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# A Way to innovate Schroeder Type Diffusers

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**ABSTRACT:** Ways to innovate the long existing designs of Schroeder type diffusers have been investigated. The objective was primarily to extend the low frequency response. Subsequently, a shallower diffuser with the equivalent performance of a deeper diffuser was also developed. The designs involved adding acoustic mass by using perforated plates and thin membranes fixed inside the wells. The diffusers are approximated as surface with variable impedance. The wave motion in the wells is prescribed by plane waves forming a rectangular waveguide, from which is calculated the impedance at one end. A prediction model based on boundary element methods (BEM) was used to estimate the performance of the diffusers and compare them with existing designs. The suggested designs did show a lower frequency response but hardly any diffusion improvement at mid frequencies and in some cases worse results at higher frequencies. Shallower diffusers matched very closely the performance of equivalent deeper diffusers. Numerical results are compared with measurements made to several diffuser designs.

## 1. INTRODUCTION

Almost three decades ago, Schroeder[1] proposed a systematic approach to design a profiled diffuser with well defined acoustic performance, based on mathematical number sequences. Since then, diffusers have been primarily used to improve the acoustic performance of spaces such as concert halls, recording studios and listening rooms. The most popular Schroeder diffuser is the so-called quadratic residue diffuser (QRD), Figure 1, which employs a mathematical number sequence to determine the well depth.

The downside is that the QRDs used in most practical situations are of limited size or the space available for application is often limited. Consequently, diffusion does not start at low enough frequency. Building on the revolutionary QRD designs, means for extending the low frequency response and reduce the in-depth size of diffusers are investigated. The aim is to lower the frequency of resonance of the wells by increasing mostly the acoustic mass of the wells. This was attained by fixing perforated plates and thin membranes inside the wells.

## 2. THE QRD

The QRD is a profiled structure constant in one direction. The depth of the wells, $d_n$, are determined by the quadratic residue sequence, $S_n$, and a chosen design frequency, $f_d$, as described by Refs. [1,2]. An example is given in Figure 1, QRD with seven wells ($N$=7) and $S_n = \{0;\ 1;\ 4;\ 2;\ 2;\ 1;\ 4\ 0\}$. Basically, the Schroeder theory assumes plane waves travelling

down and back up the wells. Since the wells have different depths, the scattered waves have different phases. However, precise phase shifts only occur at design frequency, where diffusion is maximum. Moreover, good diffusion is mainly obtained at multiples of $f_d$ only, and it decreases with increasing frequency. There is a lower frequency limit below which the wells will be too shallow to have effect on the incident wave and so the diffuser will be seen as a flat surface. There is also a high frequency limit, above which the plane wave propagation within the wells breaks down and more complicated propagation modes will dominate. In terms of frequency, these limits are roughly[1] $f_{min} = f_d$ and $f_{\max} = c/2W$, where $W=w+t$ is the total well width, $w$ is the width of the well cavity and $t$ is the thickness of the diving slit.
From the many authors[2] that have analysed and reviewed the design of QRDs, it is of interest to refer Cox[3]. This author suggested an optimised QRD, which offers a broader frequency range of diffusion. However, the enhancement is mainly verified at frequencies above $f_d$. The optimised QRD is taken as a reference for the comparisons made in this study.

| | |
|---|---|
| **Optimised QRD[3], *N=7* and $f_d$=490*Hz*** | $l_n$={0.034, 0.154, 0, 0.101, 0, 0.154, 0.034}*m*<br>*w*=0.061; *d*=0.57; *t*=0.004; *T*=0.4210*m* |

*Table 1 – Parameters of the optimised QRD.*

## 3. MODEL OF A SINGLE WELL

### 3.1 Perforated plate and thin membrane acoustic impedances

Expressions to predict the acoustic impedance of perforated plates were derived by Guess[4]. These are based on lumped acoustic parameters. The perforated plate is seen as a lattice of short tubes of length $t_p$ with both ends flanged and radius $a_p$. The specific acoustic impedance $Z_p$ is given by the sum of resistance, $R_p$, and mass, $M_p$,

$$Z_p = R_p + j\omega M_p$$
$$R_p = \frac{\rho_0}{\varepsilon}\sqrt{8\eta\omega}\left(1+\frac{t_p}{2a_p}\right);\ M_p \approx \frac{\rho_0}{\varepsilon}\left[t_{eff} + \sqrt{\frac{8\eta}{\omega}}\left(1+\frac{t_p}{2a_p}\right)\right] \quad (1)$$

where $\varepsilon$ is the porosity of the plate, $t_{eff}$ is the effective length, $\eta \approx 15\times10^{-6}\,m^2/s$ is the kinemetric velocity of air and $\omega$ is the wavelength of frequency of interest. A series of experimental tests were also made by the author to check the validity of the formulae.
From a comparative analysis of experimental results, Voronina[5] introduced an empirical formulation for thin membrane impedance, in terms of measurable physical parameters. It essentially corrects the commonly used mass law approximation. Good agreement was shown between measured data and predicted numerical results. The specific acoustic impedance $Z_m$, is again given by the sum of lumped acoustic parameters $R_m$, resistance, and $M_m$, mass,

$$Z_m = R_m + j\omega M_m$$
$$M_m = \frac{m_m}{\rho_0 c}\cdot\left(1-\frac{1}{B}\cdot\sqrt[12]{\frac{k\cdot t_m}{y_m \times 10^{-4}}}\right);\ R_m = \omega M_m/6 \qquad (2)$$

where $m_m$ is the physical mass (kg/m$^2$) of the thin membrane, *k* is the wavenumber and *B* is a coefficient that depends on measurable dimensionless physical parameters $y_m$ and $p_m$. Table 2 resumes the properties of thin membranes and perforated plates added to the wells of the investigated diffusers.

| **Added element** | **$t_p$ or $t_m$ [m]** | **ε** | **$a_p$ [mm]** | **$\rho_m$ [kg/m$^3$]** | **$y_m$** | **$p_m$** |
|---|---|---|---|---|---|---|
| Perforated plate | 2 x10$^{-3}$ | 0.20 | 1.5 | -- | -- | -- |
| PE thin membrane | 56x10$^{-6}$ | -- | -- | 1000 | 0.07 | 125 |
| Foil thin membrane | 50x10$^{-6}$ | -- | -- | 1900 | 0.95 | 140 |

*Table 2 – Physical and acoustic characteristics of perforated plates and thin membranes used in investigated diffusers.*

## 3.2 Impedance at the entrance of the well

Since the surface structure of the QRD is constant along its length *L*, the acoustic impedance of the well is assumed constant in that direction. So, the propagation of plane waves in the wells can be analysed as rectangular waveguides of constant cross-section, Figure 2. If the slits in the diffuser are considered hard surfaces and is assumed that there are no losses in the wave propagation within the well, then the wave velocity *u* and the wavenumber *k* are both real.

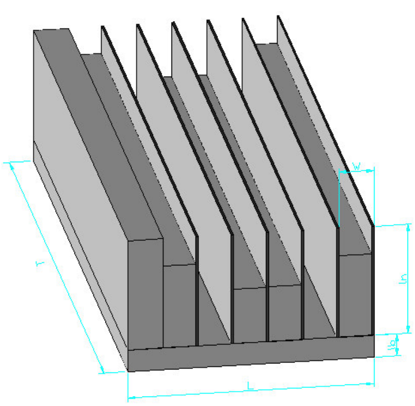

*Figure 1 – Quadratic residue diffuser.*

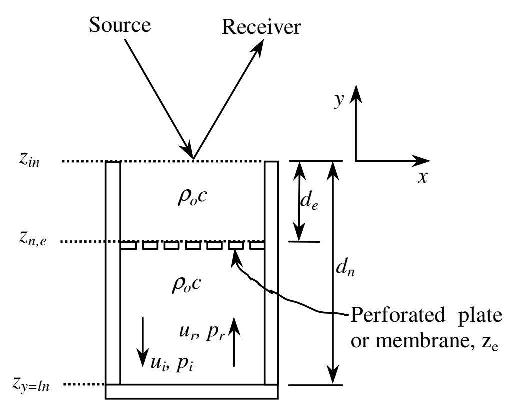


*Figure 2 – Diagram of diffuser well with element fixed at $d_e$.*

Since the end of the wells is usually made of hard dense material, and so it can be assumed perfectly reflective ($z_{y=d_n} \to \infty$), then, the specific acoustic impedance at the entrance of empty wells (at *y*=0) is simply,

$$z_{in} = -j\tan(kd_n)/\rho_0 c \tag{3}$$

Now consider that a perforated plate or a thin membrane is fixed inside the well. If all parts down and along the well share the same volume velocity, in principle, the specific acoustic impedance at the outer surface of the element, $z_{n,e}$, equals the sum of the impedance of the element alone, $z_e$, and that of the air cavity behind it, Equation (3). The specific acoustic impedance at the entrance of the well can then be derived by the multiplayer transfer matrix as,

$$\begin{aligned} z_{in} &= \frac{z_{n,e} + j\rho_0 c\tan\left(k\left(d_n - d_e\right)\right)}{1 + jz_{n,e}\tan\left(k\left(d_n - d_e\right)\right)/\rho_0 c} \\ z_{n,e} &= -j\tan\left(k\left(d_n - d_e\right)\right)/\rho_0 c + z_e \end{aligned} \tag{4}$$

where $d_e$ is the distance of the element to the top of the well. In the case of the element being located at the top of the well, the input impedance $z_{in}$ is simply that given by $z_{n,e}$.

The inspection of well impedance behaviour is focused primarily on the imaginary part. From Equation (4) it can be seen that the position of the added element is important for the change in resonance. The longer the cavity behind the element, the lower is the first resonance of the well. Typically, the thin membrane induces more acoustic mass than the perforated plate, and so it is more efficient in lowering the resonance of the well. However, in both cases it was verified that the typical evenly spaced resonances of an empty well is destructed, being actually worse in the case of the well with thin membrane. The positioning of resonances in frequency is the key factor in the design process, especially the first resonance and those up to the theoretical upper frequency limit of the diffuser.

Radiation effects must also be accounted. The radiation impedance of a well can be approximated to that of a rectangular piston in an infinite rigid baffle. The closed form solution suggested by Mechel[6] has been used in this work.

# 4. EVALUATING DIFFUSION

## 4.1 Prediction process

Since the diffuser designs investigated are 1-dimensional, a 2D Boundary Element Method (BEM) prediction technique[3] was used. The diffuser is modelled as a closed box the same size as the diffuser, with a variable admittance on the front surface. Hence, the calculated normalised specific acoustic impedance at the entrance of each well was converted to admittance – the boundary condition of surface elements. The slits, sides and back of the diffuser assume zero admittance. The far-field hemispherical polar response was predicted at different discrete random frequencies. The fixed source was centred with and normal to the diffuser at twice the distance of the receiver arc (source at (0,100)*m* and receiver arc *r*=50*m*). Receiver arc had an angular resolution on 1º, i.e 180 receiver points.

### 4.2 Practical measurements

The measurement of polar responses from test samples closely followed the AES information document[7] for the measurement of surface scattering. Polar responses were measured using an MLS measuring system with 37 microphone positions in an arc of 1,5m and a test source at 1,9m normal to and centred with the diffusers. Diffuser samples were scaled to ½ the original size. The wells were terminated by MDF varnished three times, and the slits were made of aluminium sheets. All parts have been glued and the cracks properly sealed with silicone (mastic) to avoid excess in absorption.
The diffusion coefficient suggested in Ref. [7] was calculated from the set of predicted polar sound pressure levels. The coefficient varies between 1, when the diffuser scatters sound completely (good diffusion) and 0, when scattering is concentrated in one location.

### 4.3 QRD with lower bass response

Intuitively, the lower the resonances of the wells the lower can be the frequency response of the diffuser. Thus, the added elements are positioned on top of the wells in order to provide the lowest possible frequency of resonances. From Figure 3(a) and (b) it can be seen that at lower frequencies the scattered responses of the optimised QRDs with added elements are in both cases flatter than those of the original optimised QRD.

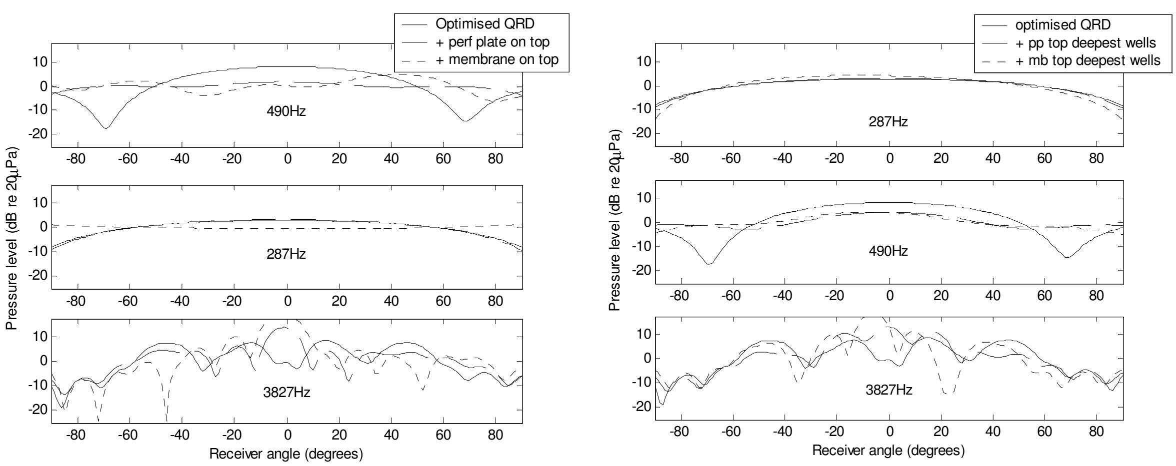


*Figure 3 – Far-field polar responses of optimised QRDs with lower frequency response (a) all wells with added elements and (b) only deepest wells with elements.*

Typically this is verified for frequencies up to the design frequency or just above it. However, it is also clear that in comparison to the original optimised QRD the deviations in the polar response increase at higher frequencies. Diffusion is clearly extended to frequencies lower than the $f_d$, but at mid and high frequencies diffusion performance is considerably lower than in the reference QRD. First, it could be believed that high frequencies see the wells with elements as flat plates and consequently more specular scattering would be present.

Nonetheless, another interpretation may be that diffusion at high frequencies is affected by: the corruption of the evenly spaced resonances typical of empty wells and the loss of impedance amplitude at these resonances. Another problem is that the desired phase grating properties, especially at higher frequencies, are destroyed by the unproportional lowering of resonances between wells. Since these effects tend to worsen with increasing frequency, this may explain the rapid loss of diffusion with increasing frequency, particularly in the case of the QRDs with elements fixed at the entrance of all the wells. With elements positioned only at the top of the deepest wells, the unwanted effects are restricted to two wells, which have the lowest resonances, and so the loss of diffusion at mid-high frequencies is not as marked. Nevertheless, if there is the need for lower diffusion response maintaining an existing QRD design, then this may be a solution.

### 4.4 Shallower diffuser designs

The method to design shallower diffusers is based on impedance matching of resonances. It consists in finding the correct position for the element to fix inside a shallower well, in order to produce the same first resonance of a deeper empty well. For that, the input impedance Equation (4) is rearranged into a system of two equations to two unknowns, i.e. in terms of $d_e$ and $z_{n,e}$ respectively,

$$\begin{aligned} d_e &= \frac{\arctan\left( z_{n,e} \big/ j\left( z_{n,e} \cdot z_{in} - 1\right)\right)}{k} \\ z_{n,e} &= -\rho_0 c \big/ \tan\left( k\left( d_n - d_e\right)\right) + z_e \end{aligned} \tag{5}$$

First, one needs to calculate the input impedance of the original well to be shortened. Then substitute the values of $z_{in}$, $z_e$ and $k$ at resonance into Equation(5). Given $d_n$ for the shallower well, the correct position of the element inside the shallower well can be determined to precisely match the first resonant frequency of the original deeper empty well.

In some cases, one element alone does not provide sufficient inertance to the well, so that the well can be as short as desired. Hence, the use of two elements inside the same well was also considered. The design procedure is similar to the previous one, but with a system of three equations to three unknowns, two of which are the depth positions to fix the elements. Moreover, with more than one element inside the same well, these can be positioned in order to match more than the first resonance and so, obtain a better approximation of the impedance of the original deeper empty well. Although the resistance offered by these elements alone is small, their combination with an air cavity behind, results in a relative increase in resistance at the entrance of the well. A consequent increase in absorption is verified. Since high absorption is usually undesirable in sound diffusers, possibly a compromise must be made between a shorter well with many elements and the consequent excess in absorption.

The shallower diffusers investigated are essentially optimised QRDs with the deepest wells shortened to the next well up, i.e. with wells $d_3$ and $d_6$ reduced to $d_4$ (=0.101). Only the wells that have been shortened had elements fixed inside. The foil thin membrane was positioned at $d_e$=39*mm*, the thin black plastic membrane at $d_e$=32*mm* and the two perforated plates at $d_{e1}$=17*mm* and $d_{e2}$=27*mm*.

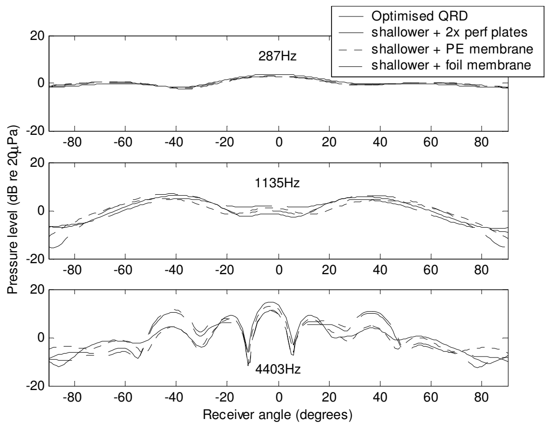


*Figure 4 – Polar responses of shallower diffusers.*

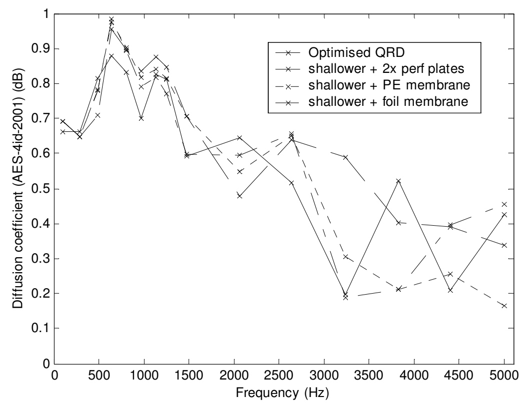


*Figure 5 – Diffusion coefficients of shallower diffusers.*

Figure 4 shows that the polar responses of the shallower diffusers tend to approximate that of an optimised QRD across most of the frequency range. However, this behaviour tends to fall with increasing frequency. Bellow, around and above $f_d$ up to mid frequencies, the diffusion coefficients (Figure 5) of the shallower diffusers assume values considerably close to the optimised QRD. At high frequencies and particularly after around 3800Hz, diffusion is severely affected, but around $f_d$, results are expected to be very good. It was decided to shorten only the deeper wells and apply elements only to these wells, for several reasons. First, for the same reasons stated in the case of the previously investigated low frequency QRDs. Secondly, since there are more resonant frequencies in a deeper well, more harmonics are closer to match those of the equivalent deeper empty well. Hence, there will be a better impedance curve, more phase properties are preserved and consequently better diffusion is attained. Since the aim is to construct a diffuser with smaller dimensions, the most significant in-depth reduction is obtained in reducing the depth of the deeper wells of the original QRD.

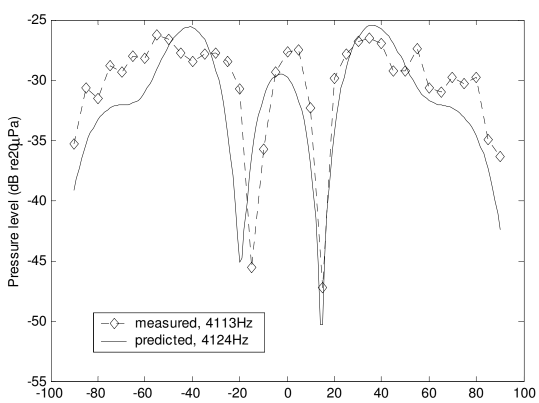


*Figure 6 – Polar response of shallower diffuser with foil membrane.*

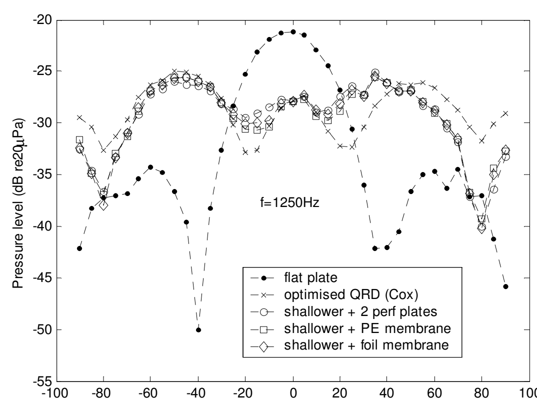


*Figure 7 – Comparison of polar responses of investigated shallower diffusers.*

Reasonably good agreement can be seen in Figure 6 for the scattered pressure levels between predictions and measurements. Although it was verified that results tend do deviate from predictions at high frequencies and with increasing frequency, at low and mid frequencies, the measurement results demonstrated the validation of the prediction model for this kind of diffusers. Predictions are limited by the well impedance model (due plane wave assumption) rather than the BEM technique. Figure 7 shows that the measured polar responses of shallower diffusers approximate quite well the optimised QRD. This was verified at most frequencies tested but, as expected, there is the tendency to lack diffusion earlier in frequency. Curiously, all shallower diffusers have surprisingly similar behaviour.

## 5. CONCLUSION

It was shown how the positioning of perforate plates and thin membranes inside the wells and the number of wells with these elements, influence the surface impedance of QRDs and consequently can enhance their diffusion. QRDs with extended bass response showed increased diffusion at low frequencies, but to the price of worse performance at mid at high frequencies. As predicted, the shallower diffuser designs showed very similar behaviour to the original optimised QRD.

## 6. REFERENECES

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