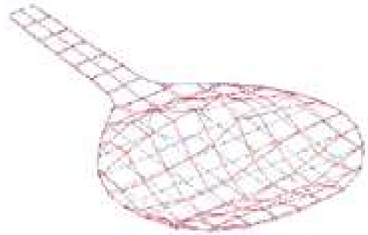
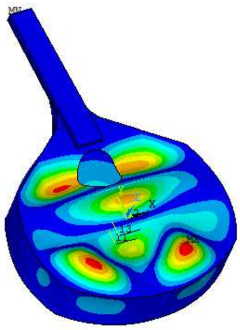
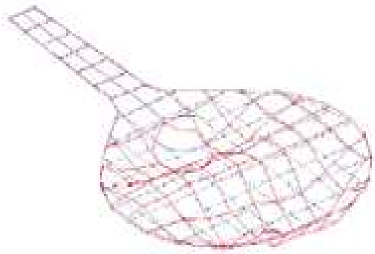
As, mentioned before the temperature across the string needs to be uniform so that the tension along the string will be uniform as well. The twelve hard steel strings are tensioned to resonate at a natural frequency corresponding to the guitar tuning frequencies. The constraints change from the first to the guitar model with the twelve strings. In this case all the constraints were applied to the guitar and it was necessary to modeling the bridge. This component is where the strings are supported. Posteriorly input we did a static analysis with prestress in order to introduce a tension in the string. To confirm the natural frequency of the string, we did a modal analysis with the prestressed analysis on.

4. RESULTS

In this section we’ll present the results obtained for the modal response of the Portuguese guitar.

Firstly we present the results obtain by the FE model of only the guitar body. In order to compare the experimental results with the numerical ones it was done a study of convergence varying the mesh refinement. Table 3 presents the structural natural frequencies and mode shapes obtained with the numerical structural model and the experimental measurements on the actual Portuguese guitar.

Table 3 – Numerical and experimental natural frequencies and mode shapes of the guitar body

No.	Finite Element Model	Experimental Model [3]	Frequency deviation [%]
Mode 6	 258.839 Hz	 $f_{(0,0)} = 275$ Hz	5.88
Mode 12	 379.014 Hz	 $f_{(0,1)} = 360$ Hz	5.28
Mode 18	 656.983 Hz	 $f_{(0,2)} = 635$ Hz	3.46

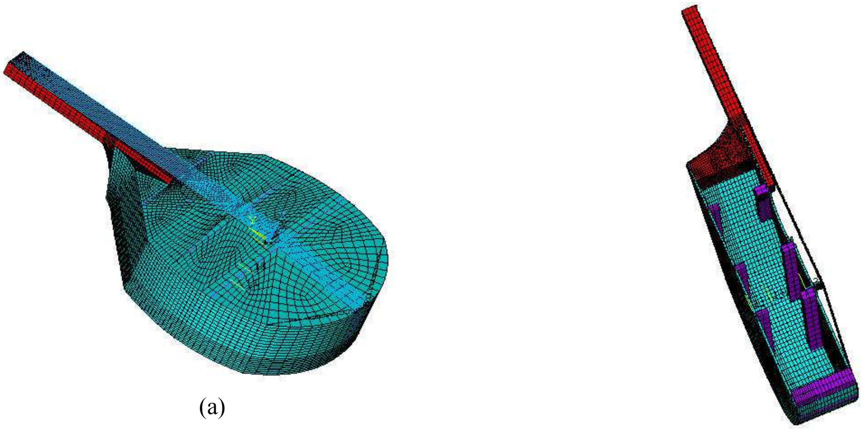
The results presented in Tab.3 differ slightly. Beside the approximations in the FE model, there is also a difference in one material as mentioned before table 1.

A separated analysis was done to tune the strings fixed at the three points indicated. The table 4 shows the structural natural frequencies required to each one. They have to be tuned, one by one in order to respond with the desired frequency.

Table 4 – Numerical natural frequencies and prestress of each strings.

String	Diameter [mm]	Frequency values [Hz] – FEM	Frequency values [Hz] - from [4]
Si (b4)	0,24	493,880	493,880
Lá (a4)	0,25	440,000	440,000
Mí (e4)	0,32	329,630	329,630
Si bordão (b3)	0,50	246,940	246,940
Lá bordão (a3)	0,64	220,000	220,000
Ré (d4)	0,44	293,660	293,660
Ré bordão (d3)	0,79	146,830	146,830

The twelve chords where added to the model of the guitar, as illustrated by the Fig.4 and are being tuned now with the 3 supporting points (extremities and bridge) fixed at the body which is deformable. This turns the tuning of the12 strings a challenger.



(a) (b)

Figure 4 - FE model of the guitar with twelve strings. a) Perspective view; b) Perspective with a cut.

The results obtained with the guitar depend on the correct tuning of the strings.

5. CONCLUSIONS

Authors present in this text their first attempt to model the Portuguese guitar with strings. As far as their knowledge it is a pioneer attempt, and a step forwards in the actual knowledge it this field. The work stills in progress and results obtained are preliminary and require some comparison with experimental values.

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